Speckle reduction of SAR images using ICA basis enhancement and separation

Yutong Li (李昱彤) and Yue Zhou (周 越)

Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University, Shanghai 200240

Received April 6, 2007

An approach for synthetic aperture radar (SAR) image de-noising based on independent component analysis (ICA) basis images is proposed. Firstly, the basis images and the code matrix of the original image are obtained using ICA algorithm. Then, pointwise Hölder exponent of each basis is computed as a cost criterion for basis enhancement, and then the enhanced basis images are classified into two sets according to a separation rule which separates the clean basis from the original basis. After these key procedures for speckle reduction, the clean image is finally obtained by reconstruction on the clean basis and original code matrix. The reconstructed image shows better visual perception and image quality compared with those obtained by other traditional techniques.

OCIS codes: 100.0100, 030.6140, 280.6730, 350.4600.

Synthetic aperture radar (SAR) sensors can produce range imagery of high spatial resolution under different conditions. However, the images suffer from the effects of speckle noise. Thus, speckle reduction is a key step for desirable image quality. For this application, Lee developed a linear approximation filter based on the minimum mean-square error (MMSE) criterion in 1980[1], and Kuan presented a generalized filter of Lee's in 1985[2]. Wavelet thresholding shrinkage was proposed in 1995[3]. Taking advantage of the excellent performance of independent component analysis (ICA), a method namely sparse code shrinkage was proposed as a novel improvement[4]. However, conventional algorithms lack consideration on the basis information obtained by independent component analysis (ICA)[5], which usually contains substantial knowledge for our denoising work. Based on extensive study on these meaningful basis, we propose a novel theory for SAR image dispeckling in this paper. The proposed method contains two key procedures, basis enhancement and basis separation. As to the former, we extend the signal enhancement algorithm by Vehel[6] for image application and use pointwise Hölder exponent as a criterion for basis enhancement; as to the latter, based on the proposed separation theory and separation rule, we further classify the enhanced basis into two types respectively called 'clean' basis and 'noise' basis, which belong to corresponding subspaces, 'clean' and 'noise'. After these procedures, the clean image is obtained by reconstruction on the clean basis.

ICA is a statistical method for transforming an observed multi-dimensional random vector into components that are mutually independent. Denoted by X the observed matrix: $X = \{x_1, x_2, x_3, \cdots, x_m\}^{\mathrm{T}}$, by S the independent components matrix (or sparse code matrix): $S = \{s_1, s_2, s_3, \cdots, s_n\}^{\mathrm{T}}$, and by $A(m \times n, m > n)$ the mixed matrix, the linear representation can be given by

$$X = AS \text{ or } S = WX, \tag{1}$$

where x_i $(i = 1, \cdots, m)$ is the observed signal and s_i $(i = 1, \cdots, n)$ is the independent component, W namely demixed matrix or transformation matrix is the pseudo inverse of A, that is $W = A^{+}$. In the following content, we replace x_i, s_i with x, s for short.

The independent components in ICA are obtained by maximizing non-Gaussianity measure, which is equivalent to searching for sparse representation. Thus, ICA gives sparse codes for natural images. According to sparse coding theory[4], the localized and compact distribution of energy in images suggests that they have a "sparse structure", which means any image can be represented by a relatively small number of descriptors out of a much larger set to choose from. Thus, an image $I(x, y)$ can be modelled as a linear superposition of basis ϕ_k, which can be defined as

$$I(x, y) = [\sum_k s_{k,b}\phi_k], \quad k = 1, \cdots, n, \tag{2}$$

where $s_{k,b}$ is the element of code matrix S at row k, column b. And ϕ_k is the kth column vector of mixed matrix $A = [\phi_1, \cdots, \phi_k, \cdots, \phi_n]$.

If we use symbol '$\leftrightarrow$' to define the mapping association between a basis and the corresponding independent component, we obtain $s_k \leftrightarrow \phi_k$, $k = 1, \cdots, n$.

Measuring the local smoothness of functions is proved to be an important task for many applications in mathematical analysis and in signal and image processing. Such a characterization is vital in multi-fractal analysis, and is an instrumental tool for image segmentation and denoising. Hölder exponent is considered as a powerful parameter for studying the structure of singular signals[7].

