Entanglement dynamics of non-interacting two-qubit system under a squeezed vacuum environment

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Entanglement dynamics of two non-interacting atoms in a squeezed vacuum reservoir is studied. Several examples with different initial entangled states are investigated, and it is found that entangled atoms become disentangled faster in squeezed vacuum than in ordinary vacuum, and larger squeezing results in faster entanglement decay. The time evolution of the concurrence and the separability "distance" Λ can be used to explain this novel entanglement sudden death phenomenon.

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Entangled states are an indispensable ingredient of quantum computation and quantum information[1]. The entanglement between distant atoms can be prepared by detecting cavity decays[2,3], dipole-dipole interactions[4] and so on. However, quantum entanglement is fragile due to the interaction with an environment, and this issue is recognized as a fundamental obstacle in practical quantum computing and quantum information processing.

In recent years, controlling the evolution of entanglement between atoms interacting with the environment has received much attention[5,6]. It has been predicted that two initial entangled and afterwards not interacting atoms become separable (completely disentangled) within a finite time due to the coupling with the vacuum noise[7,8]. Especially, the decay rate of entanglement of two qubits is faster than local decoherent rate (spontaneous emission rate) of each atom. Otherwise, Ficek and Tanaś found an unusual entanglement revival after the disentanglement which results from the presence of collective damping[9]. In a sense, the collective behaviors seem to be beneficial to alleviate the disentanglement of the system. In general, the characteristic of the environment plays an important role in the evolution of multi-particle entanglement.

We are inspired by the above works and begin to investigate the dynamical details of the entanglement evolution in squeezed vacuum. In the present paper we study the entanglement dynamics of two non-interacting entangled qubits in a squeezed vacuum environment. We present the exact solutions of density matrix elements of the system, and analyze the entanglement dynamics of the different initial states using the concurrence and the separability "distance" Λ which can serve as the measurements of the bipartite entanglement and can explain the novel disentanglement phenomenon.

We consider the situation where the atoms A and B are coupled to squeezed vacuum reservoir, independently. Initially, the two atoms are entangled, then they are separated far enough that there is no direct interaction (for example dipole-dipole interaction) between them. The relationship between the ordinary vacuum and squeezed vacuum can be expressed by means of the squeezed operator[10]

$$|0\rangle_{\rm sq} = S(\xi)\,|0\rangle\,, \tag{1}$$

where $S(\xi) = \exp(\xi^* a_{k_0+k} a_{k_0-k} - \xi a^+_{k_0+k} a^+_{k_0-k})$ with $\xi = re^{i\theta}$, r being the squeezed parameter and θ being the reference phase for squeezed field. The time evolution of the two atoms interacting with squeezed vacuum is studied using the Lindblad form of the master equation,

$$\begin{aligned}\frac{\partial\rho}{\partial t} = \sum_{i=A,B} [&\frac{\Gamma_i}{2}(N+1)(2\sigma^i_-\rho\sigma^i_+ - \rho\sigma^i_+\sigma^i_- - \sigma^i_+\sigma^i_-\rho) \\ &+\frac{\Gamma_i}{2}N(2\sigma^i_+\rho\sigma^i_- - \rho\sigma^i_-\sigma^i_+ - \sigma^i_-\sigma^i_+\rho) \\ &-\Gamma_i M\sigma^i_-\rho\sigma^i_- - \Gamma_i M^*\sigma^i_+\rho\sigma^i_+],\end{aligned} \tag{2}$$

where $N = \sinh^2 r$ and $M = {\rm e}^{-i\theta}\sinh r\cosh r$ characterize squeezed vacuum and fulfill $|M|^2 \le N(N+1)$. For simplification, we only consider the ideal squeezing, i.e., $|M| = N(N+1)$. $\Gamma_A = \Gamma_B = \gamma$ are spontaneous emission rates of the two atoms. $\sigma^i_+ = |e\rangle_i\langle g|$ and $\sigma^i_- = |g\rangle_i\langle e|$ ($i = A, B$) are atomic raising and lowering operators, respectively. The two-qubit standard bases are given by

$$\begin{aligned}&|1\rangle = |e\rangle_A|e\rangle_B\,,\quad |2\rangle = |e\rangle_A|g\rangle_B\,, \\ &|3\rangle = |g\rangle_A|e\rangle_B\,,\quad |4\rangle = |g\rangle_A|g\rangle_B\,.\end{aligned} \tag{3}$$

In order to determine the entanglement degree between the atoms, we use concurrence which is widely accepted as a measurement of entanglement in a two-qubit system[11,12],

$$C = \max(0, \Lambda),\ \Lambda \equiv \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, \tag{4}$$

where the quantities λ_i are the eigenvalues in decreasing order of the matrix

$$\xi = \rho(\sigma^A_y \otimes \sigma^B_y)\rho^*(\sigma^A_y \otimes \sigma^B_y), \tag{5}$$

where ρ^* denotes the complex conjugation of ρ in the standard basis, $\sigma^{A(B)}_y$ are usual Pauli matrices expressed in the same basis. The concurrence varies from $C = 0$ for a product (disentangled) state to $C = 1$ for a maximally entangled state. It is apparent that the concurrence is not negative for all the states (entangled and disentangled), but the quantity Λ can be negative. Furthermore, if $\Lambda(\rho)$ is strictly negative, it implies that ρ must be both mixed and separable[11,12].

