Influence of group-delay ripple on timing jitter induced by SPM and IXPM in systems with dispersion compensated by CFBG

Xi Qin (秦 曦)[1,2], Jihong Cao (曹继红)[1,2], Yong Chen (陈 勇)[1,2], Feng Zhang (张 峰)[1,2], and Shuisheng Jian (简水生)[1,2]

[1]*Key Laboratory of All Optical Network & Advanced Telecommunication Network of EMC, Beijing Jiaotong University, Beijing 100044*

[2]*Institute of Lightwave Technology, Beijing Jiaotong University, Beijing 100044*

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An analytical expression was proposed to analyze the influence of group-delay ripple (GDR) on timing jitter induced by self-phase modulation (SPM) and intra-channel cross-phase modulation (IXPM) in pseudo-linear transmission systems when dispersion was compensated by chirped fiber Bragg grating (CFBG). Effects of ripple amplitude, period, and phase on timing jitter were discussed by theoretical and numerical analysis in detail. The results show that the influence of GDR on timing jitter changes linearly with the amplitude of GDR and whether it decreases or increases the timing jitter relies on the ripple period and ripple phase. Timing jitter induced by SPM and IXPM could be suppressed totally by adjusting the relative phase between the center frequency of the pulse and the ripples.

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Pseudo-linear transmission is a regime for transmission of high-speed time division multiplexing (TDM) signals where fast variations of each channel waveform with cumulative dispersion allow important averaging of the intra-channel effects of fiber nonlinearity[1]. The main nonlinearities in this kind of systems are self-phase modulation (SPM), intra-channel cross-phase modulation (IXPM) and intra-channel four wave mixing (IFWM). IXPM together with SPM cause timing jitter between pulses, which is considered as one of the major limiting factors for systems with transmission speed reaching 40 Gb/s or beyond. This makes timing jitter become a hot area of research[2,3]. Theoretical analyses have shown that timing jitter induced by SPM and IXPM in this kind of transmission system results from a two-step process: Firstly, SPM and IXPM induce frequency shift between adjacent pulses. Then, the frequency shift is transformed to timing shift between pulses through dispersion and this finally results in timing jitter[4,5].

In this kind of transmission system, chirped fiber Bragg grating (CFBG) is a good alternative as dispersion compensator[6−8]. It has some advantages relative to other dispersion compensation methods such as low insertion loss, small package size, and reduced nonlinear effects. It is likely that the group-delay ripple (GDR) of CFBG could bring about some effects on timing jitter since it changes the lumped dispersion of the grating. However, to our knowledge, no qualitative or quantitative analysis of this problem, has been published so far. In this letter, an analytical method is proposed to analyze the influence of GDR on timing jitter induced by SPM and IXPM in pseudo-linear transmission systems using CFBG as dispersion compensator.

The transmission function of the fiber grating is modeled as[9]

$$H(\omega) = \exp\left[j\frac{g}{2}\omega^2 - j\frac{\Gamma}{T_0^2}\cos(\omega T_0+\theta) + j\frac{\Gamma}{T_0^2}\right], \quad (1)$$

where g is the average lumped dispersion of the grating. Parameters Γ, $2\pi/T_0$, and θ are the amplitude, period, and phase of the dispersion ripple, respectively. Using the Jacobi-Anger expansion,

$$\exp(jz\cos\theta) = \sum_{n=-\infty}^{\infty} j^n J_n(z)\exp(jn\theta), \quad (2)$$

where $J_n(z)$ is the Bessel function of the first kind. Leaving out the terms containing high-order Bessel function, we get

$$\begin{aligned}&\exp\left[-j\frac{\Gamma}{T_0^2}\cos(\omega T_0+\theta)\right]\\ &\approx J_0(-\frac{\Gamma}{T_0^2}) + jJ_1(-\frac{\Gamma}{T_0^2})\exp(j\omega T_0 + j\theta)\\ &\quad -jJ_{-1}(-\frac{\Gamma}{T_0^2})\exp(-j\omega T_0 - j\theta). \end{aligned} \quad (3)$$

Substituting Eq. (3) into Eq. (1), then

$$\begin{aligned}H(\omega) &= \exp(j\frac{g}{2}\omega^2)\exp(j\frac{\Gamma}{T_0^2})\\ &\times\left\{J_0(-\frac{\Gamma}{T_0^2}) + jJ_1(-\frac{\Gamma}{T_0^2})\exp(j\omega T_0 + j\theta)\right.\\ &\left.-jJ_{-1}(-\frac{\Gamma}{T_0^2})\exp(-j\omega T_0 - j\theta)\right\}. \end{aligned} \quad (4)$$

