Influence of a high-speed dielectric spherical particle's movement on its scattering characteristics

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The scattered field and differential scattered section (DSS) of a moving spherical particle with a high speed are investigated numerically. The coordinate and vector transformations are used to establish a theoretical basis for studying the laser scattering of a moving particle. The DSS of a moving spherical particle is explained by the electric and magnetic field from Mie scattering theory. Assuming the laser wavelength of 1.06 μm, we compute the ratio of the laser DSS of the moving dielectric spherical particle to that of the static dielectric spherical particle, which changes with radii, speeds and scattering angles of the particle. The numerical results show that the laser DSS of the moving spherical particle is tightly connected with its speed and scattering zenith angle. If a spherical particle moves with high speed, the laser DSS due to movement of the particle could not be neglected. If the speed of the dielectric spherical particle is fluctuating, the Doppler effect and the frequency spectrum expansion play important roles.

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Important physical characteristics are shown when the light interacts with all kinds of particles, and a lot of correlative information can be gained on these scattering bodies[1−4]. Some algorithms such as PMM, DDA, TMM, have been developed according to the particle's geometric dimension[5−7] or clustering particles for studying this subject[8−13]. In recent years, with the development of the technology in space, there is an increasing interest in the light scattering of particles such as cosmic dust, interstellar atomies and particles, and so on, all with a high moving speed. However, most literatures only discussed light scattering characteristics of static or low-speed particles[1−13].

In order to study the scattered field and differential scattered section (DSS) of the spherical particle with a high speed, we need to establish a theoretical basis of the coordinate and vector transformations, which can be used to derive the relation between electromagnetic fields in the moving coordinate system and those in the static one (see Fig. 1). The coordinate system Σ' moves along the z axis of another one Σ, and $\vec{v}$ is a constant vector by neglecting its fluctuation ($\vec{v}_{\mathrm{f}} = 0$). The general representation for the four-dimensional vectors may be written as

$$a_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\beta\gamma \\ 0 & 0 & -i\beta\gamma & \gamma \end{bmatrix}, \quad (1)$$

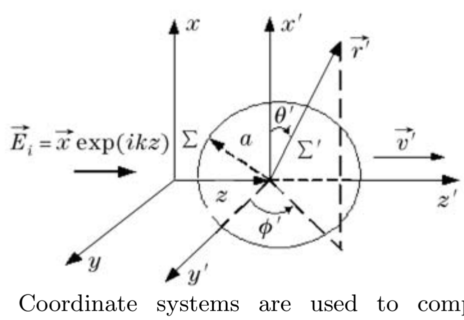

Fig. 1. Coordinate systems are used to compute the differential scattered sections of spherical particles.

where $\gamma = \left(1-\beta^2\right)^{-1/2}$, $\beta = v/c$. $x_\mu = (r, ict)$, c is the speed of light in free space and r is the distance from the origin of the coordinate system to the observing point. The relations of the four-dimensional wave vectors are obtained as

$$k'_x = k_x, \quad k'_y = k_y, \quad k'_z = \gamma(k_z - \frac{v}{c^2}\omega),$$

$$\omega' = \gamma\left(\omega - v k_z\right). \quad (2)$$

The electromagnetic fields make up two-step tensors in the four-dimensional space. The relation of the tensors can be written as

$$F'_{\mu\nu} = a_{\mu\lambda} a_{\nu\tau} F_{\lambda\tau}, \quad (3)$$

where

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_z & -B_y & -\frac{i}{c}E_x \\ -B_z & 0 & B_x & -\frac{i}{c}E_y \\ B_y & -B_x & 0 & -\frac{i}{c}E_z \\ \frac{i}{c}E_x & \frac{i}{c}E_y & \frac{i}{c}E_z & 0 \end{bmatrix}. \quad (4)$$

Then the scattering field $\vec{E}_{\mathrm{s}}$ in the system Σ and $\vec{E}'_{\mathrm{s}}$ in the system Σ' are related by

$$E'_x = \gamma\left(E_x - vB_y\right), \quad E'_y = \gamma\left(E_y + vB_x\right), \quad E'_z = E_z,$$

$$B'_x = \gamma\left(B_x + \frac{v}{c^2}E_y\right), \; B'_y = \gamma\left(B_y - \frac{v}{c^2}E_x\right), \; B'_z = B_z. \quad (5)$$

