Dynamics of dark-bright vector solitons in a birefringent fiber

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Coupled dark-bright vector solitons are considered in a birefringent fiber, and their dynamics are studied by the variational approach. The stationary states are analyzed, and it is found that their intensity distribution depends strictly on the ratio of the group-velocity delay strength to the effective total energy. Finally, the propagation of the coupled dark-bright vector solitons is investigated by the numerical method.

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Optical solitons in fibers are formed by a balance of group dispersion and Kerr non-linearity. Numerical simulation and experiments have demonstrated that solitons can propagate an extended distance without distortion, so they may become the ideal message carrier in long distance communication[1].

Dark solitons propagate in the normal dispersion region. They have robust features such as low intrinsic transmission loss, resistance to perturbations, and weak interaction between neighboring solitons. The amplified spontaneous emission (ASE) noise in the dark soliton system is only half of that corresponding to the bright counterpart. The techniques for generating and detecting dark soliton pulses have been developed[2], and dynamics of the dark solitons propagating in an optical fiber exhibiting birefringence have been investigated[3−5].

Optical fibers are birefringent in reality, and pulses travel along two orthogonal polarizations of the fiber at slightly different speeds. A technique called polarization-division multiplexing has been proposed, and this technique doubles the transmission rate compared with the launching of pulses along the same polarizations[6,7]. In fact, theoretical and experimental results show that the single-wavelength bit-rate capacity of an ultra-long distance soliton transmission system can be doubled by the polarization-division multiplexing of orthogonal polarization solitons, and the capacity can be furtherly increased by using a combination of polarization and other multiplexing (such as time-division multiplexing)[8].

So-called coupled dark-bright vector solitons, where a bright optical solitary wave exists in a system with defocusing nonlinearity because it is trapped within a co-propagating dark soliton, have some interesting and distinguishing dynamics different from those of the bright soliton and the dark soliton. The interaction of dark and bright solitons may be strongly repulsive. In this case the cross-phase modulation has an important influence on both the formation and interaction of the solitons[9,10]. Switching and self-trapping effects on dynamics of the coupled dark-bright solitons have not been studied in detail yet under the classic theory (such as the variational approach).

In this letter, the coupled dark-bright vector solitons are considered in a birefringent fiber, their dynamics are investigated by the variational approach and the numerical method, and some novel results are obtained.

In a real birefringent fiber with normal dispersion, the envelop of field can be described by the coupled nonlinear Schrödinger equation[11,12]

$$j\frac{\partial u}{\partial Z}-j\eta\frac{\partial u}{\partial \tau}-\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+(|u|^2+\frac{2}{3}|v|^2)u=0,$$

$$j\frac{\partial v}{\partial Z}+j\eta\frac{\partial v}{\partial \tau}-\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+(|v|^2+\frac{2}{3}|u|^2)v=0, \quad (1)$$

where u and v are normalized elliptically polarized components along two orthogonal directions, respectively. $Z=z/z_{\mathrm{d}}$, $z_{\mathrm{d}}=\tau_0^2/\left|\bar{d}\right|$ and $\tau=t/\tau_0$. z and t are actual distance coordinate and time, respectively. τ_0, z_{d}, $\bar{d}$, and η are the pulse width, the dispersion length, the path-average dispersion, and the group-velocity delay strength caused by the birefringence, respectively.

Treating the dark soliton as an effective particle, the variational approach for the dark soliton can be used. To adopt Lagrangian variational approach, the dark soliton wave function u is rescaled to be $u\exp(jE^2Z/4)$ to remove the background wave. The renormalized equation can be expressed as[9,10]

$$j\frac{\partial u}{\partial Z}-j\eta\frac{\partial u}{\partial \tau}-\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+[(|u|^2-\frac{E^2}{4})+\frac{2}{3}|v|^2]u=0,$$

$$j\frac{\partial v}{\partial Z}+j\eta\frac{\partial v}{\partial \tau}-\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+(|v|^2+\frac{2}{3}|u|^2)v=0. \quad (2)$$

We adopt trial functions below as the dark-bright vector soliton solutions to Eq. (2)

$$u(Z,\tau)=j\frac{E\sin\theta}{2}+\frac{E\cos\theta}{2}\tanh[\frac{E}{2}(\tau+\frac{\Delta}{2})],$$

$$v(Z,\tau)=\frac{E\sin\theta}{2}\mathrm{sech}[\frac{E}{2}(\tau-\frac{\Delta}{2})], \quad (3)$$

where θ is the distributing angle, which determines the depth of the dark soliton and the energy distribution between two dark-bright vector solitons. Δ is the time

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spacing between two center positions of the two dark-bright vector solitons. $E_1(Z) = \int_{-\infty}^{\infty} (E^2/4 - |u|^2)\mathrm{d}\tau = E\cos^2\theta$ is the effective energy of the dark soliton without the background wave, $E_2(Z) = \int_{-\infty}^{\infty} |v|^2\mathrm{d}\tau = E\sin^2\theta$ is the effective energy of the bright soliton, and the effective total energy is $E = E_1 + E_2$ (a conserved quantity).