For the sake of analyzing convenience, we set $\exp(j\frac{g}{2}\omega^2)$ as $H_1(\omega)$, and $J_0(-\frac{\Gamma}{T_0^2}) + jJ_1(-\frac{\Gamma}{T_0^2})\exp(j\omega T_0 + j\theta) - jJ_{-1}(-\frac{\Gamma}{T_0^2})\exp(-j\omega T_0 - j\theta)$ as $H_2(\omega)$. Then, $H(\omega)$ can be expressed as

$$H(\omega) = H_1(\omega)\exp(j\frac{\Gamma}{T_0^2})H_2(\omega). \quad (5)$$

Supposing that the electrical field of the signal input into CFBG is $u(L,t)$ and the average lumped dispersion of CFBG can compensate the accumulated dispersion exactly. L represents the position of CFBG. Then, the output signal after the grating is

$$\tilde{u}_{\mathrm{FBG}}(L,t) = \tilde{u}(L,t)H(\omega) = \exp(j\frac{\Gamma}{T_0^2})\tilde{u}(L,t)H_1(\omega)H_2(\omega), \quad (6)$$

where superscript '∼' represents Fourier transform, $\tilde{u}_{\mathrm{FBG}}(L,t)$ is the signal output from the grating.

The part of $\tilde{u}(L,t)H_1(\omega)$ denotes dispersion compensation of distorted pulses with ideal grating without GDR. We set its Fourier inverse transform as $u_D(L,t)$. When $\tilde{u}(L,t)H_1(\omega)$ multiplies with $H_2(\omega)$ in frequency area, the three parts of $H_2(\omega)$ make $u_D(L,t)$ be timing shifted by 0, $-T_0$ and T_0 with different amplitude, respectively. Then we get the time area expression of the output signal after compensating grating,

$$u_{\mathrm{FBG}}(L,t) = \exp(j\frac{\Gamma}{T_0^2})\left\{u_D(L,t)J_0(-\frac{\Gamma}{T_0^2}) + u_D(L,t-T_0)J_1(-\frac{\Gamma}{T_0^2})\exp(j\frac{\pi}{2}+j\theta) + u_D(L,t+T_0)J_{-1}(-\frac{\Gamma}{T_0^2})\exp(-j\frac{\pi}{2}-j\theta)\right\}. \quad (7)$$

If the input signal is Gaussian-shaped pulses with peak power of A_0^2 and 3-dB width of τ, the timing jitter after compensating grating can be obtained directly from Eq. (7) by using

$$\delta T = \left(\sqrt{\pi}A_0^2\tau\right)^{-1}\int_{-\infty}^{\infty} t\,|u|^2\,\mathrm{d}t. \quad (8)$$

To discuss the influence of GDR on timing jitter, we deduce the analytical expression of $|u_{\mathrm{FBG}}|^2$ and get

$$|u_{\mathrm{FBG}}(L,t)|^2 \approx |u_D(L,t)|^2 + |u_\Delta|^2, \quad (9)$$

where

$$|u_\Delta|^2 = \Gamma\mathrm{Re}\Big\{\frac{1}{T_0^2}[u_D(L,t)u_D^*(L,t-T_0)\exp(j\frac{\pi}{2}-j\theta) + u_D(L,t)u_D^*(L,t+T_0)\exp(j\frac{\pi}{2}+j\theta)]\Big\}, \quad (10)$$

the superscript '∗' denotes complex conjugating. During the above procedure, we leave out the terms containing the product of two first-order Bessel functions for their far less contribution to $|u_\Delta|^2$, compared with other terms. Numerical simulation shows that this approximation is reasonable. In fact, by assuming a sine distribution of GDR as

$$\tau_{\mathrm{GDR}}(f) = \tau_{\mathrm{p}}\sin(\frac{2\pi f}{f_{\mathrm{p}}}+\theta), \quad (11)$$

where τ_{GDR} stands for GDR, f_{p} and τ_{p} are the period and amplitude of GDR, respectively.

