

An improved method of angle measurement with a position sensitive detector

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An improved method of angle measurement is proposed based on a parallel plate interferometer. A position detection system is incorporated into a parallel plate interferometer in order to realize large deflection angle measurement. A reflecting mirror is introduced for increasing the measurement resolution. In experiments, a deflection angle of a measured target was measured within $\sim 3^\circ$ with high accuracy. And as a phase modulating interferometer, it was used to measure a small angular displacement with a repeatability of 5.5×10^{-8} rad.

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The angle measurement has widespread applications in the fields of alignment, assembly, and precision control. It is conventionally performed by using either autocollimators^[1,2] or interferometers^[3,4]. Besides, other techniques^[5-7] are often used. Generally, the angle measurement needs to be done with high stabilization and high accuracy in order to increase the automated production level and satisfy the demands of high efficiency in precision control.

A parallel plate interferometer for measuring angular displacement has been proposed by our research group^[8]. The plane-parallel plate has a simple structure itself, and it converts the angular displacement of a measured target into the phase difference of interference signal. Owing to the fact that a standard reference surface is not required in this optical configuration, the interferometer is stable and compact. But this interferometer can only measure the dynamic angle variation within a small measurement range, and it is not suitable to measure the deflection angle of a measured target within a relatively large range. In addition, it is not convenient for the field testing because the charge coupled device (CCD) is sensitive to the stray light. The angle measurement needs to be done in a dark room.

In this paper, an improved angle measurement method is proposed based on the above situation. An optical system using a position sensitive detector (PSD) and a lens is incorporated in order to enlarge the angle measurement range. And a reflecting mirror is introduced for increasing the measurement resolution. The improved interferometer not only can measure dynamic small angle variation with higher accuracy, but also can measure a relatively large deflection angle whose range is far thousand times larger than the variation range of small angle described in Ref. [8]. The analysis and experimental results prove that the improved interferometer can measure the angle with high accuracy.

The setup for angle measurement is shown in Fig. 1. The light beam from the laser diode (LD) is collimated by a micro objective, and then is reflected several

times in the system composed of a measured target and a reflecting mirror. The light beam is incident on the plane-parallel plate where the two reflected beams interfere with each other. The interference signal is detected by a photo diode. The transmitted beam from the plane-parallel plate is focused into a spot by a lens. The measurement setup consists of two measurement units. One is used for measuring relatively large angle based on position detection technique; the other is used for measuring dynamic small angle variation based on phase modulating interferometry.

In the process of assembly and alignment, the deflection angle can be measured as follows. The collimated beam *A* is incident on the measured target shown in Fig. 2. Let *i* and *m* be the incident angle on the plane-parallel plate and the number of the reflections,

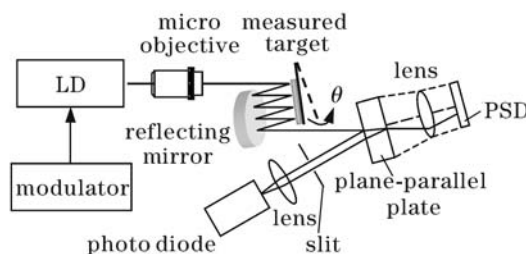


Fig. 1. Experimental setup for angle measurement.

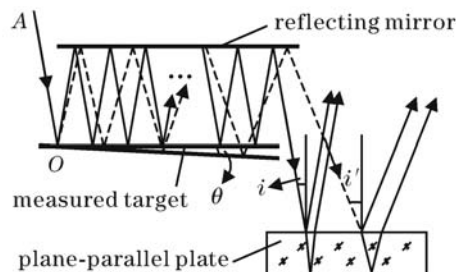


Fig. 2. Amplification of the incident angle of the beam and two beams reflected from the plane-parallel plate.

