Study of the normalized intensity correlation function of a single-mode laser system with colored cross-correlated noises

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Received December 8, 2006

A single-mode laser system with colored cross-correlated additive and multiplicative noise terms is considered. By the means of projection operator method, we study the effects of the cross-correlation time τ and the cross-correlation intensity λ between noises on the normalized intensity correlation function $C(s)$. It is found that if $\lambda > 0$ ($\lambda < 0$), the normalized intensity correlation function $C(s)$ increases (decreases) with increasing the cross-correlation time τ, and at large value of τ, the variation of the normalized intensity correlation function $C(s)$ becomes small. With the increase of the net gain a_0, $C(s)$ exhibits a maximum when λ is larger. However, a minimum and a maximum appear on $C(s)$ curves with the increase of a_0 when λ becomes smaller and smaller.

OCIS codes: 140.3570, 140.0140, 140.3430.

According to the study of laser system, the statistical properties of laser system can be regarded as a particular prototype of a nonlinear problem in non-equilibrium statistical mechanics[1−6]. These previous studies assumed that the additive and multiplicative noises have different origins and are uncorrelated with each other. In 1991, Fulinski *et al.* first introduced a correlation between an additive white noise and a multiplicative white noise[7]. From then on, a lot of work on laser system has been made in the investigation of nonlinear systems with cross-correlated noises[8−11]. Those studies have revealed that the consideration of additive and multiplicative noises simultaneously is of importance to deeply understand statistical properties of single-mode laser system. In recent years, the effects of cross-correlation between additive and multiplicative noises on statistical fluctuation of the single-mode laser model have attracted close attention[12−17]. In 2004, Zhang *et al.* studied the stationary intensity distribution of the single-mode laser cubic model driven by colored pump noise with cross-correlation between the real and imaginary parts of the quantum noise[18]. Xie *et al.* have used the projection operator method to study the normalized intensity correlation function of single-mode laser driven by cross-correlated pump noise and quantum noise[19]. But this study only considered the case of the cross-correlation time $\tau = 0$. In some instance, the cross-correlation time of the additive and multiplicative noises is not zero ($\tau \neq 0$). So in this paper, we investigate the effects of cross-correlation time τ and cross-correlation intensity λ on the normalized intensity correlation function.

The complex field-amplitude E of the cubic model of a single-mode laser system can be described by the Langevin equation (LE),

$$\frac{\mathrm{d}E}{\mathrm{d}t} = a_0 E - A\left|E\right|^2 E + \tilde{p}(t)E + \tilde{q}(t), \tag{1}$$

where a_0 and A are real and respectively stand for the net gain and the self-saturation coefficient, $\tilde{p}(t)$ is the pump noise (multiplicative noise) and $\tilde{q}(t)$ is the quantum noise (additive noise). By performing the polar coordinate transform $E = r\mathrm{e}^{i\varphi}$, Eq. (1) can be transformed into two coupling LEs about the field-amplitude r and the phase φ. By decoupling them, the LE of the field-amplitude r can be obtained as[12]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = a_0 r - Ar^3 + \frac{D}{2r} + rp(t) + q(t). \tag{2}$$

Assume I is the laser intensity ($I = r^2$),

$$\frac{\mathrm{d}I}{\mathrm{d}t} = (2a_0 - AI)I + D + 2I^{1/2}q(t) + 2Ip(t). \tag{3}$$

We usually consider that the multiplicative noise $p(t)$ and the additive noise $q(t)$ are Gaussian-type noises,

$$\langle q(t)\rangle = \langle p(t)\rangle = 0, \tag{4}$$

$$\langle q(t)q(t')\rangle = D\delta(t-t'), \tag{5}$$

$$\langle p(t)p(t')\rangle = Q\delta(t-t'), \tag{6}$$

and

$$\langle p(t)q(t')\rangle = \langle q(t)p(t')\rangle = \frac{\lambda\sqrt{QD}}{2\tau}\exp\left[-\left|t-t'\right|/\tau\right] \rightarrow \lambda\sqrt{QD}\delta(t-t') \quad \text{as } \tau \rightarrow 0, \tag{7}$$

where Q and D are the multiplicative and the additive noise intensities, respectively. τ and λ are the cross-correlation time and the cross-correlation intensity, respectively. Applying the Novikov theorem[20] and the Fox's approach[21], the approximate Fokker-Planck equation corresponding to Eq. (3) reads