Let $\alpha \in (0, 1)$, and $x_0 \in K \subset R$. A function $f : K \to R$ is in $C^{\alpha}_{x_0}$ (or has a pointwise Hölder exponent α at x_0), if for all in a neighborhood x of x_0,

$$|f(x) - f(x_0)| \leq c\,|x - x_0|^{\alpha}\,, \tag{3}$$

where c is a constant independent of x_0 and α. The Hölder exponent α is computed as

$$\alpha_f(x) = \liminf_{h \to 0} \frac{\log|f(x+h) - f(x)|}{\log|h|}. \tag{4}$$

Clearly, a function that is differentiable at $x = x_0$ has a Hölder exponent $\alpha \geq 1$. Geometrically, this means that the magnitude of oscillations of the function near x_0 decreases faster than the distance to x. In general, if this

 © 2007 Chinese Optics Letters

inequality holds for α_0, then it will hold for all $\alpha \le \alpha_0$. Thus, the Hölder exponent of a function is the upper bound of α. For images, this exponent characterizes the edges and the features.

Suppose we have obtained the sparse description of the observed matrix X, $X = AS$. According to the above analysis, we have $A = [\phi_1, \cdots, \phi_k]$, where ϕ is a vector namely basis vector corresponding to its basis image φ. So, we derive from the ICA sparse coding that input image can be spanned by the basis and each basis corresponds to its independent component which has a 'sparse' distribution. With the basis images enhanced or smoothed, the original image can get recovery with speckles reduced. Therefore, our first key step is focused on basis image enhancement. Vehel developed an algorithm for signal enhancement based on Hölder regularity[6], we now extend and apply this approach to image processing.

Consider the basis image to be full of singularity or nonsingularity characteristics. Based on the multi-fractal theory[7], unorderly distributed points represent points with nonregular nonsingularity, meaningful context represents singularity, and background represents regular nonsingularity. For image smoothing and enhancement, the key is to decrease the nonregular nonsingularity which is equivalent to retaining points with singularity or with regular nonsingularity. This means singularity of every pixel should be increased to different extent or be regulated to be more uniform. Wavelet theory[8] and 2-microlocal analysis[9] are used for this procedure. Specially, wavelet is applied for the regulation of coefficients based on its multi-scale property, and 2-microlocal is used to define the regulating correlation between wavelet coefficients and Hölder exponent.

Firstly, 'Harr' wavelet which has an orthonormal wavelet basis is used to decompose each basis image and the computed coefficients are denoted by c_k^j, where as usual j denotes scale and k position.

Next, we shall modify wavelet coefficients with the desired Hölder exponent α, then reconstruct the enhanced image from them. According to 2-microlocal analysis, we can define this regulating correlation as

$$c_k^j \to c_k^j \times 2^{-j(\Delta\alpha)}, \tag{5}$$

where, $\Delta\alpha = \alpha^* - \alpha$, α^* must be adjusted in particular experiment. Too large value of α^* results in oversmoothness, and too small value lacks intensity of enhancement. Optimal tradeoff can be achieved when α^* lies between 2.95 and 3.0.

Basis enhancement verifies that 'noise' part can be merged into the 'clean' part, forming approximately uniform singularitiy, but the internal characteristics of both 'noise' and 'clean' are not differentiated clearly. For superior image quality, the second key procedure, enhanced basis images separation is necessary.

Suppose we have obtained $X = AS$ using ICA technique and the basis images have been enhanced with the first key step mentioned above. From the view of signal separation, we believe that 'noise pattern' in the image comes from another kind of signal source which is different from the 'clean' signal ones. Therefore, an original image space can be separated into two subspaces. If we can separate the original one into these two subspaces using some criterion, we can effectively obtain the clean version extracted from the original image by reconstruction on the clean subspace. Let us denote the original image space by S_{origin}, 'clean' subspace by S_{clean}, and 'noise' subspace by S_{noise}, we obtain $S_{\text{origin}} = S_{\text{clean}} + S_{\text{noise}}$, which is equivalent to $X_{\text{origin}} = X_{\text{clean}} + X_{\text{noise}}$. Because each space comprises basis images and corresponding independent components, separating basis implies separation of space.

