Parallel finite difference beam propagation method based on message passing interface: application to MMI couplers with two-dimensional confinement

Chaojun Yan (严朝军), Wenbiao Peng (彭文标), and Haijun Li (李海军)

College of Electrical Engineering and Information Technology, Three Gorges University, Yichang 443002

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The alternate-direction implicit finite difference beam propagation method (FD-BPM) is used to analyze the two-dimensional (2D) symmetrical multimode interference (MMI) couplers. The positions of the images at the output plane and the length of multimode waveguide are accurately determined numerically. In order to reduce calculation time, the parallel processing of the arithmetic is implemented by the message passing interface and the simulation is accomplished by eight personal computers.

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There has been a growing interest in two-dimensional (2D) multimode interference (MMI) couplers[1,2] recently. For the practical 2D MMI coupler design, the important parameters concerned are the positions of input or output waveguides and the length of multimode waveguide (MMWG). Jiang *et al.* have analyzed 2D MMI couplers with guided mode propagation analysis (MPA) method[3−5] which is approximate to some extent. In some cases, the modal parameters, such as the propagation constants and even the modal field profiles of modes, must be determined before the MPA method is used. It is unfortunate that the analytical solutions for three-dimensional (3D) waveguide devices do usually not exist, so, these demands for modal parameters will be obstacles for the usage of MPA method and limit its precision. Rajarajan first obtained the propagation constants and spatial field profiles of modes by the finite element method (FEM) and then analyzed the 2D MMI coupler by least square boundary residual (LSBR) method[6]. His method is purely numerical, accurate, but complicated.

The finite difference beam propagation method (FD-BPM)[7,8] is a widely used important numerical simulation method for optical waveguide devices. The alternate directions implicit (ADI) scheme was constructed to improve computing efficiency for 3D devices[8]. In this paper, the 2D MMI couplers will be analyzed by 3D ADI FD-BPM which is also a sort of purely numerical method and it does not need any predefined modal parameters.

Although FD-BPM is efficient, there still exist new problems. The grid counts at a calculation window must be large enough in order to guarantee the precision of the used difference schemes. For example, we can use a grid mesh of 1600×50 to simulate a one-dimensional (1D) MMI coupler. Although the two numbers which represent x and y axes respectively are so different, they still work well because the modelling target is a 1D device whose properties of imaging mainly concentrate on one direction (x or y). For 2D MMI devices, we must use a grid mesh of 1600×1600 for simulation if the same calculation window is used. The main trouble is that the calculation burden for the 2D device exceeds far than that for the 1D one.

Parallel computing is one of the optional methods to save the calculation time. If we have no super parallel computer, we can use personal computers instead, which can switch messages with each other through message passing interface (MPI). MPI is a sort of specification and the release of MPI has been extended to MPI-2 now. MPICH2, which has been bound with C++, C or Fortran and integrated into Microsoft Visual Studio developing environment (or Linux), is an implementation of MPI-2 from the group at Argonne National Laboratory. So, we will stress the parallel processing of arithmetic, that is, the total burden of simulation is divided and allocated to more than one computers simultaneously.

Consider an optical beam propagating along z axis in MMI couplers. According to the finite difference method, the transverse window is divided by a mesh grid with spacing of Δx and Δy. The longitudinal step is Δz and the field at the lattice point of $x = m\Delta x$, $y = n\Delta y$, and $z = s\Delta z$ is represented by $E^s_{m,n}$. The electric field is expressed as $E(x,y,z) = \phi(x,y,z)\mathrm{e}^{ik_\mathrm{r}z}$, where k_r is called reference wave vector and the choice of its value must ensure the slowly varying envelope approximation condition. The scalar wave equation is $\partial\phi/\partial z = A(\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2) + B\phi$, where $A = i/2k_\mathrm{r}$, $B = i[k_0^2 n^2 - k_\mathrm{r}^2]/2k_\mathrm{r}$, n is the refractive index, which is spatially varying.

In the ADI approach, each propagation step from z to $z + \Delta z$ is split into two steps: $z \to z + \Delta z/2$ and $z + \Delta z/2 \to z + \Delta z$. The difference equation of scalar Helmholtz equation of electric field is

$$-A\frac{\Delta z}{\Delta x^2}\phi^{s+1/2}_{m+1,n} + (2 + 2A\frac{\Delta z}{\Delta x^2} - \frac{\Delta z}{2}B^{s+1/2}_{m,n})\phi^{s+1/2}_{m,n} - A\frac{\Delta z}{\Delta x^2}\phi^{s+1/2}_{m-1,n}$$

$$= A\frac{\Delta z}{\Delta y^2}(\phi^s_{m,n-1} + \phi^s_{m,n+1}) + (2 - 2A\frac{\Delta z}{\Delta y^2} + \frac{\Delta z}{2}B^{s+1/2}_{m,n})\phi^s_{m,n}, \quad (1a)$$