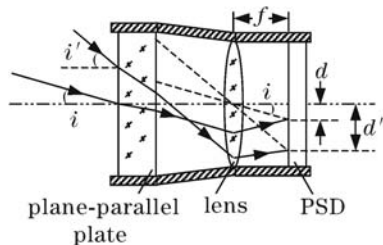


Fig. 3. Measurement of the incident angle using a PSD.

respectively. If the measured target rotates θ from the original place, the incident angle i' can be expressed as

$$i' = i + 2m\theta. \tag{1}$$

In Eq. (1), the incident angles i and i' can be measured by the system shown in Fig. 3. The photo surface of the PSD just locates on the focal plane of the lens. According to geometrical optics, the incident angle is respectively given by

$$\begin{aligned} \sin i &= 1/[1 + (f^2/d^2)]^{1/2}, \\ \sin i' &= 1/[1 + (f^2/d'^2)]^{1/2}, \end{aligned} \tag{2}$$

where f is the focal length of the lens, d and d' are the offsets of light spot deviating from the center of the PSD's photo surface. The offsets d and d' can be obtained according to the output voltage of the PSD. It follows from Eqs. (1) and (2) that the measured deflection angle is given by

$$\begin{aligned} \theta = \frac{\Delta i}{2m} = \frac{1}{2m} \left\{ \arcsin \left[\frac{1}{(1 + f^2/d^2)^{1/2}} \right] \right. \\ \left. - \arcsin \left[\frac{1}{(1 + f^2/d'^2)^{1/2}} \right] \right\}, \end{aligned} \tag{3}$$

where $\Delta i = i - i'$. Figure 4 shows the relationship between $i/2m$ and offset d , d' in the case of $m = 3$ and $f = 30$ mm. In Fig. 4, $\Delta d = d' - d$. The value of θ depends on the value of light spot's shifting on the PSD. Owing to the fact that the intensity of the light spot is very high, the method based on the PSD to

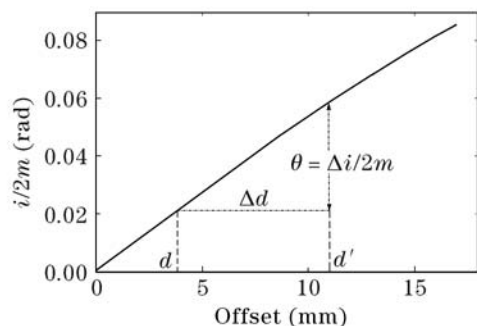


Fig. 4. Relationship between $i/2m$ and the offset of light spot on the PSD.

measure the angle of incidence is insensitive to the stray light.

In particular, the improved interferometer can be used to measure the dynamic small angle variation with higher accuracy. Assume that the reflecting mirror is not introduced into the measurement system in Fig. 1. Let the reflecting beam be directly incident on the plane-parallel plate. According to Ref. [8], the angle $\theta(t)$ is expressed as

$$\theta(t) = \frac{\alpha(t)\lambda}{8\pi h} \frac{(n^2 - \sin^2 i)^{1/2}}{\sin i (1 - \sin^2 i)^{1/2}}, \tag{4}$$

where λ is the central wavelength of LD, $\alpha(t)$ is the phase of interference signal, and n , h are the refractive index and thickness of plane-parallel plate, respectively. Substituting Eq. (2) into Eq. (4), and taking the number of the reflections m into consideration, finally the expression of angle measurement is given by

$$\theta(t) = \frac{\alpha(t)\lambda}{8m\pi h} \left[\left(\frac{nd}{f} + \frac{nf}{d} \right)^2 - \frac{d^2}{f^2} - 1 \right]^{1/2}. \tag{5}$$

It is obvious from Eqs. (3) and (5) that the measurement resolution can be increased by enlarging the number m of reflections.

