Rotation-invariant texture analysis using Radon and Fourier transforms

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Texture analysis is a basic issue in image processing and computer vision, and how to attain the rotation-invariant texture characterization is a key problem. This paper proposes a rotation-invariant texture analysis technique using Radon and Fourier transforms. This method uses Radon transform to convert rotation to translation, then utilizes Fourier transform and takes the moduli of the Fourier transform of these functions to make the translation invariant. A k-nearest-neighbor rule is employed to classify texture images. The proposed method is robust to additive white noise as a result of summing pixel values to generate projections in the Radon transform step. Experiment results show the feasibility of the proposed method and its robustness to additive white noise.

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Texture analysis is a basic issue in image processing and computer vision. In many practical applications, it is difficult or impossible to ensure that obtained images have the same translation, rotation or scaling. This requires that the texture analysis should be ideally invariant to viewpoints, as it is always perceived as the same texture image by a human observer. More and more attention has been paid to invariant texture analysis[1]. Recently, multi-resolution approaches such as Gabor filters, wavelet transforms, and wavelet frames have been widely studied and used for texture characterization. Porter *et al.* employed the wavelet transform, a circularly symmetric Gabor filter and a Gaussian-Markov random field to achieve rotation-invariant texture classification and gave experiment results[2]. Jafari-Khouzani *et al.* utilized the Radon and wavelet transforms to attain the invariant features[3]. Haley *et al.* presented a rotation-invariant texture classification method based on polar two-dimensional (2D) Gabor wavelet[4]. Liu *et al.* proposed a texture classification method based on fractal dimension using self-similar texture characterization[5]. All of these methods have achieved considerable success in texture analysis. However, a large number of features are commonly used, which can lead to an unmanageable size of feature space. Furthermore, the feature extraction technique employed may be computationally very demanding.

We present a new technique for rotation-invariant texture classification using Radon and Fourier transforms. This method is proved to be feasible and robust to additive white noise.

The Radon transform of an image $f(x,y)$, denoted as $R(r,\theta)$ hypothetically, is defined as its linear integral along a line. The integration along a particular line defined by a normal distance r from the origin and a normal angle θ will generate the corresponding Radon transform point $R(r,\theta)$. Mathematically, it is written as

$$R(r,\theta)[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(r - x\cos\theta - y\sin\theta)\mathrm{d}x\mathrm{d}y, \quad (1)$$

where $-\infty < r < +\infty$, $0 < \theta < \pi$. According to the Fourier slice theorem, this transformation is invertible. Fourier slice theorem states that for a 2D image $f(x,y)$, the one-dimensional (1D) Fourier transforms of the Radon transform along r are the 1D radial samples of the 2D Fourier transform of $f(x,y)$ at the corresponding angles[7]. Materially, Radon transform projects the image to the other parameter space. And with useful property in the scope of this paper, the rotation of an image by an angle θ_r, causes the Radon transform to be shifted through the same amount, i. e.,

$$f(x\cos\theta_\mathrm{r} - y\sin\theta_\mathrm{r}, x\sin\theta_\mathrm{r} + y\sin\theta_\mathrm{r}) \leftrightarrow R_f(r, \theta - \theta_\mathrm{r}). \quad (2)$$

It goes without saying that wavelet transform is more popular than Fourier transform. However, Fourier transform is more convenient and speedy for some applications, such as image retrieval based on contents, machine vision, vision robots, and so on. According to practice experience, it is obvious that Fourier transform can be easily calculated by hardware using fast Fourier transform (FFT).

Fourier transform is a most commonly used tool in the image processing field. Transforming the image to the spectrum gives a possibility to further work with other disposing methods. The Fourier transform of an image $f(x,y)$ is denoted as $F(u,v)$, whose discrete form is written as

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\mathrm{e}^{-j2\pi(ux/M+vy/N)}. \quad (3)$$

And in this paper, the property most interested is[6]

$$f(x-x_0,y) \leftrightarrow F(u,v)\exp(-j2\pi ux_0/N), \quad (4)$$

$$|F(u,v)\exp(-j2\pi ux_0/N)| \Rightarrow |F(u,v)|\,|\exp(-j2\pi ux_0/N)|. \quad (5)$$

To develop a rotation-invariant texture-analysis method, capturing the features invariant to rotation is most important. In this paper we propose a new method using Radon and Fourier transforms to attain the features, this method also is robust to additive white noise.