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$$-A\frac{\Delta z}{\Delta y^2}\phi_{m,n+1}^{s+1}+(2+2A\frac{\Delta z}{\Delta y^2}-\frac{\Delta z}{2}B_{m,n}^{s+1/2})\phi_{m,n}^{s+1}-A\frac{\Delta z}{\Delta y^2}\phi_{m,n-1}^{s+1}$$

$$=A\frac{\Delta z}{\Delta x^2}(\phi_{m-1,n}^{s+1/2}+\phi_{m+1,n}^{s+1/2})+(2-2A\frac{\Delta z}{\Delta x^2}+\frac{\Delta z}{2}B_{m,n}^{s+1/2})\phi_{m,n}^{s+1/2}, \tag{1b}$$

$$m=1,2,\cdots,N_x-2; \qquad n=1,2,\cdots,N_y-2.$$

For the 3D ADI FD-BPM, the paralleling of arithmetic is direct. Let's consider Eq. (1a). The matrix equation, which corresponds to certain integer n concerned, is constructed by the field at the lattice points in the lines flagged by integer $n-1$, n, and $n+1$ on the plane of z. So, there will be N_y matrix equations to be constructed and solved. This property is used for the paralleling of program. According to the MPI terms, the unit which takes part in the paralleling program is called a processor or a thread. In this case, suppose the number of threads, which will share the task of solving N_y matrix equations, is N. The thread with the serial number p will be responsible for the solution of N_y/N matrix equations whose corresponding flags of lines are from $n=p\times N_y/N$ to $(p+1)\times N_y/N-1$. Every thread will switch a few messages with adjoining threads so as to construct its matrix equations and then all N threads will begin operation of solving matrix equations simultaneously. The similar process is hold for Eq. (1b). The threads for Eq. (1a) are called x-threads, and those for Eq. (1b) are called y-threads. When completing the calculation of Eq. (1b), y-threads will switch messages with x-threads in order to take the next step of BPM. The quantity of messages being switched, whether from x- to y-threads or from y- to x-threads, is huge and time-consuming because all the memory held by every thread to construct the matrix equations must now be updated. In the actual implementation, for the convenience of programming, each computer offers two threads, one is x-thread and the other is y-thread. The serial numbers of all x-threads are even and those of y-threads are odd.

For the practical design of MMI couplers, the positions of input and output waveguides and the length of MMWG must be determined. According to the theory of MPA method, the positions of input or output waveguides, which are actually the positions of the input and output images, are only dependent on the equivalent width W_{eq} of the MMWG. The fact is, W_{eq} is not equal to the physical width W of the MMWG though there exists a good approximation of $W_{\text{eq}}\approx W$ when the waveguide is strongly confined. As mentioned before, the modal characteristics of MMWG must be solved first to get W_{eq} and it is usually difficult to obtain the accurate analytical solution for 3D waveguide. There exists an alternate method to obtain W_{eq}. If the image is input from the geometrical center of the cross section of MMWG, this kind of MMI couplers is called symmetric interference MMI. We can obtain W_{eq} from the space between the images at output plane of a symmetric interference MMI coupler. For example, when a 1×4 symmetric interference MMI coupler is considered and the average distance between four images is d, we will have $W_{\text{eq}}=4d$. Due to the complicated dependence on many factors, such as the position of the input image, the symmetry or asymmetry of the input field, the shape of the cross section of MMWG, and the parameters of modes supported by the waveguide, the design of the length of MMWG is more difficult. This just proves the necessity and value of simulation. In the given example, the lengths of some typical MMI devices will be numerically designed.

The considered MMWG had symmetrical structure, i.e., $W_x=W_y=48$ μm. It was called a symmetrical MMI coupler with 2D confinement. The refractive index in the guide and cladding sections of MMWG were $n_{\text{core}}=1.46$ and $n_{\text{clad}}=1.45$, respectively, and the wavelength was $\lambda=1.55$ μm. The schematic diagram of the MMI coupler is shown in Fig. 1. The origin of coordinates, i.e., $O(0,0)$ in Fig. 1, was located at the geometrically symmetrical center of the cross section of the waveguide. The center of the input waveguide was $P(x_1,y_1)$ and it was the position of input image too. It was sometimes convenient to use another reference frame whose origin of coordinates was located at point O', as also shown in Fig. 1, and it was called absolute reference frame. The computing window in every transverse direction (x and y) covered form -35 to 35 μm. The number of x- or y-threads was $N=8$ and the grid counts was $N_x=N_y=1600$. The longitudinal propagation step was $\Delta z=0.2$ μm. 1×4 symmetric interference was considered and the field profiles of four images at output plane were shown in Fig. 2(a) when the length of MMGW was 624 μm. From the space between the images, we can obtain that $W_{x\_\text{eq}}=W_{y\_\text{eq}}=W_{\text{eq}}=50.76$ μm. It should be emphasized that the obvious difference between W_{eq} and W in this example was observed and it meant that the positions of input and output waveguides of MMI couplers must be adjusted accordingly. According to the theory of MPA method, if the image is not input at point $(iW_{x\_\text{eq}}/M, jW_{y\_\text{eq}}/N)$, where i, j are integers and the point is expressed in absolute reference frame, the $M\times N$-fold images with equal intensities can be observed at the output plane. Supposing $M=N=3$

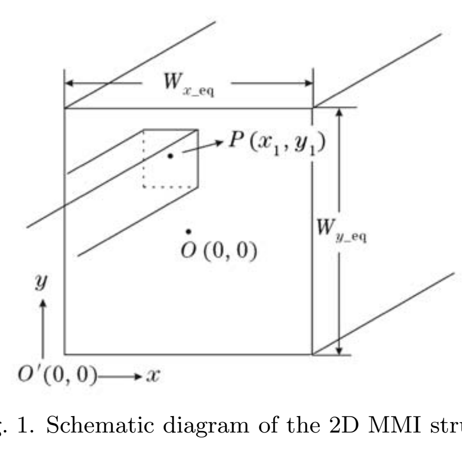

Fig. 1. Schematic diagram of the 2D MMI structure.

