Whiteness formula in CIELAB uniform color space

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Many attempts have been made to standardize the calculation of whiteness. Whiteness formulas currently in use satisfactorily characterize the appearance of commercial whiteness. However, they have poor correlations with the observers' evaluations, and are often unsuccessful in assessing tinted white samples. A whiteness formula in the CIELAB uniform color space is developed in this paper. Several whiteness formulas are analyzed and compared. The experimental results show that the whiteness formula in the CIELAB uniform color space agrees well with the visual ranking, and it is superior to the CIE whiteness formula and the others in visual correlativity, uniformity and applicability.

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With the advent of fluorescent whitening agents, the general idea of whiteness has changed. Many whiteness formulas for evaluating whiteness of white materials have been proposed. Although whiteness is an object of study for a long time, there is little agreement by users. The CIE whiteness formula W_{CIE10} was recommended as an assessment method of white materials in 1986[1]. However, it has been found that the CIE whiteness formula is poorly correlated to visual estimation for many white samples, and its white region is too narrow[2−5]. Whiteness formula W_{G} in CIELAB uniform color space was obtained by transforming CIEXYZ into CIELAB color space[6]. The W_{G} values in CIELAB are nearly the same numeric values in CIEXYZ. Also it is not recommended to evaluate dingy and shaded white fabrics. Differences on a^* values (red-green) have no influence on final whiteness value. It has the same limitation of CIE formulas. A modified CIE whiteness formula W_{10} in CIEXYZ dealing with the tint and the excitation purity was proposed[7]. The formula can be divided into two types, one type serves for an in-base point sample, and the other type serves for an out-base point sample. The base point is the chromaticity point on the baseline that is obtained by $5Y_{10} - 275$ for the luminance factor of each sample. It enlarges the region of evaluating white on the color space, but its whiteness index is non-uniform in vision. Whiteness W_{uv} in the CIELUV uniform color space was also developed[8−10]. It improves on visual correlativity, visual uniformity and applicability. Since CIELAB uniform color space is used widely, it is urgent to establish a whiteness formula in CIELAB uniform color space that has well correlations with observers' evaluations.

The whiteness index in the CIELUV uniform color space is given by[10]

$$W_{\mathrm{uv}} = W_{\mathrm{H}} - 2(T_{\mathrm{uv}})^2,\ 40 < W_{\mathrm{H}} < 3.37L^*_{10} - 185.35, \quad (1)$$

$$W_{\mathrm{uv}} = P_{\mathrm{uv}} - 2(T_{\mathrm{uv}})^2,\ W_{\mathrm{H}} > 3.37L^*_{10} - 185.35, \quad (2)$$

where $W_{\mathrm{H}} = L^*_{10} + 260(u'_{n,10} - u'_{10}) + 1294(v'_{n,10} - v'_{10})$, $P_{\mathrm{uv}} = 5.74L^*_{10} + 260(u'_{10} - u'_{n,10}) + 1294(v'_{10} - v'_{n,10}) - 382.73$, $T_{\mathrm{uv}} = 1294(u'_{n,10} - u'_{10}) - 260(v'_{n,10} - v'_{10})$. u'_{10}, v'_{10} are the chromaticity coordinates in the CIELUV uniform color space of sample, $u'_{n,10}$, $v'_{n,10}$ are the chromaticity coordinates of the perfect diffuser, and L^*_{10} is the lightness of sample for CIE 10° standard observer and CIE standard illuminant D65.

Idea of establishing the whiteness formula in CIELAB uniform color space is that Eqs. (1) and (2) in CIELUV uniform color space are transformed into ones in CIELAB uniform color space. The key is to get approximate linear transform between (u', v') and (a^*, b^*) near the perfect diffuser.

It is known that the coordinates (u', v') in the CIELUV uniform color space are defined as

$$u' = 4X/(X + 15Y + 3Z), \quad (3)$$

$$v' = 9Y/(X + 15Y + 3Z), \quad (4)$$

where X, Y, and Z are the tristimulus values of the sample.

The coordinates (a^*, b^*) in the CIELAB uniform color space are defined as

$$L^* = 116(Y/Y_n)^{1/3} - 16, \quad (5)$$

$$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}], \quad (6)$$

$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}], \quad (7)$$

where X_n, Y_n, and Z_n are the tristimulus values of the perfect diffuser under CIE standard illuminant D65.