The relations between the electric fields in the moving coordinate system and those in the static one are given as

$$E_{\theta\mathrm{s}} = \left((\gamma^2\cos^2\theta + \sin^2\theta)/\left(1+\beta^2\gamma^2\cos^2\theta\right)^{1/2} + \gamma\beta\cos\theta\right)E'_{\theta'\mathrm{s}},$$

$$E_{\phi\mathrm{s}} = \gamma\left(1 + \beta\gamma\cos\theta/\left(1+\beta^2\gamma^2\cos^2\theta\right)^{1/2}\right)E'_{\phi'\mathrm{s}}. \quad (6)$$

Let us suppose that a dielectric sphere whose center is located at the origin of the coordinate system Σ', with a being its radii and m the relative refractive index. The scattered fields of a dielectric sphere have been obtained by the Mie theory when the incident wave propagates along the z' axis, polarizes in the x' direction, and the center of the sphere is located at the origin of the coordinate system Σ' attached to it. The scattered electric fields by a sphere are expressed as

$$E'_{\theta' \mathrm{s}} = \frac{jE'_0 \mathrm{e}^{-j(k'r'-\omega' t')}}{k'r'} \cos\varphi' s_2(\theta'),$$

$$E'_{\varphi' \mathrm{s}} = \frac{-jE'_0 \mathrm{e}^{-j(k'r'-\omega' t')}}{k'r'} \sin\varphi' s_1(\theta'),$$

$$B'_{\theta \mathrm{s}} = -\frac{E'_{\phi' \mathrm{s}}}{c},\ B'_{\phi \mathrm{s}} = \frac{E'_{\theta' \mathrm{s}}}{c},$$

$$r' = \sqrt{r^2 + \gamma^2 \left(\frac{\vec{v}\cdot\vec{r}}{c}\right)^2}, \tag{7}$$

where

$$S_1(\theta') = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n(\cos\theta') + b_n \tau_n(\cos\theta')),$$

$$S_2(\theta') = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n(\cos\theta') + b_n \pi_n(\cos\theta')).$$

The DSS is defined by[1,4]

$$\sigma_{\mathrm{d}} = \lim_{r\to\infty} \frac{r^2 s_{\mathrm{s}}}{s_i}. \tag{8}$$

Substituting Eqs. (2), (6) and (7) into Eq. (8), we obtain the DSS of a moving dielectric spherical particle as

$$\sigma_{\mathrm{d}v} = \frac{(1-\beta)^2 \left[\cos^2\phi S_2^2(\theta') (\gamma\cos\theta\cos\theta' + \gamma\beta\cos\theta + \sin\theta\sin\theta')^2 + \sin^2\phi S_1^2(\theta') \gamma^2 (1+\beta\cos\theta')^2\right]}{2\eta (1+\gamma^2\beta^2\cos^2\theta) \left(k - \frac{\beta}{c}\omega\right)^2}. \tag{9}$$

where η is the ratio to the laser DSS of a moving dielectric spherical particle to that of a static one.

By using the coordinate and vector transformations, the coordinates θ, θ' and ϕ, ϕ', corresponding to the coordinate systems Σ' and Σ respectively, can be written as

$$\cos\theta' = \frac{\gamma\cos\theta}{(1+\beta^2\gamma^2\cos^2\theta)^{\frac{1}{2}}}, \quad \sin\theta' = \frac{\sin\theta}{(1+\beta^2\gamma^2\cos^2\theta)^{\frac{1}{2}}}, \quad \sin\phi' = \sin\phi, \quad \cos\phi' = \cos\phi. \tag{10}$$

Inserting Eq. (10) into Eq. (9) leads the DSS to

$$\sigma_{\mathrm{d}v} = \frac{(1+\beta\cos\theta')^2 \left[\cos^2\phi' S_2^2(\theta') + \gamma^2 \sin^2\phi' S_1^2(\theta') \left(1-\beta^2\cos^2\theta'\right)\right]}{2\eta k^2}. \tag{11}$$

For a dielectric particle with a low speed $v \ll c$, $\vec{v}_{\mathrm{f}} = 0$, we have

$$\sigma_{\mathrm{d}} \to \frac{\cos^2\phi' S_2^2(\theta') + \sin^2\phi' S_1^2(\theta')}{2\eta k^2}. \tag{12}$$

As $v = 0$, it is easy to obtain $\vec{E}' = \vec{E}$, $\vec{k} = \vec{k}'$, $\sigma_{\mathrm{d}v} = \sigma_{\mathrm{d}}$,

$$\eta = \frac{\sigma_{\mathrm{d}v}}{\sigma_{\mathrm{d}}} = \frac{(1+\beta\cos\theta')^2 \left[\cos^2\phi' S_2^2(\theta') + \gamma^2 \sin^2\phi' S_1^2(\theta') \left(1-\beta^2\cos^2\theta'\right)\right]}{\cos^2\phi' S_2^2(\theta') + \sin^2\phi' S_1^2(\theta')}. \tag{13}$$