For an initial state in the more specialized class of bipartite "X" density matrix[13,14]

$$\rho(0) = \begin{pmatrix} a & & & w \\ & b & z & \\ & z^* & c & \\ w^* & & & d \end{pmatrix}, \tag{6}$$

the time evolution does not change the structure of density matrix, i.e., all initial zero elements of the matrix are always zero. The concurrence of the above density matrix can be expressed by

$$C[\rho(t)] = \max(0, \Lambda_1, \Lambda_2),$$

$$\Lambda_1 = 2(|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}),\ \Lambda_2 = 2(|\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}). \tag{7}$$

For the case of squeezed vacuum, we list the time-dependent solutions of the density matrix elements related to the concurrence as

$$\rho_{11}(t) = (2N+1)^{-2}\{[N+(N+1)x]^2 a + N^2(1-x)^2 d$$
$$+N(1-x)[N+(N+1)x](b+c)\}, \tag{8a}$$

$$\rho_{22}(t) = (2N+1)^{-2}\{(N+1)(1-x)[N+(N+1)x]a$$
$$+N(1-x)(Nx+N+1)d$$
$$+(Nx+N+1)[N+(N+1)x]b$$
$$+N(N+1)(1-x)^2 c\}, \tag{8b}$$

$$\rho_{33}(t) = (2N+1)^{-2}\{(N+1)(1-x)[N+(N+1)x]a$$
$$+N(1-x)(Nx+N+1)d + N(N+1)(1-x)^2 b$$
$$+(Nx+N+1)[N+(N+1)x]c\}, \tag{8c}$$

$$\rho_{44}(t) = (2N+1)^{-2}[(N+1)^2(1-x)^2 a$$
$$+(Nx+N+1)^2 d$$
$$+(N+1)(1-x)(Nx+N+1)(b+c)], \tag{8d}$$

$$\rho_{23}(t) = \frac{x}{2}[(\cosh 2y+1)z + (\cosh 2y-1)z^*$$
$$-\sinh 2y(\sqrt{\frac{M}{M^*}}w + \sqrt{\frac{M^*}{M}}w^*)]. \tag{8e}$$

$$\rho_{14}(t) = \frac{x}{2}[(\cosh 2y+1)w + \frac{M^*}{M}(\cosh 2y-1)w^*$$
$$-\sqrt{\frac{M^*}{M}}\sinh 2y(z+z^*)], \tag{8f}$$

here $x = \exp[-(2N+1)\gamma t]$ and $y = |M|\gamma t$. Apparently, the time evolutions of the diagonal elements of the density matrix are affected only by N, i.e., squeezing parameter r, while $\rho_{14}(t)$ and $\rho_{23}(t)$ are dependent on not only N, but also $M/M^* = \mathrm{e}^{-2i\theta}$ which includes a reference phase θ. It is easily found that steady-state solutions are $\rho_{11}(\infty) = N^2/(2N+1)^2$, $\rho_{22}(\infty) = \rho_{33}(\infty) = N(N+1)/(2N+1)^2$, and $\rho_{44}(\infty) = (N+1)^2/(2N+1)^2$. In contrast to the common vacuum case (no squeezing), in which the final state of the two-atom system must be their ground state $|4\rangle$, the populations of $|1\rangle$, $|2\rangle$, and $|3\rangle$ of the total system are not zero, which is dependent on the degree of the squeezing. Here, we are interested in the influences of the squeezing parameter on the two-particle entanglement when the two atoms interact independently with squeezed vacuum. It is difficult to give a simple analytic expression for the concurrence and the quantity Λ of two-atom entanglement in squeezed vacuum, so we study the details with the help of the numerical simulations (θ is set to zero for all figures below).

First, we study a special class of entangled states with a single parameter λ,

$$\rho(0) = \frac{1}{3}\begin{pmatrix} \lambda & & & \\ & 1 & 1 & \\ & 1 & 1 & \\ & & & 1-\lambda \end{pmatrix}. \tag{9}$$

The dependence of the concurrence of this state on the parameter λ is shown in Fig. 1. When $N = 0$, finite-time disentanglement takes place for $\lambda > 1/3$; in other words, for a certain parameter regime, entanglement of the initial state only disappears asymptotically[7]. However, when $N = 0.1$, entanglement goes to zero for all the values of λ. This surprising result shows that the squeezing environment prevents the existence of entanglement drastically and the disentanglement is little related to the initial state.