In the evolution of the dark-bright vector solitons, the wave functions u and v retain the functions given by Eqs. (3), but the distributing angle, the spacing, and the effective energy of each soliton become functions of the propagation distance.

The averaged Lagrangian of Eq. (2) can be defined with the variational approach[13,14]

$$\begin{aligned}L(Z) = \int_{-\infty}^{\infty} \Big\{ &\frac{j}{2}(u^*\frac{\partial u}{\partial Z} - u\frac{\partial u^*}{\partial Z}) - \frac{j\eta}{2}(u^*\frac{\partial u}{\partial \tau} - u\frac{\partial u^*}{\partial \tau}) \\ &+\frac{1}{2}\left|\frac{\partial u}{\partial \tau}\right|^2 + \frac{1}{2}(|u|^2 - \frac{E^2}{4})^2 \\ &+\frac{j}{2}(v^*\frac{\partial v}{\partial Z} - v\frac{\partial v^*}{\partial Z}) + \frac{j\eta}{2}(v^*\frac{\partial v}{\partial \tau} - v\frac{\partial v^*}{\partial \tau}) \\ &+\frac{1}{2}\left|\frac{\partial v}{\partial \tau}\right|^2 + \frac{1}{2}|v|^4 + \frac{2}{3}|u|^2|v|^2 \Big\}\mathrm{d}\tau \\ = &\frac{E^2\sin 2\theta}{8}(\frac{\mathrm{d}\Delta}{\mathrm{d}Z} - 2\eta) \\ &+\frac{E^3(4 + 4\sin 2\theta + 14\sin^2\theta - \sin^4 2\theta)}{48} \\ &-\frac{E^3\sin^2 2\theta}{24}\exp(-\frac{aE^2\Delta^2}{8}),\end{aligned} \tag{4}$$

where $a \approx 0.21$.

The equations of motions for the distributing angle and the spacing are obtained from the averaged Lagrangian using $\mathrm{d}L(Z)/\mathrm{d}\sigma - \mathrm{d}[\mathrm{d}L(Z)/\mathrm{d}(\mathrm{d}\sigma/\mathrm{d}Z)]/\mathrm{d}Z = 0$ $(\sigma = \Delta, \theta)$, and two important equations are obtained

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}Z} &= \frac{aE^3\sin^2 2\theta\Delta}{24\cos 2\theta}\exp(-\frac{aE^2\Delta^2}{8}), \\ \frac{\mathrm{d}\Delta}{\mathrm{d}Z} &= \frac{E}{6\cos 2\theta}[(\frac{12\eta}{E} - 4)\cos 2\theta - 7\sin 2\theta \\ &+2\sin^2 2\theta\sin 4\theta + 2\sin 4\theta\exp(-\frac{aE^2\Delta^2}{8})].\end{aligned} \tag{5}$$

We can see that the dynamics of the coupled dark-bright solitons depend strictly on the group-velocity delay strength and the effective total energy.

The stationary state can be obtained by setting the distance derivatives in Eq. (5) to zero. The condition for the stationary state can be obtained as

$$\begin{aligned}&\Delta_0 = 0, \\ &(\frac{12\eta}{E} - 4)\cos 2\theta_0 - 7\sin 2\theta_0 \\ &+2\sin^2 2\theta_0\sin 4\theta_0 + 2\sin 4\theta_0 = 0.\end{aligned} \tag{6}$$

where Δ_0 and θ_0 are the spacing and the distributing angle of the stationary state.

These features show that the stationary state may exist when two solitons are of the same central position ($\Delta = 0$), and the energy may be distributed within both the dark and bright vector solitons. Figures 1 and 2 are the intensities ($|u|^2$ and $|v|^2$) of two vector solitons for the stationary states versus time with different ratios of the group-velocity delay strength to the effective total energy, and the distributing angles of the stationary states are obtained by numerically solving Eq. (6). The effective total energy is selected as $E = 4$, and the group-velocity delay strengths are selected as $\eta = 1.0$ (namely, for $\eta/E = 0.25$), 2.0 (namely, for $\eta/E = 0.5$), and 4.0 (namely, for $\eta/E = 1.0$). We can see the effective intensity distribution of the dark or bright vector solitons for the stationary states strictly depends on the ratio of the group-velocity delay strength to the effective total energy.

Equation (1) can be solved numerically by using the split-step Fourier algorithm to study the dynamics of the coupled dark-bright vector solitons. The simulation parameters are: the soliton pulse width $\tau_0 = 10$ ps, the path-average dispersion of the birefringent fiber $\bar{d} = -1.00$ ps^2·km^{-1}, and the dispersion length is about 100 km corresponding to the average dispersion. Figure 3 is the normalized soliton intensity versus the propagation distance. The initially input polarized components (soliton pulses) are given by Eq. (3), the initial distributing angle is $\theta = \pi/4$ and the initial time spacing

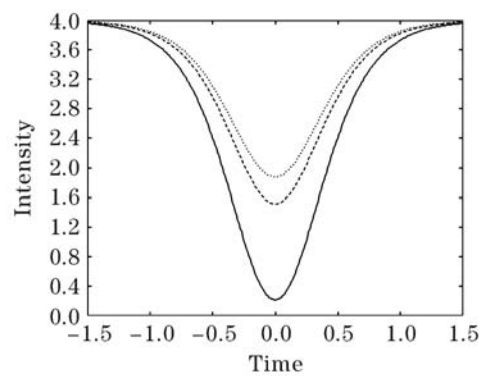

Fig. 1. Intensity ($|u|^2$) of the dark soliton for the stationary states versus time with $E = 4$. Solid line: $\eta/E = 0.25$; dashed line: $\eta/E = 0.5$; dotted line: $\eta/E = 1.0$.