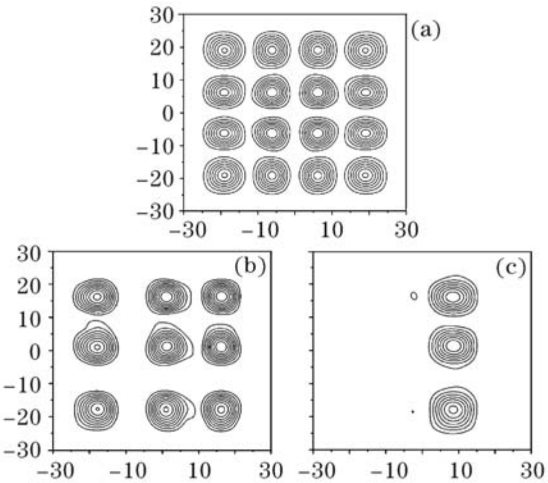

Fig. 2. Field profiles at the output plane of the MMI coupler. (a) 4×4 images of equal intensity when the input field is at the center of waveguide and the length of MMWG is $L = 624$ μm; (b) 3×3 images of equal intensity when the input field is at (-16 μm, -16 μm) and $L = 3300$ μm; (c) 1×3 images when the input field is at (-16.92 μm, -16 μm), the condition of overlapping images is satisfied in x direction and the images at the output plane overlap from three columns to one column, $L = 3275$ μm.

Table 1. Positions of Images at the Output Plane of the MMI Coupler, x Direction only, the Input Position Is at $x = -16$ μm

	x_1 (μm)	x_2 (μm)	x_3 (μm)
Modelling Value	−17.807	0.965	16.228
Theoretical Value ($W \neq W_{\rm eq}$)	−17.84	0.92	16
Theoretical Value ($W = W_{\rm eq}$)	−16	0	16

in our example, the input point (8.38, 8.38) apparently satisfies the specified condition. Figure 2(b) shows the 3×3-fold images at the output plane when the MMWG length is 3300 μm. Now, the overlapping images of MMI couplers should be considered. If an image is input exactly at $(iW_{x\_\rm eq}/M, jW_{y\_\rm eq}/N)$, the output images will overlap. For example, let the input image be at point $(W_{x\_\rm eq}/3, 8.38)$, the overlapping of images only exists in the x direction and the number of images will reduce to one in this direction. The 1×3-fold images at the output plane are shown in Fig. 2(c) when the MMGW length is 3275 μm.

The comparison of the positions of images when the 3×3-fold images are of equal intensity is shown in Table 1. The data in this table are only for x-direction, and those for y-direction are similar. The light field was input at $x = -16$ μm and the data in Table 1 represented the positions of three images at the output plane. The theoretically expected values were obtained according to the theory of MPA method[3] for two cases, 1) the difference between $W_{\rm eq}$ and W was considered and $W_{\rm eq}$ was determined by our numerical, 2) the condition of $W_{\rm eq} \approx W$ was supposed. We can find from the table that the modelling results are close to those expected for $W_{\rm eq} \neq W$, and the assumption of $W_{\rm eq} \approx W$ is imprecise in this example. When designing a weak MMI coupler, one will consider the assumption of $W_{\rm eq} \neq W$, but the 3D modelling is one way by which more precise result can be got. The length of the MMWG was also determined by the 3D simulation in this example and it was well known that it was usually a difficult task for the normal design methods.

How about the efficiency of this paralleling program? In our example, eight x-threads and eight y-threads were used, so it was theoretically expected that the simulation time would be decreased to 1/8 of the serial program. In practice, it was only about 1/4. The most important reason was that numerous messages needed to be transmitted between the x- and y-threads. How to improve the efficiency of the paralleling program is another problem to be solved. After all, we have realized the simulation with desktop computers by the MPI. FD-BPM has been proved a powerful method with the convenience and the extensibility of application. In our example, the configuration of desktop computers is CPU P4 3.0 GHz, 512-MB memory and the network card integrated in the mainboard. It takes about four minutes per hundred steps of calculation.

In this letter, the design of MMI couplers with 2D confinement is illustrated. The MMI parameters, including the positions of the input and output waveguides and the length of MMWG, are accurately designed numerically without any previous knowledge of modes. The method of design is 3D ADI FD-BPM. A parallel approach is proposed to improve the running efficiency of FD-BPM and eight personal computers are used to accomplish the design.

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