Next, we study two examples, which are the well-known Bell states $|\Psi^\pm\rangle = (|eg\rangle \pm |ge\rangle)/\sqrt{2}$ and $|\Phi^\pm\rangle = (|ee\rangle \pm |gg\rangle)/\sqrt{2}$. The time evolutions of the concurrence and two quantities $\Lambda_{1,2}$ are plotted in Fig. 2. Similar to the above single-parameter model, the concurrence goes to zero in both cases of two kinds of Bell states. Disentanglement time in squeezed vacuum is much shorter than that in ordinary vacuum. Entanglement dynamics of $|\Psi^\pm\rangle$ is determined by Λ_1 which is positive before

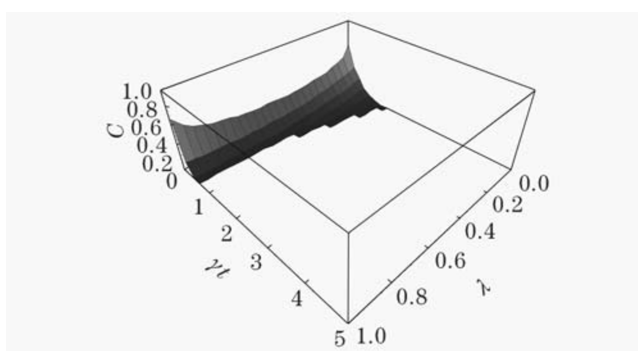

Fig. 1. Entanglement decay via vacuum noise starting from the initial entangled state (9) with λ between 0 and 1. Finite-time complete disentanglement occurs for all the values of λ. Squeezing parameter N is 0.1.

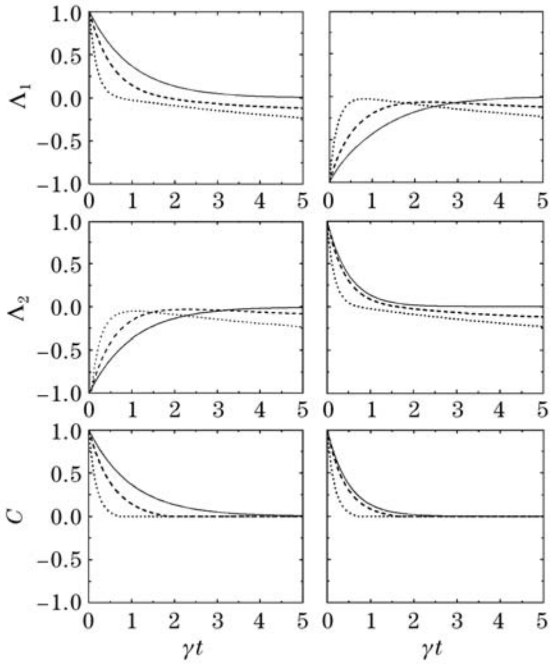

Fig. 2. Concurrence and separability quantity Λ of Bell states $|\Psi^{\pm}\rangle$ (left) and $|\Phi^{\pm}\rangle$ (right). Solid, dashed, and dotted lines represent the time evolutions for different squeezing $N = 0$, 0.1, 1, respectively.

disentanglement, and Λ_2 is negative for all the time. It is interesting that it is opposite to $|\Phi^{\pm}\rangle$, in which Λ_2 is positive in the beginning and Λ_1 keeps negative. This result can be explained from Eq. (7) that when the initial state is $|\Psi^{\pm}\rangle$, $\rho_{11}(0) = \rho_{44}(0) = \rho_{14}(0) = 0$ results in $\Lambda_1(0) > 0$ and $\Lambda_2(0) < 0$. Afterwards the values of ρ_{11}, ρ_{44}, ρ_{14} begin to be nonzero, Λ_1 decreases along with the time and Λ_2 becomes close to zero but cannot be positive. When the initial state is $|\Phi^{\pm}\rangle$, the case is just the opposite which can be observed clearly in Fig. 2. Another remarkable phenomenon is that the time evolution of the concurrence for the two classes of Bell states is not identical completely, although the trends of them are similar. It is related to non-identical evolutions of the diagonal elements of density matrix. Compared with $|\Psi^{\pm}\rangle$, $|\Phi^{\pm}\rangle$ is more sensitive to the atomic decay whether the environment is ordinary vacuum or squeezed vacuum. Qualitatively, the entanglement decay of $|\Phi^{\pm}\rangle$ is only dependent on the influence of the vacuum noise on the combined state when both atoms stand on excited state $|ee\rangle$, not on the ground state $|gg\rangle$. On the other hand, $|\Psi^{\pm}\rangle$ is coherent superposition of two product states $|eg\rangle$ and $|ge\rangle$. These result in the difference of finite-time disentanglement between Bell states.

In summary, we have investigated the role of squeezed vacuum playing in the entanglement dynamics of two-atom system in which the atoms interact with their reservoirs independently. The single-parameter density matrix and Bell states are checked and it is found that squeezing quickens the disentanglement of the system, and the separability quantity Λ is helpful to understand the details of entanglement sudden death. These may be beneficial to a deep understanding of the relation between decoherence and disentanglement and will be of importance for both the foundation of quantum mechanics and quantum information processing.

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