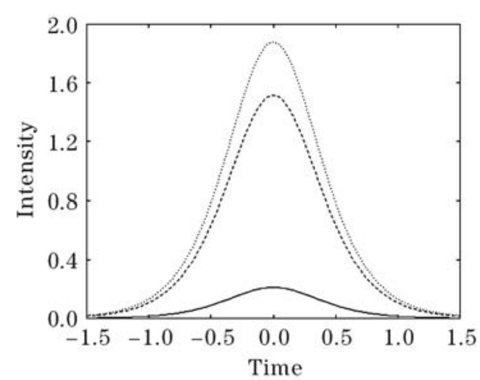

Fig. 2. Intensity ($|v|^2$) of the bright soliton for the stationary states versus time with $E = 4$. Solid line: $\eta/E = 0.25$; dashed line: $\eta/E = 0.5$; dotted line: $\eta/E = 1.0$.

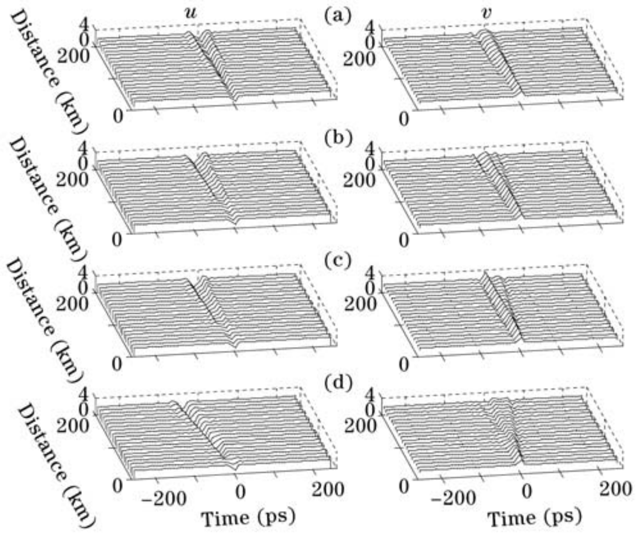

Fig. 3. Normalized vector soliton intensity versus propagation distance with $E = 4$. (a) $\eta/E = 0.05$; (b) $\eta/E = 0.25$; (c) $\eta/E = 0.5$; (d) $\eta/E = 1.0$.

is $\Delta = 0$. The effective total energy is selected as $E = 4$, and the group-velocity delay strengths are selected as $\eta = 0.2$ (namely, for $\eta/E = 0.05$), $\eta = 1.0$ (namely, for $\eta/E = 0.25$), 2.0 (namely, for $\eta/E = 0.5$), and 4.0 (namely, for $\eta/E = 1.0$). We can see that the birefringence causes the energy transfer between the bright and dark solitons, leading to the disintegration of the bright soliton and the submergence of the dark soliton. The disintegration distance or the submergence distance, which is defined as the propagating distance until disintegration or submergence, depends strictly on the ratio of the group-velocity delay strength to the effective total energy. For example, the stability of the vector soliton propagation is alike when the ratio is small enough (such as $\eta/E < 0.05$) under the given effective total energy ($E = 4$). When the ratio becomes large (such as $\eta/E > 0.05$), the stability of the vector soliton propagation is reduced. Furthermore, the effective propagation distance of the soliton system is determined by the propagation of the bright soliton because the dark soliton has the longer submergence distance than the disintegration distance of the bright soliton. So the performance of the dark-bright vector soliton system is determined by the bright soliton because the dark soliton has robust features.

In the bright or dark soliton system with the birefringence, we find that the birefringence causes change of the time relative displacement between normalized elliptically polarized components, which leads to the disintegration of the bright soliton or the submergence of the dark soliton[11,12]. We can see the effects on the propagation of the coupled dark-bright vector solitons also are principally caused by the time relative displacement between normalized elliptically polarized components in the evolution of the dark-bright vector solitons, which causes the energy transfer between the bright and dark solitons. The effecting mechanism is that the birefringence causes the oscillation of the soliton central position, and the large birefringence may lead to the stochastic dynamics of the oscillation, so the elliptically polarized components take place disaccord along two orthogonal directions in the birefringent fiber[15].

In summary, the coupled dark-bright vector solitons are considered in a birefringent fiber, and their dynamics are investigated by the variational approach. The stationary states are discussed, and their intensity distribution depends on the ratio of the group-velocity delay strength to the effective total energy. The propagation of the coupled dark-bright vector solitons is investigated by the numerical method, their dynamics are analyzed, and the effecting mechanism of the birefringence is demonstrated.

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