Application of phase-diverse phase retrieval to wavefront sensing in non-connected complicated pupil optics

Heng Mao (毛 珩), Xiao Wang (王 潇), and Dazun Zhao (赵达尊)

School of Information Science and Technology, Beijing Institute of Technology, Beijing 100081

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Baseline algorithm, as a tool in wavefront sensing (WFS), incorporates the phase-diverse phase retrieval (PDPR) method with hybrid-unwrapping approach to ensure a unique pupil phase estimate with high WFS accuracy even in the case of high dynamic range aberration, as long as the pupil shape is of a convex set. However, for a complicated pupil, such as that in obstructed pupil optics, the said unwrapping approach would fail owing to the fake values at points located in obstructed areas of the pupil. Thus a modified unwrapping approach that can minimize the negative effects of the obstructed areas is proposed. Simulations have shown the validity of this unwrapping approach when it is embedded in Baseline algorithm.

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The phase retrieval is an inverse problem in optics, and generally involves estimating a complex-valued phase distribution from known intensity distributions at some properly selected planes. The original phase retrieval algorithm was constructed by Gerchberg and Saxton in 1972, known as Gerchberg-Saxton (G-S) algorithm[1]. It only uses the Fourier-transform relationship between in-focus image plane and pupil plane and suffers from the problem of non-unique solution. In the following decades, phase retrieval algorithm was modified by Fienup and Gonsalves *et al.*[2−4], but the non-unique problem remains unsolved. In 1993, Roddier *et al.*[5] proposed a modified Misell algorithm to recover the exact spherical aberration of the Hubble Space Telescope. In that algorithm, the phase-diverse method was incorporated with the G-S algorithm for the first time though the beacon used there was still supposed to be a point object. Then in the late 1990s, Jet Propulsion Laboratory (JPL) proposed the modified G-S (MGS) algorithm, which was classified as an approach of phase-diverse phase retrieval (PDPR) and served as the preferred wavefront sensing (WFS) methodology in the Next Generation Space Telescope (namely James Webb Space Telescope)[6]. It uses the in-focus image and some de-focus images to make a joint estimate of the wavefront after the G-S iteration of each image, so it greatly enhances the speed of convergence and successfully ensures the uniqueness of the estimate. But once the wavefront's peak-valley (PV) value exceeds one wavelength, corresponding to a phase PV of 2π, the phase ambiguity problem may appear. So JPL further modified the MGS algorithm and proposed the Baseline algorithm[7], in which some phase unwrapping algorithms were introduced to resolve the 2π ambiguity problem.

Essentially, the algorithm based on G-S iteration is a geometric projection algorithm which is only robust on the convex sets. But in certain cases, the pupil shape is of nonconvex sets, for example, is obstructed by gaps due to the segmented primary mirror or by shadows resulted from mechanical supports in the optical path, as shown in Fig. 1. We define this kind of pupil as a complicated pupil. Based on the overall continuity of phase at the pupil, all the complicated pupils can be divided into two types, where type I has an overall continuous phase distribution across the whole pupil, only the values in the obstructed areas are lost; type II loses the overall continuity but keeps the continuity within each mirror segment. For aberrations in small dynamic range, usually less than one wavelength, the Baseline algorithm still works no matter which type of the pupil belongs to. But in the case of high dynamic range, the Baseline algorithm would fail due to the phase-unwrapping problem across the obstructed areas.

In this paper, considering the high dynamic range phase-unwrapping on the type I pupil, we introduce a modified PDPR algorithm based on Baseline algorithm. A flowchart of the Baseline algorithm is presented as shown in Fig. 2. It makes use of the pupil image and a few defocus images. Each defocus image has its own G-S iteration, called inner iteration, and each inner iteration will produce a wrapped phase estimate for the pupil. After the defocus phase corrections ($\theta_{\mathrm{div}\,i}$, known *a priori*), all these wrapped phase estimates $\theta_{\mathrm{output}\,i}$ will respectively be experienced by a hybrid-unwrapping to produce respective unwrapped phase estimate point by point. Then a weighted average is obtained, and serves as the starting phase distribution for the next inner iterations. The process is repeated several times, resulting in hopefully an estimate very close to the true wavefront.