As shown in Fig. 1, in the proposed method the Radon transform of the image is first calculated, and then the Fourier transform and its corresponding module are computed to extract the corresponding rotation-invariant features. Since the Radon transform is invertible, any texture information is not lost. Rotation of the input image corresponds to the translation of the Radon transform along θ. To obtain the uniformity of the Radon transform in different orientations, the Radon transform is calculated for a disk area of the texture image. As shown in Fig. 2, the rotation of the texture sample corresponds to a circular shift along horizontal axis in this figure. Therefore, the translation also is eliminated by taking the moduli of the Fourier transform of these functions, so rotation-invariant features can be produced. Finally, a k-nearest-neighbor rule is applied to properly classify the texture.

By Radon transform, the rotated image is transformed to an image with a translation change by mapping the image to the other parameter space. So we define $R_f(\theta, r)$ as the image after Radon transform and with a rotation to image $f(x, y)$, the Radon transform will generate a translation $\Delta\theta$ along θ; its function may be defined as $R_f(\theta - \Delta\theta, r)$.

We apply the Fourier transform to the expressions (1) and (2) respectively and obtain

$$R_f(\theta, r) \leftrightarrow F(u, v), \tag{6}$$

$$R_f(\theta - \Delta\theta, r) \leftrightarrow F(u, v)\exp(-j2\pi u\Delta\theta/N). \tag{7}$$

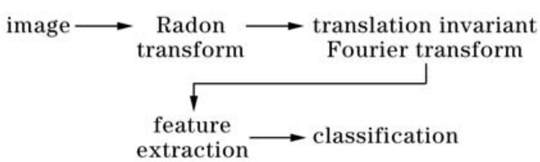

Fig. 1. Diagram of the proposed method.

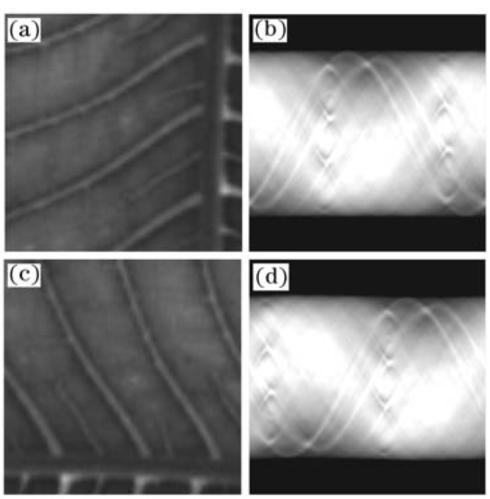

Fig. 2. (a) Texture image example and (b) its Radon transform; (c) rotated texture image example and (d) its Radon transform.

Then we take the modulus of the expressions (1) and (2) and get

$$F(u, v) \leftrightarrow |F(u, v)|, \tag{8}$$

$$F(u, v)\exp(-j2\pi u\Delta\theta/N) \leftrightarrow |F(u, v)|\,|\exp(-j2\pi u\Delta\theta/N)| = |F(u, v)|. \tag{9}$$

And for $\exp(-j2\pi u\Delta\theta/N)$, its value equals 1, i.e., the expressions (8) and (9) are equal indeed. So finally we get the rotation-invariant feature. This can be seen in Fig. 3.

We get an image of size 128×400 after these two transforms. But due to the error generated by the algorithm, the data obtained in practice do not totally accord with the theoretical result. It is believed that the more pixels are calculated, the more accurate result will be attained. But, in order to accelerate the calculation speed, we finally select a matrix size of 16×16. By the way, the matrix can provide the global and basic information about image, which is enough for classification. And for each vector, we calculate the following features:

$$e_i^1 = \max \mu_i(a_j), \quad i, j = 1, 2, \cdots, 16, \tag{10}$$

$$e_i^2 = \frac{1}{16}\sum_{j=1}^{16} \mu_i(a_j), \quad i = 1, 2, \cdots, 16, \tag{11}$$

$$e_i^3 = \sum_{j=1}^{15} (\mu_i(a_j) - \mu_i(a_{j+1}))^2, \quad i = 1, 2, \cdots, 16, \tag{12}$$

where μ_i denotes a 16-dimensional row vector and a_j is the value of the vector.

The k-nearest-neighbor rule is a well-established and nonparametric pattern-classification method. We use it to classify the texture image into an appropriate class. In this algorithm, the unknown pattern is assigned to the class label which is most highly represented among these neighbors[7]. However, the features may not have the same significance for classification. Here, we utilize weights to the features to select the best combination of

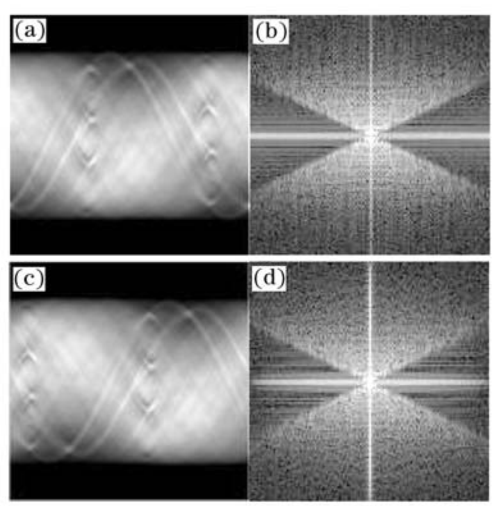

Fig. 3. (a) Texture after Radon transform and (b) its modulus of Fourier transform; (c) rotated texture after Radon transform and (d) its modulus of Fourier transform.

features. The weight for each feature is calculated as the correct classification rate in the training set using only that feature.