$$\frac{\partial P(I,t)}{\partial t} = L_{\mathrm{FP}} P(I,t), \tag{8}$$

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial I}f(I) + \frac{\partial^2}{\partial I^2}G(I), \tag{9}$$

where the drift coefficient $f(I)$ and the diffusion coefficient $G(I)$ are given by

$$f(I) = 2(a_0 - AI + Q)I + 3\frac{\lambda}{1+2a_0\tau}\sqrt{DQ}I^{1/2} + 2D, \tag{10}$$

$$G(I) = 2QI^2 + 4\frac{\lambda}{1+2a_0\tau}\sqrt{DQ}I^{3/2} + 2DI. \tag{11}$$

It should be pointed out that the above approximate Fokker-Planck equation is valid only for the case of $1 + 2a_0\tau > 0$. We only consider the stationary state. The steady-state probability density of Eq. (8) can be obtained as

$$P_{\mathrm{st}}(I) = NW(I)^{m_1(\lambda)}\exp(m_2(\lambda)), \tag{12}$$

where N is the normalization constant, and

$$W(I) = QI + 2\frac{\lambda}{1+2a_0\tau}\sqrt{DQ}I^{1/2} + D, \tag{13}$$

$$m_1(\lambda) = \frac{a_0}{Q} - \frac{AD(4\frac{\lambda^2}{(1+2a_0\tau)^2} - 1)}{Q^2} - 1, \tag{14}$$

$$m_2(\lambda) = -\frac{A}{Q}I + 4A\frac{\lambda}{Q^2(1+2a_0\tau)}\sqrt{DQ}I^{1/2} - m_3(\lambda), \tag{15}$$

$$m_3(\lambda) = m_4(\lambda)\arctan\frac{\sqrt{\frac{QI}{D}} + \frac{\lambda}{1+2a_0\tau}}{\sqrt{1 - \frac{\lambda^2}{(1+2a_0\tau)^2}}}, \tag{16}$$

$$m_4(\lambda) = \frac{2\lambda}{(1+2a_0\tau)\sqrt{1 - \frac{\lambda^2}{(1+2a_0\tau)^2}}} \times \left(\frac{a_0}{Q} - AD\frac{\frac{4\lambda^2}{(1+2a_0\tau)^2} - 3}{Q^2} + \frac{1}{2}\right). \tag{17}$$

The expectation values of the nth power of I is defined by

$$\langle I^n\rangle_{\mathrm{st}} = \int_0^{+\infty} I^n P_{\mathrm{st}}(I)\mathrm{d}I. \tag{18}$$

Our prime concern in this paper is the normalized intensity correlation function $C(s)$, which characterizes the decay behavior of the laser intensity fluctuations[19]. The normalized intensity correlation function of intensity variation in a stationary state can be defined by[22]

$$C(s) = \frac{\langle \delta I(t+s)\delta I(t)\rangle_{\mathrm{st}}}{\langle(\delta I)^2\rangle_{\mathrm{st}}}, \tag{19}$$

where $\delta I(t) = I(t) - \langle I\rangle$, in terms of the adjoint operator L^+_{FP} of the operator given by Eq. (9), $\delta I(t+s) = \exp(L^+_{\mathrm{FP}}s)\delta I(t)$. Thus, one can rewrite Eq. (19) and get the associated Laplace transform,

$$\tilde{C}(\omega) = \int_0^{\infty}\exp(-\omega s)C(s)\mathrm{d}s = \frac{1}{\langle(\delta I)^2\rangle_{\mathrm{st}}}\left\langle \delta I\frac{1}{\omega - L^+_{\mathrm{FP}}}\delta I\right\rangle_{\mathrm{st}}. \tag{20}$$

With the projection operator method used in Ref. [23] to deal with the Laplace resolvent $\omega - L^+_{\mathrm{FP}}$ in Eq. (20), we get the following continued fraction expression[22,23],

$$\tilde{C}(\omega) = \frac{1}{\omega + \mu_0 + \frac{\eta_1}{\omega+\mu_1+\frac{\eta_2}{\omega+\mu_2+\cdots}}}, \tag{21}$$

in which

$$\mu_i = -\frac{\langle\delta I_i L^+_{\mathrm{FP}}\delta I_i\rangle_{\mathrm{st}}}{\langle(\delta I_i)^2\rangle_{\mathrm{st}}}, \tag{22}$$

$$\eta_i = -\frac{\langle(\delta I_i)^2\rangle_{\mathrm{st}}}{\langle(\delta I_{i-1})^2\rangle_{\mathrm{st}}}, \tag{23}$$