The hybrid-unwrapping approach includes raster unwrapping, minimum L^p-Norm unwrapping and sometimes other path-following unwrapping[8]. The raster unwrapping algorithm is the simplest one but is quite

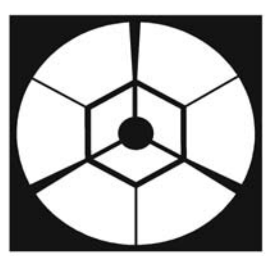

Fig. 1. Outline of a complicated pupil.

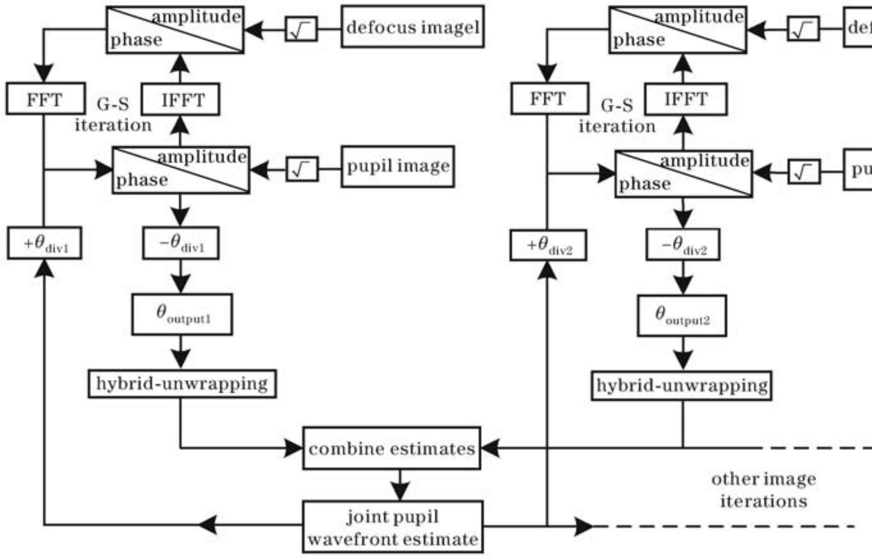

Fig. 2. Flowchart of the Baseline algorithm.

vulnerable by the residues (the obvious uncontinuous points in the discrete phase map). Differently, the path-following unwrapping algorithm intelligently selects some unwrapping paths to avoid passing through the residues. The minimum L^p-Norm unwrapping algorithm (generally $p = 2$) is essentially an unweighted least-squares algorithm so its solution is smoother than the former two algorithms.

Now we take a typical complicated pupil of type I (see Fig. 3) as an example to introduce the proposed modified PDPR algorithm. On the pupil, there is a central circular hole to allow the light reflected back from a secondary mirror passing through, and there are also three obstructed areas caused by the shadows of secondary mirror supports. So the whole pupil is divided into three separated segments, Seg1, Seg2, and Seg3. When the original Baseline algorithm shown in Fig. 2 is applied to this case, the hybrid-unwrapping processes, which should use the phase output of G-S iterations, including the fake values in the obstructed areas, to produce a continuous unwrapped estimation, would lead to the stagnation of the overall iteration convergence and the failure of the phase retrieval. We attribute it to the use of fake phases

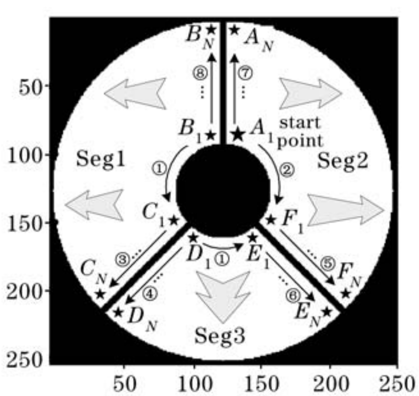

Fig. 3. A complicated pupil of type I.

in the obstructed areas during unwrapping. Thus, while we still need use the raster unwrapping to accelerate the convergence, the unwrapping paths should be carefully selected to prevent or minimize the negative effects of fake wrapped values.