The approximate linear transforms between (u', v') and (a^*, b^*) near the perfect diffuser are

$$u' - u'_n \approx \left(\frac{\partial u'}{\partial X}\frac{\partial X}{\partial a^*} + \frac{\partial u'}{\partial Y}\frac{\partial Y}{\partial a^*} + \frac{\partial u'}{\partial Z}\frac{\partial Z}{\partial a^*}\right) a^* + \left(\frac{\partial u'}{\partial X}\frac{\partial X}{\partial b^*} + \frac{\partial u'}{\partial Y}\frac{\partial Y}{\partial b^*} + \frac{\partial u'}{\partial Z}\frac{\partial Z}{\partial b^*}\right) b^*, \quad (8)$$

$$v' - v'_n \approx \left(\frac{\partial v'}{\partial X}\frac{\partial X}{\partial a^*} + \frac{\partial v'}{\partial Y}\frac{\partial Y}{\partial a^*} + \frac{\partial v'}{\partial Z}\frac{\partial Z}{\partial a^*}\right) a^* + \left(\frac{\partial v'}{\partial X}\frac{\partial X}{\partial b^*} + \frac{\partial v'}{\partial Y}\frac{\partial Y}{\partial b^*} + \frac{\partial v'}{\partial Z}\frac{\partial Z}{\partial b^*}\right) b^*, \quad (9)$$

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where $\frac{\partial u'}{\partial X} = \frac{60Y_n+12Z_n}{(X_n+15Y_n+3Z_n)^2}$, $\frac{\partial u'}{\partial Y} = -\frac{60X_n}{(X_n+15Y_n+3Z_n)^2}$, $\frac{\partial u'}{\partial Z} = -\frac{12X_n}{(X_n+15Y_n+3Z_n)^2}$, $\frac{\partial X}{\partial a^*} = \frac{3X_n}{500}\left(\frac{a_n^*}{500}+\frac{L_n^*+16}{116}\right)^2$, $\frac{\partial Y}{\partial a^*} = 0$, $\frac{\partial Z}{\partial a^*} = 0$, $\frac{\partial X}{\partial b^*} = 0$, $\frac{\partial Y}{\partial b^*} = 0$, $\frac{\partial Z}{\partial b^*} = -\frac{3Z_n}{200}\left(\frac{L_n^*+16}{116}-\frac{b_n^*}{200}\right)^2$, $\frac{\partial v'}{\partial X} = -\frac{9Y_n}{(X_n+15Y_n+3Z_n)^2}$, $\frac{\partial v'}{\partial Y} = \frac{9X_n+27Z_n}{(X_n+15Y_n+3Z_n)^2}$, $\frac{\partial v'}{\partial Z} = -\frac{27Y_n}{(X_n+15Y_n+3Z_n)^2}$.

By substituting the data of the perfect diffuser for CIE 10° standard observer under CIE standard illuminant D_{65} ($X_{n,10} = 94.8$, $Y_{n,10} = 100$, $Z_{n,10} = 107.3$, $u'_{n,10} = 0.1930$, $v'_{n,10} = 0.4601$, $L^*_{n,10} = 100$, $a^*_{n,10} = 0$, $b^*_{n,10} = 0$) into Eqs. (8) and (9), the approximate linear transforms between (u'_{10}, v'_{10}) and (a^*_{10}, b^*_{10}) near the perfect diffuser can be obtained as

$$u'_{10} - u'_{n,10} = 1.1285 \times 10^{-3} a^*_{10} + 5.6411 \times 10^{-4} b^*_{10}, \tag{10}$$

$$v'_{10} - v'_{n,10} = -1.3935 \times 10^{-4} a^*_{10} + 1.1828 \times 10^{-3} b^*_{10}. \tag{11}$$

By substituting Eqs. (10) and (11) into Eqs. (1) and (2), the whiteness formula in CIELAB uniform color space can be obtained as

$$W_{\text{LAB}} = W_{ab} - 2(T_{ab})^2,\ 40 < W_{ab} < 3.37L^*_{10} - 191, \tag{12}$$

$$W_{\text{LAB}} = P_{ab} - 2(T_{ab})^2,\ W_{ab} > 3.37L^*_{10} - 191, \tag{13}$$

where $W_{ab} = L^*_{10} - 0.1131a^*_{10} - 1.6772b^*_{10}$, $P_{ab} = 5.74L^*_{10} + 0.1131a^*_{10} + 1.6772b^*_{10} - 382.73$, and $T_{ab} = -1.4965a^*_{10} - 0.4224b^*_{10}$. When sample's whiteness W_{LAB} is less than 40, it is considered that the sample is not white color.

CIBA-GEIGY white plates, whitened fabric samples, and whitened pottery samples are used in our experiments. Their whiteness indices are measured with the color measuring and matching system (Datacolor SF600 plus). Whiteness indices by different formulas and the visual estimation for same samples are analyzed and compared.

The experimental results show that five whiteness formulas (W_{CIE10}, W_{G}, W_{10}, W_{uv}, and W_{LAB}) agree well with the visual ranking VR for a great deal of white samples. It is shown that these whiteness formulas are adequate for general use. Some typical examples are shown in Tables 1 – 3.