Let $M_1 = (1+\beta\cos\theta')^2$, $M_2 = \gamma^2 (1+\beta\cos\theta')^2 \times \left(1-\beta^2\cos^2\theta'\right)$, and M_1, M_2 are defined as the moving influence functions. It is obvious that $v \to 0$, $M_1 \to 1$, $M_2 \to 1$. Thus

$$\eta = \frac{M_1 \cos^2\phi' S_2^2(\theta') + M_2 \sin^2\phi' S_1^2(\theta')}{\cos^2\phi' S_2^2(\theta') + \sin^2\phi' S_1^2(\theta')}. \tag{14}$$

The functions M_1 and M_2 change with the scattering angle and speed, as shown in Fig. 2.

When the speed of a moving particle is $\vec{v} = \vec{U}$ ($\vec{U}$ is a constant vector), by neglecting its fluctuating speed ($\vec{v}_{\mathrm{f}} = 0$), we obtain the Doppler frequency shift of the moving particle from Eq. (2) in the coordinate system Σ,

$$\begin{aligned} \omega_{\mathrm{d}} &= -2\frac{v\cos\theta}{c}\omega \cdot \gamma(1 - \frac{v}{c}\cos\theta)\sin(\theta/2) \\ &= -2\frac{v_{\mathrm{d}}}{c}\omega \cdot \gamma(1 - \frac{v_{\mathrm{d}}}{c})\sin(\theta/2), \end{aligned} \tag{15}$$

where θ is the zenith angle in the coordinate system Σ, and v_{d} is the components of $\vec{v}$ in the $\vec{k}$ direction.

If $v \ll c$, the laser DSS and frequency spectrum expansion of the moving particle become

$$W_{\sigma\omega}(\omega) = 2\int_{-\infty}^{\infty} \sigma_{\mathrm{d}} \exp(i\omega\tau)\mathrm{d}\tau = 2\sigma_{\mathrm{d}}\delta(\omega + \vec{k}_{\mathrm{s}}\cdot\vec{U}). \tag{16}$$

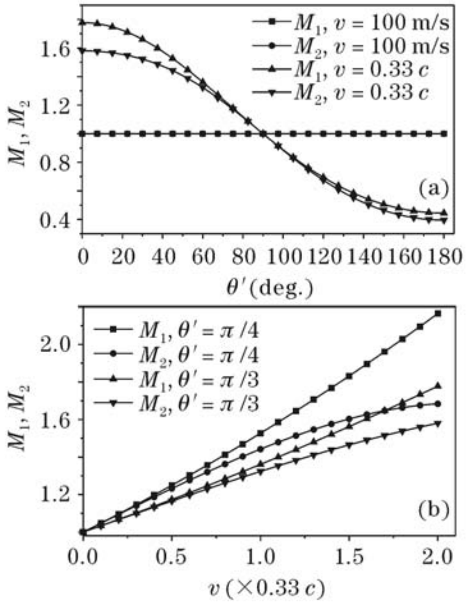

Fig. 2. Moving influence functions M_1 and M_2 change with (a) scattering angle θ' and (b) speed v.

Note that $\vec{k}_{\mathrm{s}} = k(\hat{i} - \hat{o})$, $k = \omega/c$, ω is the carrier frequency of the incident wave, then we can obtain the Doppler frequency shift

$$\omega_{\mathrm{d}} = -(\hat{i} - \hat{o}) \cdot (\vec{v}/c) \cdot \omega = -2\sin(\theta/2)(v_{\mathrm{d}}/c) \cdot \omega. \quad (17)$$

If the fluctuating speed of the dielectric spherical particle $\vec{v}_{\mathrm{f}} \neq 0$, we suppose that $\vec{v}_{\mathrm{f}}$ is satisfied with the normality distribution, then the frequency spectrum expansion is given as

$$W_{\sigma\omega} = 2\sigma_{\mathrm{d}}(\hat{o}, \hat{i})[2\pi/k^2\sigma_{\mathrm{f}}^2]^{1/2} \exp[-(\omega + \vec{k} \cdot \vec{v})^2/2k^2\sigma_{\mathrm{f}}^2)]. \quad (18)$$

As pointed out, due to the fluctuating speed of the dielectric spherical particle, the frequency spectrum has an expansion of $\Delta\omega = \left|\sqrt{2}k\sigma_{\mathrm{f}}\right|$.