In order to evaluate the performance of the technique proposed, two different kinds of experiments were investigated. Data set consists of 15 texture images of size 512×512 from Brodatz album[8], as shown in Fig. 4. In all of the experiments, we used texture images of size 128×128 with 256 gray levels.

The first experiment was to test the feasibility of the proposed method. Each texture image was divided into 16 non-overlapping subimages of size 128×128 at $0°$ to create a training set of 240 (15×16) images. For the test set, each image was rotated at angles $10°$ to $180°$ with $10°$ increments. In total, we created 4320 ($15 \times 16 \times 18$) samples for the testing set (240 images as training set). The experiment results are shown in Table 1.

Furthermore, to validate the robustness with the additive white noise, Gaussian random noise with zero mean and variance depending on the wanted signal to noise ratio (SNR) has been added to all of the images. Then we performed the experiment with same steps like the test one. Four SNR values have been considered (30, 20, 15, 10 dB). The experiment results are shown in Table 1.

From the experiments, we can draw a conclusion that the proposed method is very feasible in the rotation-invariant texture analysis and shows a high robustness even if the SNR is as low as 10 dB.

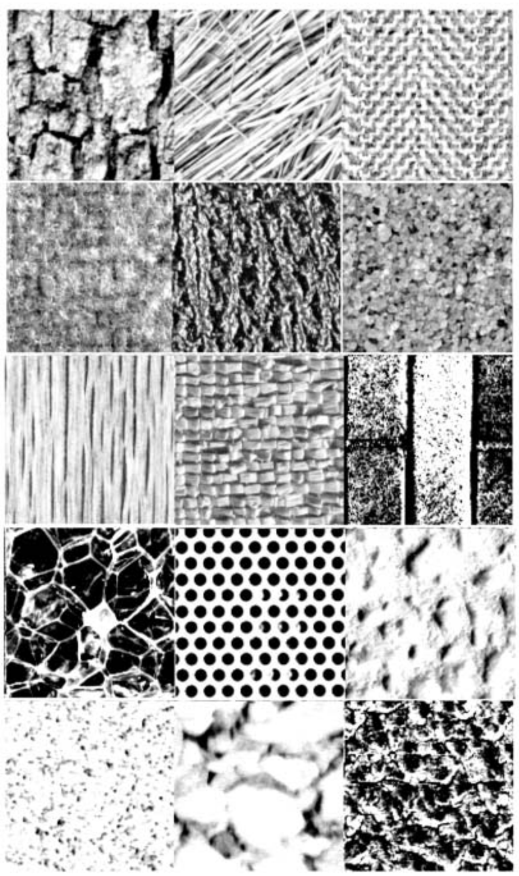

Fig. 4. Fifteen texture samples from Brodatz album for test.

Table 1. Correct Classification Percentage for Texture Data (%)

White Noise (dB)	30	20	15	10	0
T1	100	100	97.1	90.5	100
T2	100	100	97.3	90.4	100
T3	100	100	96.7	89.7	100
T4	99.5	99.4	95.5	87.3	99.5
T5	100	100	97.0	89.9	100
T6	99.3	99.1	95.6	86.9	99.3
T7	100	100	97.2	90.2	100
T8	100	100	96.9	89.7	100
T9	100	100	97.5	91.0	100
T10	100	100	97.2	90.3	100
T11	100	100	97.3	90.0	100
T12	100	100	96.9	89.9	100
T13	100	100	96.4	89.6	100
T14	100	100	97.0	90.1	100
T15	100	100	96.5	89.5	100

In this paper, we introduced a new method for rotation-invariant texture analysis utilizing Radon and Fourier transforms. The proposed technique was proved to be feasible and efficient. And we compared the classification results with the most recent rotation-invariant texture analysis methods, for example, the maximum correct classification percentage (CCP) of 97.9% achieved on 25 texture images[3], the CCP of 99% obtained on 13 texture images[4], we got over 99% CCPs, and good robustness to the additive white noise was achieved as well.

But due to the error generated by the calculation, the data we got in practice did not totally accord with the theoretical result. So we need to improve the calculation precision in the future.

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