The proposed principle on separation and reconstruction is shown in Fig. 1 for clear demonstration.

Clearly, the core is how to separate two subspaces from the original one. We define the separation rule as

$$\begin{cases} \frac{1}{N}\|s_i\|_1 > \theta \cdot \frac{1}{n}\|S\|_\infty & \Rightarrow s_i \in S_{\text{clean}} \text{ and } \phi_i \in S_{\text{clean}}, \\ \frac{1}{N}\|s_i\|_1 < \theta \cdot \frac{1}{n}\|S\|_\infty & \Rightarrow s_i \in S_{\text{noise}} \text{ and } \phi_i \in S_{\text{noise}}, \end{cases} \tag{6}$$

For more clarity, the formula (6) can be written as

$$\begin{cases} \frac{1}{N}\sum_j |S_{ij}| > \theta \cdot \frac{1}{n} \max\limits_{1\le i\le n} \sum\limits_{j=1}^{N} |S_{ij}| \Rightarrow \\ \qquad s_i \in S_{\text{clean}} \text{ and } \phi_i \in S_{\text{clean}}, \\ \frac{1}{N}\sum_j |S_{ij}| < \theta \cdot \frac{1}{n} \max\limits_{1\le i\le n} \sum\limits_{j=1}^{N} |S_{ij}| \Rightarrow \\ \qquad s_i \in S_{\text{noise}} \text{ and } \phi_i \in S_{\text{noise}}, \end{cases} \tag{7}$$

where $i = 1, \cdots, n$, n is the number of components, s_i is the ith component, ϕ_i is the ith basis with relation $\phi_i \leftrightarrow s_i$, N is the number of sample points of each component, $\theta \cdot \frac{1}{n}\|S\|_\infty$ is the separation threshold with $\theta \in (0, 1)$ which can be easily set in particular experiment.

Steps of the proposed algorithm are as follows.

1) Use a sliding window (size = 16×16) to sample the original image regularly, ensuring that each sampled block has an interval step of 8 pixels both in row and in column. Then, we obtain the observed matrix X with each row-vector as an observed signal. For example, if an image is 256×256 in size, the observed matrix X should be 256×961 by this kind of sampling.

2) To avoid huge computation complexity, we subtract

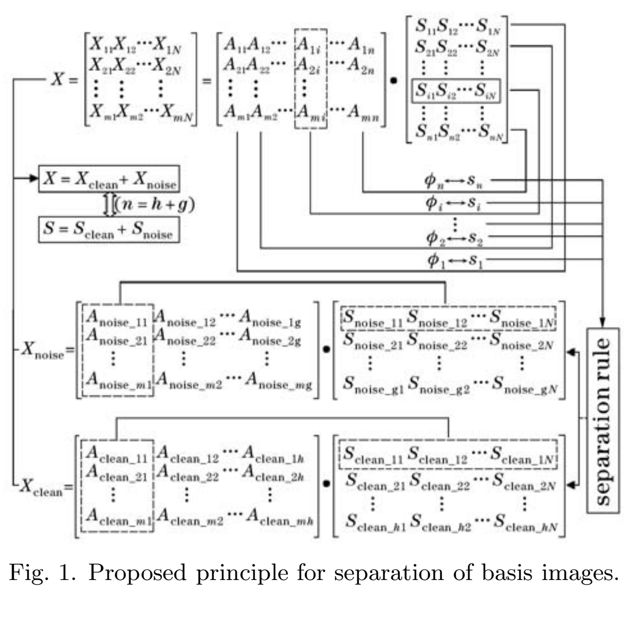

Fig. 1. Proposed principle for separation of basis images.

the mean of each signal and then apply principal component analysis (PCA) to reduce dimension of the vectors to 64, which implies 64 basis images we shall have.

3) The preprocessed X is used as the input to fast ICA algorithm[10], with 'tanh' nonlinearity.

4) The 64 basis images from A and the corresponding independent components from S are obtained after convergence of fast ICA algorithm.

5) The pointwise Hölder exponent for each of these basis images is computed.

