Generation of cluster states in thermal cavity

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A scheme is reported for generating a multi-atom cluster state in thermal cavities, which is based on the simultaneous interaction of two two-level atoms with a single-mode cavity field driven by a classical field. The photon-number-dependent parts in the evolution operator are cancelled with the assistant of a strong classical field, so the scheme is insensitive to the thermal field. In the present scheme, the detuning between the atoms and the cavity is equal to the atom-cavity coupling strength, thus the operation speed is greatly improved, which is important in view of decoherence.

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Quantum entanglement has been the focus of research in recent years due to its potential application in quantum information processing (QIP) and quantum computation (QC)[1,2]. Entanglement represents a unique quantum resource and a sort of elementary prerequisites for QC and QIP. In the past few years, great efforts have been made to understand and create entanglement. The two-atom maximally entangled states are referred to as the Einstein-Podolsky-Rosen (EPR) states[3]. Dür *et al.* have shown that there are two inequivalent classes of tripartite entanglement states, the Greenberger-Horne-Zeilinger (GHZ) class and the W class, under stochastic local operation and classical communication[4] . Recently, Briegel *et al.* have introduced another class of multiqubit entangled states, i.e., the so-called cluster states[5]. Cluster states have many interesting features. They have a high persistence of entanglement and can be regarded as an entanglement source for the GHZ states, but are more immune to decoherence than GHZ states[6]. It has been shown that a new inequality is maximally violated by the four-particle cluster states but not by the four-particle GHZ states. And the cluster states can also be used to test nonlocality without inequalities[7]. More importantly, it has been shown that the cluster states constitute a universal resource for so-called one-way quantum computation proceeding only by local measurements and feedforward of their outcomes[8].

Walther *et al.* have experimentally generated four-photon cluster states and demonstrated the feasibility of the one-way quantum computation[9]. The cluster-state violation of Bell's inequality has also been experimentally demonstrated[10]. Many schemes have been proposed for preparing cluster states[11−17]. Dong *et al.*[14] proposed a scheme to generate the cluster states based on the resonant interaction between two atoms and a single-mode cavity field, and extended it to multi-atom cluster state case. The interaction time of the scheme is very short, but it required the cavities to be prepared in the vacuum states initially. Xiang *et al.*[15] reported a one-step scheme to generate a two-atom cluster state through the simultaneous interaction of two two-level atoms with a single-mode cavity field prepared initially in an odd-coherent state under a large-detuned limit. Yang *et al.*[16] presented a scheme to generate a four-atom cluster state in thermal cavities and generalized their scheme to prepare an n-atom cluster state. The advantage of the scheme is that it allows the cavities in the thermal states. But the scheme required the detuning between the atoms and the cavity to be much larger than the atom-cavity coupling strength, so the operation time is long.

In this paper, we investigate an alternative scheme to generate a multi-atom cluster state in thermal cavities, which is based on the simultaneous interaction of two two-level atoms with a single-mode cavity field driven by a classical field. The advantage of the scheme is that the photon-number-dependent parts in the evolution operator are cancelled with the assistant of a strong classical field, so the scheme is insensitive to the thermal field. Unlike the previous scheme[16], in the present scheme, the detuning between the atoms and the cavity is equal to the atom-cavity coupling strength, thus the operation speed is greatly improved, which is important in view of decoherence.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$)[18,19]

$$H = \omega_0 S_z + \omega_a a^+ a + \sum_{j=1,2}\left[\frac{g}{2}(a^+ S_j^- + a S_j^+) + \frac{\Omega}{2}(S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t})\right], \quad (1)$$

where $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, $S_z = \frac{1}{2}\sum_{j=1,2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, with $|e_j\rangle$ and $|g_j\rangle$ $(j = 1, 2)$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity mode, and g is the atom-cavity coupling strength, Ω is the Rabi frequency of the classical field, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Assuming that $\omega_0 = \omega$, the interaction Hamiltonian, in the interaction picture, is

$$H_{\mathrm{i}} = \sum_{j=1,2}\left[\frac{g}{2}(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+) + \frac{\Omega}{2}(S_j^+ + S_j^-)\right], \quad (2)$$

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where $\delta = \omega_0 - \omega_a$ is the detuning between the atoms and the cavity. Define the new atomic basis[18,19]

$$|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)\,, \quad |-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle). \quad (3)$$

Then we can rewrite H_{i} as

$$H_{\mathrm{i}} = \sum_{j=1,2} \frac{g}{2} \left\{ \left[\mathrm{e}^{-i\delta t} a^+ (\sigma_{z,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^-) + \mathrm{e}^{i\delta t} a (\sigma_{z,j} + \frac{1}{2}\sigma_j^- - \frac{1}{2}\sigma_j^+) \right] + \Omega \sigma_{z,j} \right\}, \quad (4)$$

where $\sigma_{z,j} = \frac{1}{2}(|+_j\rangle \langle +_j| - |-_j\rangle \langle -_j|)$, $\sigma_j^+ = |+_j\rangle \langle -_j|$, $\sigma_j^- = \frac{1}{2}|-_j\rangle \langle +_j|$.