Based on this understanding, a modified PDPR algorithm is proposed. The basic points are: the flowchart shown in Fig. 2 is adopted, the only modification is on the selection of raster unwrapping paths, and when a selected path passes through an obstructed area, a process of phase transition across that area instead of using any phase values will be operated. A more detailed step by step description is as follows.

Step 1: A set of reference points located at pairwise across and nearby the obstructed areas are selected (A_1, B_1; A_N, B_N; etc., Fig. 3), and the unwrapped phase transition between two points in a pair is implemented by using a proposed method called least difference phase transition (LDPT), this method requires that the two points in a pair, such as A_1 and B_1, are selected such that they have minimum possible distance between them, and the phase transition from A_1 to B_1 would guarantee a least difference between their unwrapped phases, i.e., if the wrapped phase at B_1 is φ'_{B_1}, the unwrapped phase at A_1 is φ_{A_1}, then the unwrapped phase at B_1 (φ_{B_1}) will be $\varphi_{B_1} = \varphi'_{B_1} + m \cdot 2\pi$, where m is an integer that would minimize the difference of $|\varphi_{B_1} - \varphi_{A_1}|$.

Step 2: A set of unwrapping paths are set up. For the pupil of Fig. 3, they could be: Path 1: A_1 TR B_1 TO C_1 TR D_1 TO E_1, where 'TR' means transition, 'TO' means ordinary unwrapping path. Path 2: A_1 TO F_1, Path 3: C_1 TO C_N, Path 4: D_1 TO D_N, Path 5: F_1 TO F_N, Path 6: E_1 TO E_N, Path 7: A_1 TO A_N, and Path 8: B_1 TO B_N.

Step 3: Starting from $A_1$and along those eight paths, the unwrapped phases of all points at those paths are obtained, the correctness of these phases would depend on whether residues exist in those paths or not, the presence of residue points can be checked by calculating the phase differences of the paired points at the ends of those paths, such as E_1 and F_1; E_N and F_N; etc., to see if they are greater than π. Anyway, with the increase of the number of iterations, the number of residues would be reduced to zero.

Step 4: Unwrap the phases of all points within three closed segments respectively by setting ordinary horizontal and/or vertical paths starting from the unwrapped points at the eight paths.

The simulation is performed under the following conditions. The sampling sizes for the pupil and the images are all 256×256 points. The assumed aberrated wavefront is created using the first 15 terms of Zernike polynomials, so it is dominated by low spatial frequency components.

The simulation results given here are for a typical case shown in Fig. 4, where Fig. 4(a) shows the pupil shape (same as that in Fig. 3) and the assumed phase distribution with a PV of 35.7 radians (or 5.68λ at 632.8 nm), Fig. 4(b) shows the intensity map at the focal plane, and two intensity maps selected arbitrarily from a total of four defocus planes are shown in Figs. 4(c) and (d). The PDPR algorithm starts from the five known intensity maps to try to recover the assumed pupil phase.

In Fig. 4(a), the width of the obstructed gap is 5 pixels and may extend to 7 pixels as seen by an unwrapping path, the diameter of obstructed circle is 70 pixels, the defocus distances are ± 3.0 and ± 5.0 mm, the $F^{\#}$ number of the optics is 10, all the simulated intensity data have an accuracy of 12 bits and Gaussian noises with a variance of 8 gray-levels are added therein.