$$\delta I_{i+1} = S_{i+1}L^+_{\mathrm{FP}}\delta I_i. \tag{24}$$

With starting $\delta I_0 = \delta I$ and $S_0 = 1$, the operator S_i is determined by

$$K_{i-1} = S_{i-1} - S_i = \frac{\delta I_{i-1}}{\langle(\delta I_{i-1})^2\rangle_{\mathrm{st}}}\langle\delta I_{i-1}|, \tag{25}$$

where the operator $\langle\delta I_i|$ acting on $\varphi(I)$ means the scalar product

$$\langle\delta I_i|\varphi(I) = \langle(\delta I_i\varphi(I))\rangle_{\mathrm{st}} = \int_0^{\infty}P_{\mathrm{st}}(I)\delta I_i\varphi(I)\mathrm{d}I. \tag{26}$$

The projection operator K_i projects $\varphi(I)$ onto the subspace associated with the variable δI_i. The projector S_i projects onto the space orthogonal to the space containing δI_i. The basic idea behind the method used to lead a continued fraction expansion is to identify δI_i as a slow variable and in J_i space it slaves the remaining fast variables[24]. Setting $\eta_2 = 0$, the first-order approximation of the intensity correlation-function is

$$\tilde{C}(\omega) = \frac{\omega + \mu_1}{(\omega+\mu_0)(\omega+\mu_1) + \eta_1}, \tag{27}$$

$$\mu_0 = \frac{\langle G(I)\rangle_{\mathrm{st}}}{\langle(\delta I)^2\rangle_{\mathrm{st}}}, \tag{28}$$

$$\eta_1 = \frac{\langle G(I)f'(I)\rangle_{\mathrm{st}}}{\langle(\delta I)^2\rangle_{\mathrm{st}}} + \mu_0^2, \tag{29}$$

$$\mu_1 = -\frac{\left\langle G(I)\left[f'(I)\right]^2\right\rangle_{\mathrm{st}}}{\eta_1\langle(\delta I)^2\rangle_{\mathrm{st}}} + \frac{\mu_0^3}{\eta_1} - 2\mu_0. \tag{30}$$

Performing the Laplace converse transformation of Eq. (27), we get

$$C(s) = \beta\exp(-\alpha_- s) + (1-\beta)\exp(-\alpha_+ s), \tag{31}$$

in which

$$\alpha_{\pm} = \frac{\mu_0 + \mu_1}{2} \pm \frac{1}{2}\sqrt{(\mu_1 - \mu_0)^2 - 4\eta_1}, \tag{32}$$

and

$$\beta = \frac{\mu_1 - \alpha_-}{\alpha_+ - \alpha_-}. \tag{33}$$

Let $\tau = 0$, the above results fall back to Eq. (22) presented in Ref. [19].

By virtue of the expression of the normalized intensity correlation function Eq. (31), we can discuss the influences of the cross-correlation time τ and the cross-correlation intensity λ on $C(s)$. Figure 1 shows the curves of $C(s)$ as a function of the cross-correlation time τ for different values of the net gain a_0. From Fig. 1(a), we see that, $C(s)$ increases with the increase of τ for the case of positive correlation ($\lambda = 0.25 > 0$). In other words, in the case of positive correlation, the cross correlation time slows down the decay of the intensity fluctuation. However, when the correlation between noises is negative ($\lambda = -0.25 < 0$), $C(s)$ decreases with the increase of τ (see Fig. 1(b)). That is to say, the cross-correlation time speeds up the decay of the intensity fluctuation in the case of negative correlation. We can find that at larger value of the correlation time τ, there is almost no difference for $C(s)$ when τ changes. We also can find that $C(s)$ always decreases with the increase of a_0 whether the correlation is positive or negative.

The parameter $R = \frac{D}{Q}$ is the noise intensity ratio (the ratio of the additive noise intensity to the multiplicative noise intensity), and then we plot the curves of $C(s)-\tau$ as Fig. 2. It is found that $C(s)$ decreases with the increase of R whether $\lambda > 0$ (Fig. 2(a)) or $\lambda < 0$ (Fig. 2(b)),