Table 1. Whiteness Values and *VR* of CIBA-GEIGY White Plates

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
1	41.84	42.18	10.40	67.60	78.17	−3.07	59.34	12
2	52.60	52.93	39.24	78.33	82.15	−2.02	73.99	11
3	59.83	60.06	54.35	83.40	84.79	−1.30	81.41	10
4	71.82	72.01	71.82	88.70	89.31	0.03	89.31	9
5	82.51	82.57	80.15	90.77	93.28	0.84	91.85	8
6	90.40	90.36	87.69	94.37	96.12	0.85	94.69	7
7	96.02	95.94	93.25	96.95	98.23	0.82	96.89	6
8	104.00	103.86	102.23	101.00	101.10	0.53	100.55	5
9	110.24	110.09	109.41	103.63	103.39	0.20	103.31	4
10	118.58	118.42	116.79	106.87	106.39	0.33	106.17	3
11	120.64	120.49	118.57	107.69	107.17	0.36	106.91	2
12	130.94	130.86	127.28	111.84	111.18	0.53	110.62	1

Table 2. Whiteness Values and *VR* of 4 Whitened Fabrics

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
1	80.90	81.13	77.80	89.35	92.72	1.01	90.69	1
2	76.92	80.14	71.57	85.58	92.38	1.34	88.79	2
3	59.23	65.98	56.79	82.26	86.99	−1.41	83.00	3
4	49.05	61.12	42.85	75.03	85.15	−2.09	76.40	4

Table 3. Whiteness Values and *VR* of 2 Groups of Whitened Potteries

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
A_1	79.71	79.82	79.37	91.46	92.07	0.28	91.91	1
A_2	74.73	74.54	73.61	88.47	89.92	0.57	89.27	2
A_3	70.74	70.30	69.72	86.85	88.25	0.51	87.73	3
B_1	77.33	77.25	75.56	88.93	90.99	0.76	89.83	1
B_2	73.56	73.26	71.83	87.34	89.40	0.73	88.34	2
B_3	69.32	67.51	60.39	86.00	86.65	0.43	86.28	3

Table 4. Whiteness Values and *VR* of 4 Pairs of Fluorescent Whitening Fabrics

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
C_1	178.08	178.94	176.24	129.31	129.57	−0.05	129.57	High
C_2	178.02	179.01	170.88	123.70	129.43	−1.00	127.42	Low
D_1	172.38	173.25	171.13	127.71	126.75	−0.54	126.16	High
D_2	172.31	173.06	168.13	124.48	127.09	1.73	121.13	Low
E_1	171.71	174.04	171.09	128.01	127.77	0.83	126.41	High
E_2	171.37	172.17	162.84	119.56	126.74	−1.19	123.90	Low
F_1	170.88	171.47	170.54	127.89	126.75	0.49	126.27	High
F_2	171.37	172.17	162.84	119.56	127.14	1.48	123.90	Low

Table 5. Whiteness Values and *VR* of 4 Fluorescent Whitening Fabrics

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
1	185.49*	186.99*	117.83	133.24	131.56	0.17	127.61	1
2	184.93*	186.57*	100.76	124.24	131.17	−1.24	122.98	2
3	190.28*	192.21*	78.44	121.56	133.06	−1.02	119.24	3
4	194.71*	196.90*	39.36	114.97	134.27	−0.65	112.00	4

*Invalid whiteness indices.

Table 6. Whiteness Values and *VR* of 2 Groups of Whitened Potteries

Sample	W_{CIE10}	W_{G}	W_{10}	W_{uv}	W_{ab}	T_{ab}	W_{LAB}	VR
G_1	85.68	85.88	84.64	93.22	94.51	0.59	93.81	1
G_2	75.43	75.13	72.25	86.67	90.10	1.05	87.90	2
G_3	73.99*	69.78*	−82.22*	71.27	86.70	2.56	73.61	3
H_1	86.30	86.51	85.70	94.03	94.51	0.38	94.58	1
H_2	74.54	74.44	73.52	88.57	90.10	0.52	89.40	2
H_3	72.21*	69.18*	−21.83*	76.57	86.70	2.06	78.45	3

*Invalid whiteness indices.

The results also show that whiteness formulas W_{10}, W_{uv}, and W_{LAB} dealing with the tint and the excitation purity are well correlated with visual estimation. The white region extends widely as compared with the formulas W_{CIE10} and W_{G}. Some typical examples are shown in Table 4. It is found that whiteness values of 4 pairs of white fabrics with fluorescent whitening agents have significant difference in visual estimation, as well as in the whiteness indices W_{10}, W_{uv}, and W_{LAB}. However, whiteness values of 4 pairs of white fabrics with fluorescent whitening agents do not show significant difference in the whiteness indices W_{CIE10} and W_{G}.

From Table 5 it can be seen that W_{CIE10} and W_{G} are invalid for the fluorescent whitening fabrics with the whiteness indices larger than $5Y_{10} - 275$. The white region of these formulas is too small. Although the whiteness formula W_{10} expands the white region and agrees with visual ranking of whiteness for some samples, it is less than eyeballing value, even negative (see Table 6). The whiteness formulas W_{CIE10} and W_{10} are non-uniform in vision, because they choose the CIEXYZ non-uniform color space. The whiteness formulas W_{uv} and W_{LAB}, however, choose the CIE1976 uniform color space. Not only the whiteness evaluations are well correlated to visual estimation, but also the visual uniformity is improved significantly.

A whiteness formula W_{LAB} in the CIELAB uniform color space is developed. Several whiteness formulas are analyzed and compared. The experimental results show that whiteness formula in the CIELAB uniform color space agrees well with observers' assessments, and it is superior to the CIE whiteness formula and the others in visual correlativity, uniformity and applicability.

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