Then Eq. (10) could be rewritten as

$$|u_\Delta|^2 = \frac{\tau_{\mathrm{p}}}{2\pi}\mathrm{Re}\left\{f_{\mathrm{p}}[u_D(L,t)u_D^*(L,t-1/f_{\mathrm{p}})\exp(j\frac{\pi}{2}-j\theta) + u_D(L,t)u_D^*(L,t+1/f_p)\exp(j\frac{\pi}{2}+j\theta)]\right\}. \quad (12)$$

Substituting Eq. (9) into Eq. (8), we get the total timing jitter

$$\delta T = \left(\sqrt{\pi}A_0^2\tau\right)^{-1}\int_{-\infty}^{\infty} t\,|u_D|^2\,\mathrm{d}t + \left(\sqrt{\pi}A_0^2\tau\right)^{-1}\int_{-\infty}^{\infty} t\,|u_\Delta|^2\,\mathrm{d}t. \quad (13)$$

So the timing jitter influenced by the GDR of compensating grating is

$$\delta T_\Delta = \left(\sqrt{\pi}A_0^2\tau\right)^{-1}\int_{-\infty}^{\infty} t\,|u_\Delta|^2\,\mathrm{d}t. \quad (14)$$

Since $u_D(L,t)$ can be described as the sum of the injected signal and a nonlinear interference signal given by $\sum_{i,j,k}\Delta u_{i,j,k}(L,t)$[2], where $\Delta u_{i,j,k}(L,t)$ stands for the perturbation caused by the nonlinearity and its detail expression could be found in Ref. [2]. Timing jitter caused by SPM and IXPM when dispersion is compensated by CFBG with GDR could be calculated by Eqs. (11) and (13) when keeping only the SPM and IXPM terms of $\Delta u_{i,j,k}(L,t)$.

It can be concluded from Eqs. (10), (12)—(14) that the influence of GDR is not only relative to its amplitude, period and phase but also as function of the electrical field of signal and its delay when compensating with ideal CFBG. When GDR is taken into account, timing jitter induced by SPM and IXPM can be distinguished as two independent parts, one for ideal CFBG and the other for GDR. This makes it easy to distinguish the influence of GDR on timing jitter by wiping off the timing jitter of system with ideal CFBG compensator from that of system when taking GDR into account, and this is very useful in numerical simulation.

The following conclusions could be gotten through analyzing Eqs. (12) and (14).

Firstly, for fixed period and phase, the timing jitter induced by the GDR of compensating grating changes linearly with the amplitude of GDR, which means that compensating grating with larger delay ripple amplitude influences more acutely than that with smaller ripple amplitude. But it relies on the value of f_{p} and θ that whether this influence is depressing or increasing the total timing jitter.

Secondly, by setting the differential coefficient of δT_Δ with θ, we find that the optimal θ minimizing δT_Δ and results of minimized δT are as functions of f_{p}. Besides, there may exist special θ to make δT_Δ to be minus which results in decreasing of timing jitter and does well in transmission. To restrain timing jitter, there must be GDR with proper θ and proper range of f_{p}. This is because that the GDR changes the average lumped dispersion of the grating and different GDR parameters correspond to different dispersion value of the grating. When frequency shift between pulses induced by SPM and IXPM is finally converted into timing jitter by dispersion, different value of dispersion will result in different timing jitter.

Thirdly, since there is an integral in Eq. (14) and the integral function includes multiplication of $u_D(L,t)$ with its delay, if $u_D(L,t)$ is finite in time and $1/f_\mathrm{p}$ is larger than its duration, the integral function will be zero and GDR will do nothing with timing jitter. This means that if the input signal is Gaussian-shaped pulses with pulses amount of n and bit duration of T_b, when f_p is smaller than or equals $1/(nT_\mathrm{b})$, GDR will not affect the timing jitter and δT_Δ will turn to zero. And when $f_\mathrm{p} \to \infty$, $|u_\Delta|^2$ becomes zero in Eq. (12), this also makes δT_Δ be zero. These mean that when f_p is too small or too large the GDR will do nothing with timing jitter. This result is in agreement with the conclusion of Ref. [9].

Now we use numerical simulation of nonlinear Schrödinger equation with split-step Fourier method (SSFM) to validate these conclusions and take a deep insight of the influence of GDR on timing jitter induced by intrachannel interactions in systems using CFBG as dispersion compensator.