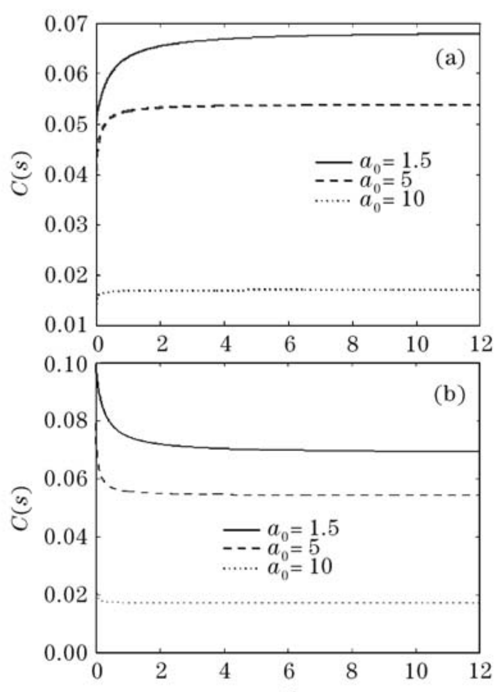

Fig. 1. $C(s)$ as a function of the cross-correlation time τ for $D = 2.5$, $Q = 2.5$, $A = 3$, and $s = 0.2$. (a) $\lambda = 0.25$; (b) $\lambda = -0.25$.

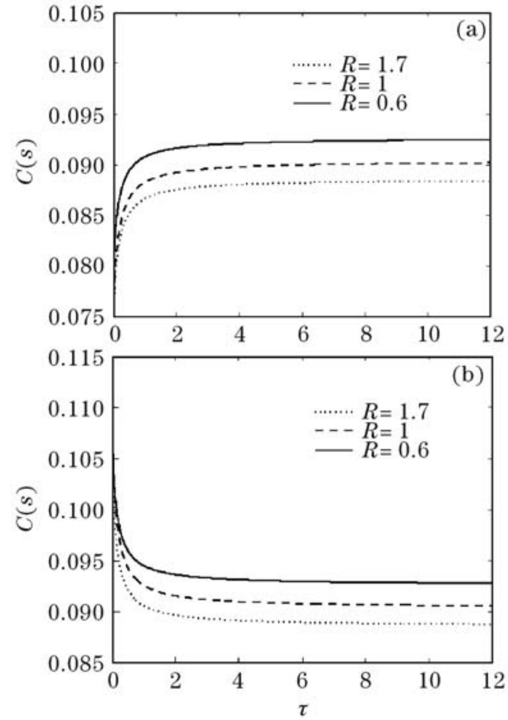

Fig. 2. $C(s)$ as a function of the cross-correlation time τ for $a_0 = 3.2$, $A = 3$, and $s = 0.2$. (a) $\lambda = 0.25$; (b) $\lambda = -0.25$.

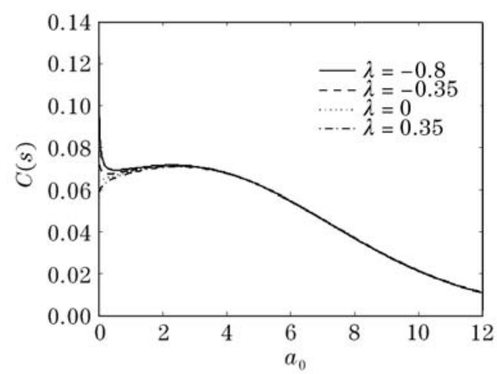

Fig. 3. $C(s)$ as a function of the net gain a_0 for $D = 1.5$, $Q = 1.5$, $A = 6.5$, $\tau = 8$, and $s = 0.2$.

i.e., the noise intensity ratio always speeds up the decay of the intensity function.

The curves of $C(s) - a_0$ with different values of λ are plotted in Fig. 3. It is obvious that $C(s)$ exhibits a maximum with the increase of a_0 when λ is larger. This means that, with the increase of a_0, the decay rate of the intensity fluctuation in the stationary state turns over, from slowing down to speeding up. However, a minimum and a maximum appear on the curves of $C(s)$ with the increase of a_0 when λ becomes smaller and smaller.

In conclusion, the intensity correlation function $C(s)$ becomes larger and larger with the increase of the correlation time τ in the case of positive correlation. But in the case of negative correlation, $C(s)$ becomes smaller and smaller with the increase of τ. With the increase of the net gain a_0, $C(s)$ exhibits a maximum when λ is larger. However, a minimum and a maximum appear on the curves of $C(s)$ with the increase of a_0 when λ becomes smaller and smaller.

This work was supported by the Natural Science Fund for Young Scholars of Anhui University of Science and Technology under Grant No. qn200628. B. Wang's e-mail address is hnitwb@163.com.

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