In the following, the relative refractive indices of two dielectric particles are $m_1 = 1.75 + i0.0005$, $m_2 = 1.533 + i0.017$, respectively, and the wavelength of the incident wave is $\lambda = 1.06$ μm. The laser DSS of the static dielectric particles versus the scattering zenith angle is shown in Fig. 3, given the scattering azimuth angel $\phi' = \pi/4$ or $\pi/3$. The speed of the dielectric particle v equals to $0.33c$. The ratio η varying with the azimuth angel is shown in Fig. 4. The ratio η as a function of the

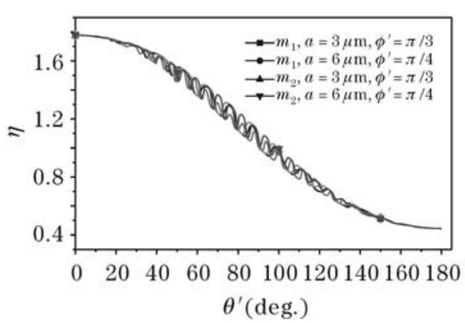

Fig. 3. Ratio η changes with scattering angle θ'. $v = 0.33c$.

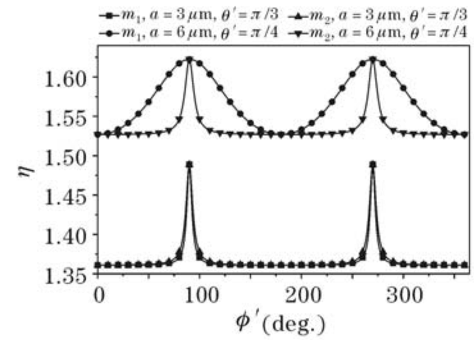

Fig. 4. Ratio η changes with azimuth angle ϕ'. $v = 0.33c$.

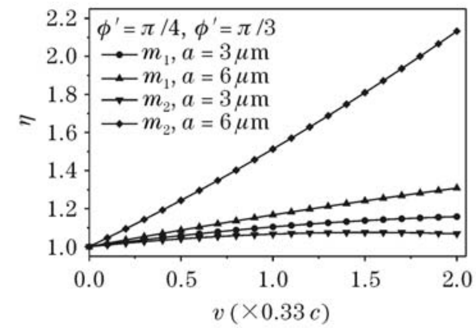

Fig. 5. ratio η changes with the speed of the particle v.

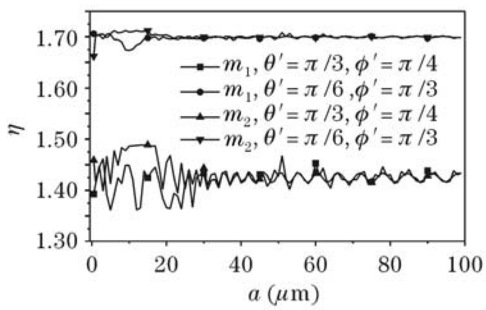

Fig. 6. Ratio η changes with the radius of the moving particle a. $v = 0.33c$.

speed of the dielectric spherical particle is plotted in Fig. 5. In order to further study and analyze the influence on the laser scattering characteristics of a dielectric particle by moving effect, the ratio η varying with radius is shown in Fig. 6.

In summary, we mainly discuss the influence on the laser scattering characteristics of a dielectric spherical particle due to moving effect. The theoretical expression of the DSS of a moving dielectric spherical particle is given by the coordinate and vector transformations. Numerical results show that the laser DSS of the moving dielectric spherical particle is tightly connected with its speed, relative refractive index and scattering angle, whether it is moving or not. The speed and scattering zenith angle of a moving dielectric spherical particle are marked factor affecting its laser DSS. The influence may be neglected as the speed $v \ll c$. But, if the speed of the particle v is close to c, its laser DSS depends on the scattering angle. Due to the moving effect, the Doppler shift plays an important role with respect to the observation point. When the speed of the dielectric spherical particle is a constant vector ($\vec{v}_{\mathrm{f}} = 0$), there is a higher Doppler frequency shift. We quantificationally give the ratio between the laser DSS of a moving spherical particle and that of the static one for analyzing the influence on the laser scattering characteristics of a dielectric spherical particle by moving effect. The results establish a base to further study cosmic dust in space. However, there arises a complex problem when a great many different moving particles exist in random medium, in which some factors including laser multiple scattering, speed fluctuation, coupled particles effect and so on must be taken into account. So it is in demand for further study on laser propagation and scattering in random medium.

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