6) Based on the Hölder exponent, enhancement of the basis images is performed (supposing the enhanced basis corresponds to $\widehat{A}_1$).

7) Enhanced basis images are separated into two types: 'clean' and 'noise' (supposing similarly the enhanced-clean basis corresponds to $\widehat{A}_2$).

8) According to $\widehat{X} = \widehat{A}_2 S$, the recovery result is obtained.

9) Compare the result with those by other conventional algorithms, and the ratio of standard deviation to mean (SD/M) is calculated for each image. SD/M, indicative of de-noising quality, is defined as

$$\mathrm{SD/M} = \frac{\frac{1}{Z-1}\sqrt{\sum\limits_{\Omega}[I(x,y) - \frac{1}{Z}\sum\limits_{\Omega} I(x,y)]^2}}{\frac{1}{Z}\sum\limits_{\Omega} I(x,y)}, \tag{8}$$

where $I(x,y)$ is the gray value of each pixel of an image Ω, Z is the number of pixels. Generally, lower value of SD/M implies less noise and better image quality.

We performed experiment on a single-look 256×256, gray-scale SAR image shown in Fig. 2. Figure 3(a) shows the basis (64 images, each 16×16) obtained by ICA, Figs. 3(b) and (c) show the pointwise Hölder exponent of each basis and the enhanced basis. Figure 4 gives the separation result (29 bases of 'noise' pattern, and 35 bases of 'clean' pattern) of enhanced basis images, with $\theta = 0.35$. The recovery result with the proposed method is shown in Fig. 5. For comparison, we also give the recovery results by other conventional methods in Fig. 6. We further calculated the SD/M of related images for ratio comparison, the results are shown in Table 1. We denote this experiment by 'EX1' in the table.

For fourther validation, we apply the proposed algorithm to two other SAR images shown in Figs. 7(a) and (c). The recovery results using the proposed algorithm are shown in Figs. 7(b) and (d), showing superior de-noising performance, with $\theta = 0.4$ and $\theta = 0.3$. Because

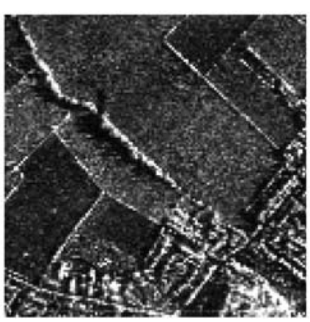

Fig. 2. Original image.

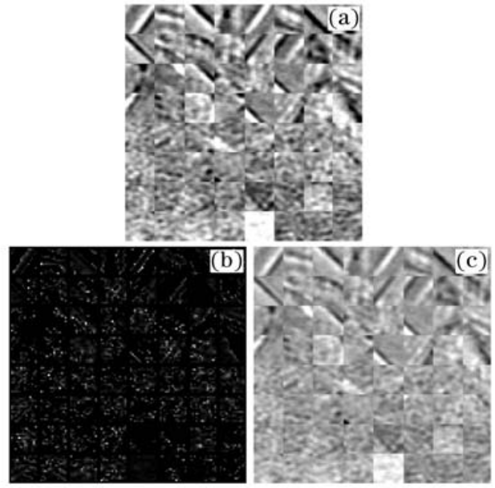

Fig. 3. (a) Basis images (64); (b) Hölder exponent of the basis images (64); (c) enhanced basis images (64).

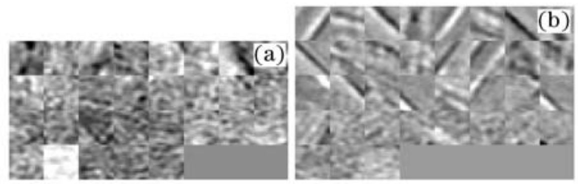

Fig. 4. (a) 29 'noise' basis images with $\theta = 0.35$; (b) 35 'clean' basis images with $\theta = 0.35$.

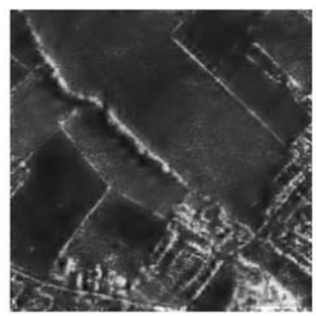

Fig. 5. Recovery result with the proposed method.