The experimental setup described in Fig. 1 was used to measure deflection angle of a mirror surface and a small angular displacement. In experiments, the central wavelength λ of the LD is 660 nm. The position resolution of the PSD is 2.1 μm and the half of the active area of a PSD's photo surface is 17 mm. $n = 1.5163$, $h = 12$ mm, $f = 30$ mm, and $m = 3$. The number of the reflecting could be counted by observing the reflected light spots on the reflecting mirror. Let modulator be offline status. The collimated light beam was obtained by adjusting the optical configuration. The measured target was shown in Fig. 5. The stiffening lever DE was fixed on the optical mount. We can deflect the reflector by rotating the micrometer head. The deflection angle can be calculated according to the length of DE and the extension length of micrometer head. The length of DE was 149.5 mm and the pitch of micrometer head was 0.5 mm. So the deflection angle produced by the rotation, which was driven by the micrometer head, was 3.34×10^{-3} rad corresponding to 360° rotation. Prior to the reflector's deflection, the offset d of light spot on the photosensitive surface of the PSD was 3057.9 μm , which was firstly recorded. When the micrometer head was rotated by 360° every time, the offset of light spot correspondingly changed. Figure 6(a) is plotted according to the values of deflection angles, which corresponds to 15 times rotation, respectively. Figure 6(b) shows the measurement

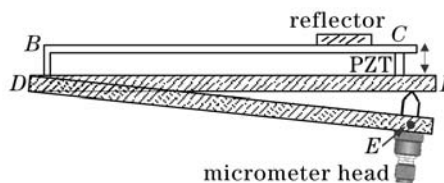


Fig. 5. Configuration of the measured target.

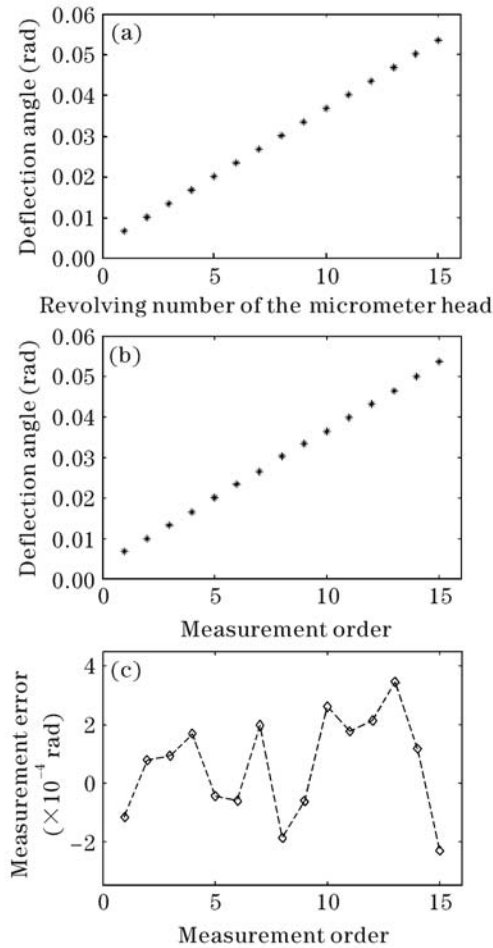


Fig. 6. (a) Relationship between deflection angle and revolving number of the micrometer head; (b) actual measurement results of deflection angle; (c) measurement error of deflection angle.

results, which are obtained according to Eq. (3). The differences of corresponding data in Figs. 6(a) and (b), which are shown in Fig. 6(c), are very small. The standard deviation of the measurement results is 1.7×10^{-4} rad ≈ 35.1 arcsec.

In Fig. 6(b), the largest offset of light spot on the photosensitive surface is $13531.2 \mu\text{m}$. Prior to the reflector's deflection, the offset of light spot is $3057.9 \mu\text{m}$. According to Eq. (3), $\theta = 5.37 \times 10^{-2}$ rad $\approx 3.1^\circ$. The measurement range of deflection angle can be increased by reducing the number of the reflections m , according to Eq. (3). Using Eq. (3), the expression of the theoretical resolution $\Delta\theta$ is given by

$$\Delta\theta = \left[\left(\frac{\partial\theta}{\partial d} \right)^2 (\Delta d)^2 + \left(\frac{\partial\theta}{\partial d'} \right)^2 (\Delta d')^2 \right]^{1/2}, \quad (6)$$

where $\Delta d = \Delta d' = 2.1 \mu\text{m}$. Under the above experimental conditions, $\Delta\theta = 3.1$ arcsec for $m = 3$.