Two return-to-zero (RZ) modulated Gaussian pulses with 3-dB bandwidth of 5 ps and peak power of 20 mW are inputted into a transmission line of 65-km single mode fiber. And then the dispersed pulses are compensated with a CFBG, whose average lumped dispersion can compensate the accumulated fiber dispersion exactly. The transmission bite rate is 40 Gb/s corresponding to time slot T_b = 25 ps. The nonlinear coefficient and dispersion coefficient of the transmission fiber are 2 $\mathrm{W}^{-1}\cdot\mathrm{km}^{-1}$ and -20 $\mathrm{ps}^2\cdot\mathrm{km}^{-1}$, respectively. Fiber loss is not taken into account in our simulation, and the timing jitter is characterized by detecting the difference between the peak time positions of the two pulses. The timing jitter induced by GDR is calculated by wiping off the timing jitter of system with ideal CFBG compensator from the total timing jitter of system when taking GDR into account. Although the input signal is set to be two Gaussian-shaped pulses to simplify calculation, it does not influence the universality of the conclusions.

Figure 1 shows the graph of δT_Δ versus ripple amplitude. It indicates that the timing jitter increases linearly as ripple amplitude increases.

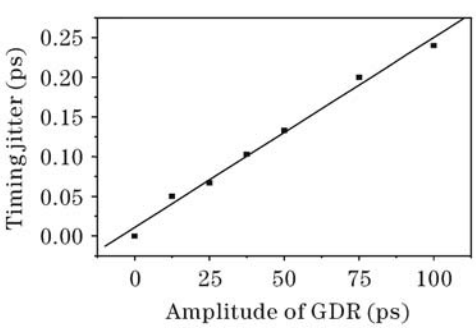

Fig. 1. Timing jitter induced by GDR as a function of ripple amplitude. Squares are results of numerical simulation, solid line is the numerical fitting. Phase = 0, period = 0.32 nm.

From Fig. 2 we can see that when period of GDR is smaller than 0.16 nm the total timing jitter of the system keeps constant to be equal to 6.71×10^{-13}, which is the timing jitter when GDR is not taken into account. Since $n = 2$ in the simulation, this is consistent with the analytical result that when f_p is smaller than or equal to $1/(nT_\mathrm{b})$ the GDR will not affect the timing jitter. And when period of GDR is larger than 32 μm which can be considered to be infinite in our simulation, the timing jitter is also 6.71×10^{-13}. Since the input pulse serial can be uninterrupted for real transmission systems and the ripple period of real used CFBG for 40-Gb/s transmission system could not be so large to reach the magnitude of micrometer, GDR will inevitably influence timing jitter induced by SPM and IXPM. Simulations also show that when ripple period is between 0.16 nm and 32 μm, timing jitter will be affected by GDR. Here we plot the curve for ripple period of 0.96 nm, which equals $3/T_\mathrm{b}$, as an example for the real case. This curve indicates that the total timing jitter is a function of GDR phase and GDR will enhance timing jitter for some ripple phase while decrease it for some others.

Since the relative phase between the center frequency of the pulse and the ripples is an important factor for determining the average lumped dispersion of the grating[9], we study the influence here. The ripple phase is changed and the center frequency of the pulse is fixed at 193.1 THz. Figure 3 is the graph of optimal GDR phase and its corresponding total timing jitter as a function of ripple period. The figure shows that the optimal phase to minimize the timing jitter is different for different value of ripple period. And their corresponding timing jitter possesses an average order of magnitude of lower than 10^{-14},

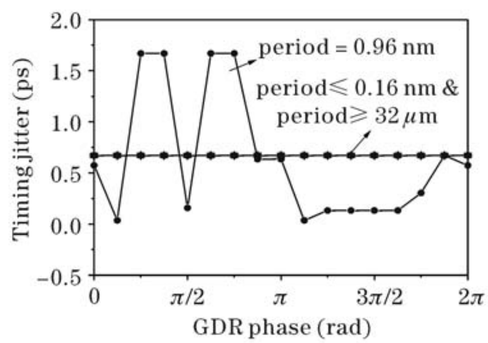

Fig. 2. Total timing jitter as a function of GDR phase, amplitude = 10 ps.