Assuming that $\Omega \gg \delta$, g, and setting

$$\delta t = 2\pi, \quad (5)$$

we can get the evolution operator of the system

$$U(t) = \mathrm{e}^{-i\Omega t S_x - i\lambda t S_x^2}\,, \quad (6)$$

where

$$\lambda = \frac{g^2}{4\delta}, \quad (7)$$

$$S_x = \frac{1}{2} \sum_{j=1,2} (S_j^+ + S_j^-). \quad (8)$$

Obviously, after an interaction time decided by Eq. (5), the evolution operator $U(t)$ is independent of the cavity field state, so the system allows the cavity to be in a thermal state. It can be shown that if we set[18−21]

$$\delta = g, \quad \Omega t = (2k + \frac{3}{2})\pi, \quad k = 1, 2, \cdots \quad (9)$$

we can get

$$|+_1\rangle |+_2\rangle \to |+_1\rangle |+_2\rangle\,, \quad (10)$$

$$|+_1\rangle |-_2\rangle \to |+_1\rangle |-_2\rangle\,, \quad (11)$$

$$|-_1\rangle |+_2\rangle \to |-_1\rangle |+_2\rangle\,, \quad (12)$$

$$|-_1\rangle |-_2\rangle \to -|-_1\rangle |-_2\rangle\,. \quad (13)$$

In this way, we can implement a controlled phase gate. The phase changes only if the two atoms are in the state $|-\rangle |-\rangle$.

Now we first consider to prepare a two-atom cluster state. We first inject two atoms initially in the state

$$|e_1\rangle |g_2\rangle = \frac{1}{2}(|+_1\rangle - |-_1\rangle)(|+_2\rangle + |-_2\rangle) \quad (14)$$

into a single-mode cavity driven by a classical field. Choosing δ, t, and Ω to satisfy Eqs. (5), (9), the two atoms undergo the transitions of Eqs. (10)—(13), we can obtain the evolution

$$\begin{aligned} |e_1\rangle |g_2\rangle \to \frac{1}{2} |+_1\rangle |+_2\rangle - |-_1\rangle |+_2\rangle \\ + |+_1\rangle |-_2\rangle + |-_1\rangle |-_2\rangle \\ = \frac{1}{2}(|-_1\rangle \sigma_z^2 + |+_1\rangle)(|-_2\rangle + |+_2\rangle), \end{aligned} \quad (15)$$

where

$$\sigma_z^2 = |-_2\rangle \langle -_2| - |+_2\rangle \langle +_2|\,. \quad (16)$$

Obviously we get a standard two-atom cluster state.

Multi-atom entanglement is a very important source in quantum information processing and quantum computation. Especially the multi-atom cluster states have attract many scientific attention recently, and some of their applications have been proposed[22−25]. Our scheme can easily be generalized to prepare a multi-atom cluster state.

We initially prepare the N atoms in the state $|e_1 g_2 g_3 g_4 \cdots g_N\rangle$, the $N-1$ cavities in the single-mode thermal states, driven by a classical field, respectively. The operation processing of preparing a multi-atom cluster state can be separated into the following steps.

Firstly, let atom 1 and atom 2 interact simultaneously with the first single-mode cavity. Choosing δ, t, and Ω to satisfy Eqs. (5), (9). Atom 1 and atom 2 undergo the transitions of Eqs. (10)—(13), we get the evolution

$$\begin{aligned} & |e_1 g_2 g_3 \cdots g_N\rangle \\ & \to \frac{1}{2}(|-_1\rangle \sigma_z^2 + |+_1\rangle)(|-_2\rangle + |+_2\rangle) |g_3 g_4 \cdots g_N\rangle \\ & = \frac{\sqrt{2}}{2}(|-_1\rangle \sigma_z^2 + |+_1\rangle) |g_2 g_3 g_4 \cdots g_N\rangle\,. \end{aligned} \quad (17)$$

Secondly, send atom 2 through the first classical field to undergo the following transition,

$$|g_2\rangle \to |e_2\rangle\,. \quad (18)$$

And send atom 2 and atom 3 through the second single-mode thermal cavity to undergo the transitions of Eqs. (10)—(13), Eq. (17) becomes

$$\begin{aligned} |e_1 g_2 g_3 \cdots g_N\rangle \to \frac{1}{2^{3/2}}(|-_1\rangle \sigma_z^2 + |+_1\rangle) \\ \times (|-_2\rangle \sigma_z^3 + |+_1\rangle)(|-_3\rangle + |+_3\rangle) |g_4 \cdots g_N\rangle\,. \end{aligned} \quad (19)$$

Thirdly, send atom 3 through the second classical field to undergo the transition of Eq. (18), then send atom 3 and atom 4 through the third single-mode thermal cavity to undergo the transitions of Eqs. (10)—(13).

Finally, send atom $N-1$ through the $N-2$th classical field to undergo the transition of Eq. (18), then send atom $N-1$ and atom N through the $N-1$th single-mode thermal cavity to undergo the transitions of Eqs. (10)—(13).

We can obtain a multi-atom cluster state

$$|\Psi\rangle_N = \frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} (|-_j\rangle \sigma_z^{j+1} + |+_j\rangle). \quad (20)$$

Now let us give a brief discussion on the experimental realization of our scheme. According to Refs. [18,22], the atom-cavity coupling strength is about $g = 2\pi \times 50$ kHz and thus the interaction time decided by Eq. (5) is on the order of $\pi/g \approx 10^{-5}$ s. The photon decay time is $T_c \approx 10^{-3}$ s, much longer than the interaction time. After the interaction, the evolution operator of the system is independent of the cavity field state, the atoms are disentangled with the cavity field and then the cavity decay will not affect the generating operation. In our scheme, the two atoms must interact simultaneously with the cavity. But in real case, we cannot achieve simultaneousness in perfect precise. Calculation on the error suggests that it only slightly affects the fidelity[18]. Thus the proposed scheme might be realizable based on the current cavity quantum electrodynamic (QED) techniques.

In conclusion, we have proposed a scheme to generate a multi-atom cluster state in thermal cavities, which is based on the simultaneous interaction of two two-level atoms with a single-mode cavity field driven by a classical field. The scheme is insensitive to the thermal state and works in a fast way, which is important from the experimental point of view.

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