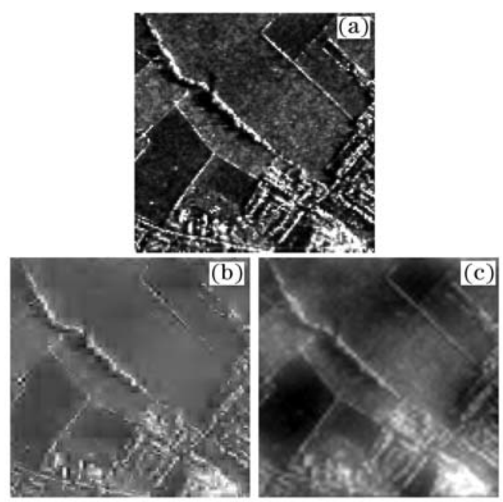

Fig. 6. Recovery results by (a) 'Wiener' filtering, (b) wavelet thresholding shrinkage, and (c) sparse coding shrinkage.

Table 1. SD/M Comparison

Algorithm	SAR Image	Wiener	Wavelet	Sparse Coding	Proposed
EX1	0.7817	0.7261	0.6624	0.6459	0.4325
EX2	0.8017	0.7856	0.6563	0.6102	0.5118
EX3	0.4020	0.4005	0.3596	0.3259	0.2624

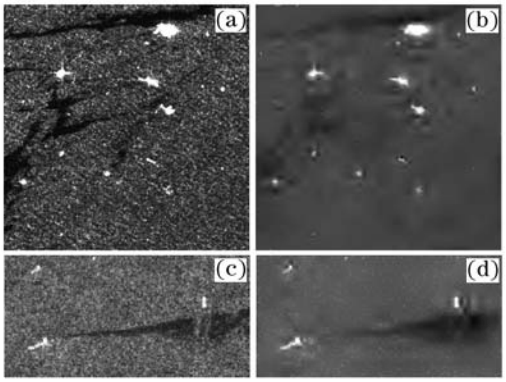

Fig. 7. (a),(c) Original images SAR_1 and SAR_2; (b),(d) recovery results with the proposed method.

of content limit, recovery results with other conventional methods are not provided in this paper, but SD/M ratios were still calculated for comparison, as also shown in Table 1. Similarly, we denote these two examples by 'EX2' and 'EX3'.

In conclusions, based on the basis information from ICA technique, a novel speckle filtering algorithm is developed using basis enhancement and separation. The recovery image obtained by this method achieves a better visual perception, which is also illustrated by the lower value of SD/M of the reconstructed image.

This work was supported by the National Natural Science Foundation of China (No. 60675023) and the Aviation Science Foundation of China (No. 04F57004). Y. Li's e-mail address is lyt.pami2005@gmail.com, and Y. Zhou's e-mail address is zhouyue@sjtu.edu.cn.

References

1. J. S. Lee, IEEE Trans. Pattern Analysis and Machine Intelligence **2,** 165 (1980).
2. D. T. Kuan, A. A. Sawchuk, T. C. Strand, and P. Chavel, IEEE Trans. Pattern Analysis and Machine Intelligence **7,** 165 (1985).
3. D. L. Donoho and I. Johnstone, J. Am. Statist. Assoc. **90,** 1200 (1995).
4. A. Hyvarinen, P. O. Hoyer, and E. Oja, in *Advances in Neural Information Processing Systems 11 (NIPS'98)* 473 (1999).
5. P. Comon, Signal Processing **36,** 287 (1994).
6. J. Lévy-Véhel, in *IMA Volumes in Mathematics and Its Applications, Fractals in Multimedia* **132,** 197 (2002).
7. J. Lévy-Véhel, in *Fractal Image Encoding and Analysis, NATO ASI Series* **159,** 299 (1998).
8. J. Lévy-Véhel, in *Fractals in the Natural and Applied Sciences* 261 (1994).
9. J. Lévy-Véhel, Fractals **3,** 755 (1995).
10. A. Hyvrinen and E. Oja, Neural Computation **9,** 1483 (1997).