The intermediate and final results for one defocus plane section (Fig. 2) are shown in Fig. 5. Figure 5(a) is a phase estimation (wrapped) produced by the first inner G-S iteration, where fake values in obstructed areas are easily seen; in Fig. 5(b), after original hybrid-unwrapping with 50 outer iterations, the convergence is poor because of the stagnation; Fig. 5(c) is the final unwrapping result based on Fig. 5(b), the recovering is failed; by modified PDPR with 40 outer iterations, the residues have decreased to zero, as shown in Fig. 5(d); Fig. 5(e) is the final unwrapping result based on Fig. 5(d); and Fig. 5(f) is the phase difference between the joint estimate of Fig. 5(e) produced respectively by four defocus plane sections and Fig. 4(a), with a root-mean-square (RMS) difference of 0.16 rad ($\lambda/40$).

Many other assumed aberrations have also been simulated for the complicated pupil shown in Fig. 3, and the results are similar to those shown in Fig. 5 and have RMS difference of $\lambda/35 - \lambda/50$. Thus the validity of the LDPT method and the modified PDPR are confirmed. The appropriate algorithm for type II pupil in high dynamic case will be discussed elsewhere.

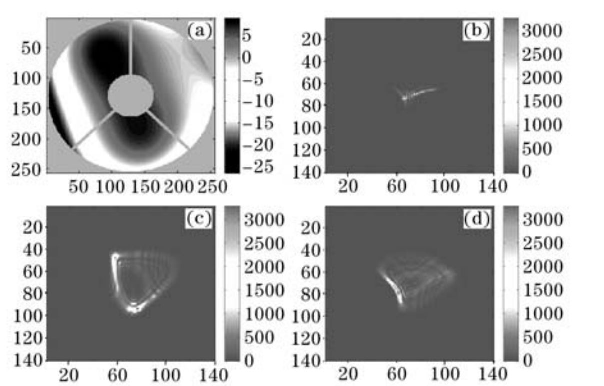

Fig. 4. (a) Assumed phase map in radian at pupil plane, and intensity maps in gray level at (b) focal plane, (c) defocus plane1, and (d) defocus plane2.

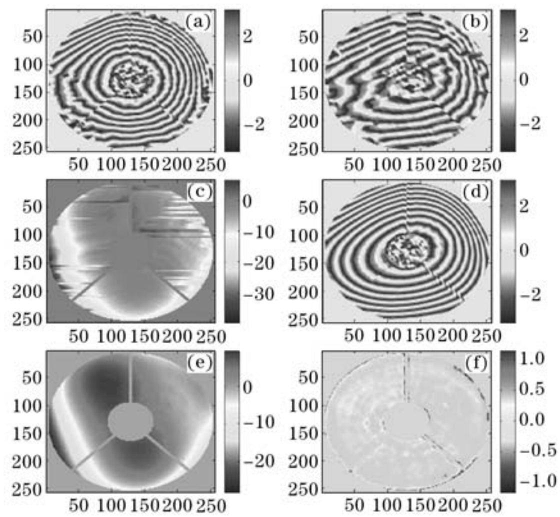

Fig. 5. Phase maps for the section of defocus plane1, resulted from iteration and unwrapping (in radian). (a) Wrapped phase estimation after the first G-S iteration; (b) final wrapped phase estimation when original Baseline unwrappings were used in outer iterations (stagnated); (c) final unwrapped phase estimation from (b), failed; (d) final wrapped phase estimation when the modified unwrappings were used in outer iterations; (e) final unwrapped phase estimation from (d), succeeded; (f) phase difference between (e) averaged and Fig. 4(a) with piston excluded (RMS = 0.16 rad).

In conclusion, by taking advantage of the overall phase continuity and avoiding the use of any phase values in the obstructed areas, the proposed LDPT method has successfully implemented a smooth unwrapped phase transition across the narrow obstructed areas in type I complicated pupil with high dynamic range aberrations. When combined with existing unwrapping techniques, the modified hybrid-unwrapping approach may make the Baseline algorithm be an effective WFS artifice for type I pupils.

H. Mao's e-mail address is alexmaomao@bit.edu.cn.

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