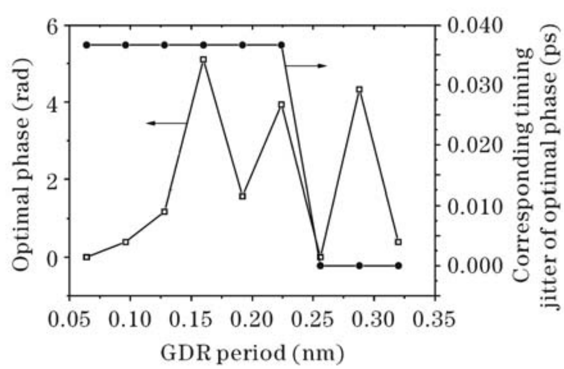

Fig. 3. Optimal phase and its corresponding total timing jitter as a function of GDR period, amplitude = 10 ps.

which is at least one order smaller than that of timing jitter when using ideal CFBG with no GDR, for the timing jitter is 6.71×10^{-13} with the above simulation parameters when GDR is not taken into account. This means that GDR does not always hurt signal transmission and restrict transmission distance, it maybe does good to transmission by weakening timing jitter for some proper ripple parameters. Then, we vary the carrier frequency of the pulse around 193.1 THz and study the total timing jitter as a function of the relative phase between the center frequency of the pulse and the ripples, with constant GDR phase. Actually, because IXPM induces different values of frequency shift to different pulses, the center frequencies of the two pulses inputted to the CFBG will not be the carrier frequency and are different to each other. But the frequency shift is negligible in comparison with the carrier frequency[3], so the center frequency of the pulse is approximated by the carrier frequency. The results are shown in Fig. 4. Both Figs. 3 and 4 show that there are points where the total timing jitter is zero, for different ripple phases and ripple periods. This means that by changing the carrier frequency of the pulse or the GDR phase, one can get proper relative phase between the center frequency of the pulse and the ripples. And this proper relative phase will result in totally cancellation of the timing jitter induced by intrachannel interactions. Such a conclusion can be understood like that the timing jitter after dispersion compensation is direct proportional to the average lumped dispersion of the CFBG[1], and the average lumped dispersion is as a cosine function of the relative phase between the center frequency of the pulse and the ripples[9]. Besides, it is reasonable that by using a proper dispersion map the timing jitter induced by intrachannel interactions can be suppressed[3]. Since the adjusting of the center frequency of the pulse or the change of the GDR phase will result in a change of the relative phase, both of them could change the average lumped dispersion of the CFBG, and thus introduce under or over compensation of the dispersion map. When such a change in dispersion map matches the condition of timing jitter cancellation, the timing jitter will be suppressed totally. In practice, the parameters of the GDR cannot be changed after fabrication. So, it is more effective to change the relative phase by adjusting the carrier frequency. Actually, as can be seen from Fig. 4, carefully adjusting the carrier frequency is necessary because a slight change in the relative phase can contribute to a significant decrease of timing jitter. For example, if the ripple period, ripple phase and ripple amplitude are 0.96 nm, 1.5π and 20 ps respectively, by changing the carrier frequency from 193.17 THz (1553.0 nm) to 193.14 THz (1553.3 nm) the total timing jitter can be suppressed from several hundred picoseconds to zero.

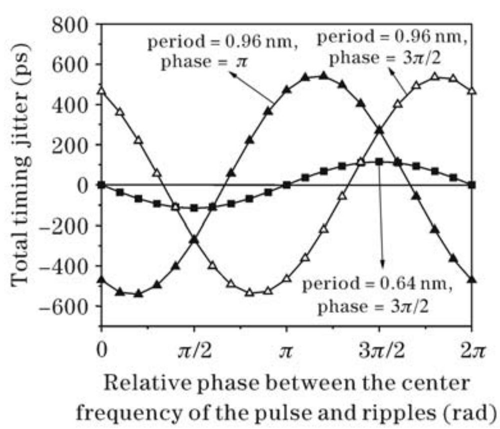

Fig. 4. Total timing jitter as a function of relative phase between the center frequency of the pulse and the ripples. The solid line represents zero total timing jitter. Amplitude = 20 ps.

The influence of GDR on timing jitter induced by SPM and IXPM in pseudo-linear transmission systems with dispersion compensated by CFBG is studied thoroughly through theoretical and numerical analysis. It is concluded that the influence of GDR on timing jitter changes linearly with the amplitude of GDR and whether it decreases or increases the timing jitter relies on the ripple period and ripple phase. Moreover, the timing jitter induced by SPM and IXPM could be suppressed totally by setting proper relative phase between the center frequency of the pulse and the ripples. In practice, by adjusting the carrier frequency of the pulse, the timing jitter can be reduced or cancelled out.

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