In Fig. 1, when the modulator is used to modulate the wavelength λ by sinusoidally changing the injection current of the LD^[9,10], the experimental setup also can be regarded as a phase modulating interferometer. The introduction of the reflecting mirror actually amplifies the optical path difference between two light beams reflected

from the plane-parallel plate. The reflector fixed on the stiffening lever shown in Fig. 5 was still used as a measured target. One side B of the stiffening lever was fixed on the lever DF , which was fixed on the optical mount. A piezoelectric transducer (PZT) was fixed on the open end C and the lever DF . The PZT was driven by a sinusoidal signal in order to let the reflector produce a small angular displacement. The length of the lever between points B and C was 147.2 mm. The amplitude and frequency of the voltage applied to the PZT were 3.45 V and 200 Hz, respectively. Since the displacement per voltage of the PZT was 85 nm. The amplitude of the angular displacement of the reflector was 1.992×10^{-6} rad.

In experiment, the initial angle of incidence on the plane-parallel plate was 0.378 rad due to d value of $11912.9 \mu\text{m}$. The sinusoidal modulation frequency was 1 kHz. The interference signal was Fourier transformed to calculate $\alpha(t)$ ^[11]. Figure 7 shows the experimental curve of the angular displacement. The frequency of the measured displacement was 200 Hz. The mean amplitude of the measured displacement over the 9 periods of the sinusoidal wave was 2.020×10^{-6} rad.

Under the same condition, the above measurements were repeated 20 times at intervals of several minutes. Twenty values of the mean amplitudes obtained from the consecutive measurements are shown orderly in Fig. 8. The average of the twenty values was 2.075×10^{-6} rad. The difference between the mean amplitude and the calculated value of 1.992×10^{-6} rad was 8.3×10^{-8} rad. This error value means that the experimental results reflect the real angular displacement of the reflector, if we take some random factors into consideration in experiments, such as external disturbance. The repeatability standard deviation of the twenty values of

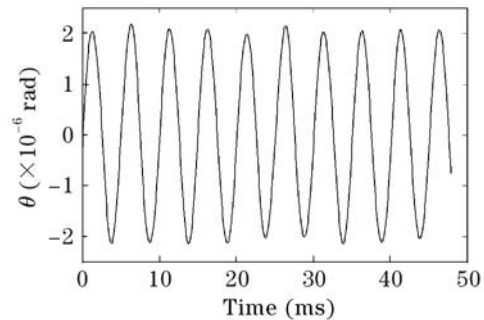


Fig. 7. Experimental curve of the angular displacement.

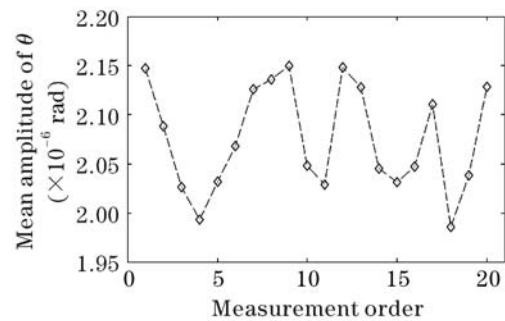


Fig. 8. Mean amplitudes of the angular displacements measured twenty times at intervals of several minutes.

the mean amplitudes was 5.5×10^{-8} rad, which was much smaller than the value of 2.7×10^{-7} rad described in Ref. [8].

In summary, a position detection system is incorporated into the system of angle measurement to enlarge the measurement range. Introduction of a reflecting mirror makes the dynamic angle measurement more sensitive and accurate. The experimental results showed that the deflection angle could be measured within $\sim 3^\circ$. The measurement repeatability of the small angular displacement was 5.5×10^{-8} rad. The measurement accuracy was up to $\sim 10^{-8}$ rad order of magnitude. The improved method provides much wider applications for angle measurement, especially in the process of assembly and precision control.

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