Problems on design of computer-generated holograms for testing aspheric surfaces: principle and calculation

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Interferometric optical testing using computer-generated hologram (CGH) has provided an approach to highly accurate measurement of aspheric surfaces. While designing the CGH null correctors, we should make them with as small aperture and low spatial frequency as possible, and with no zero slope of phase except at center, for the sake of insuring low risk of substrate figure error and feasibility of fabrication. On the basis of classic optics, a set of equations for calculating the phase function of CGH are obtained. These equations lead us to find the dependence of the aperture and spatial frequency on the axial distance from the tested aspheric surface for the CGH. We also simulate the optical path difference error of the CGH relative to the accuracy of controlling laser spot during fabrication. Meanwhile, we discuss the constraints used to avoid zero slope of phase except at center and give a design result of the CGH for the tested aspheric surface. The results ensure the feasibility of designing a useful CGH to test aspheric surface fundamentally.

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Aspheric surfaces are increasingly used in design of high-quality optical systems. They allow a reduction of the number of elements as well as an improvement of the image quality of the system. To achieve the desired surface quality, the fabrication of the aspheric must be combined with careful optical testing. The interferometric method based on a specially designed computer-generated hologram (CGH) is a well-known and high-precision way to test aspheric surfaces efficiently. In the past, a CGH element could be made by use of plotter and reduced to a small photo film. Limited by the positioning resolution of a plotter and additional distortion of the photo-reduction lens, the fringe spacing in CGH was difficult to be controlled with the precision of less than 10 μm. Recently, the world semiconductor industry is preparing itself for the next evolutionary step in the ongoing development of the integrated circuit characterized by 0.18—0.15 μm technology. The semiconductor fabrication devices have the potential to make the CGH with high precision. Most of important reports have shown new advances in testing aspheric surfaces with CGHs. The method of using CGHs as null corrector has been applied to test off-axis aspheric surfaces[1] and aspheric micro-optics[2]. In order to improve the contrast of the interferograms, the disturbing diffraction orders of the CGH must be filtered[3]. Theoretically, while position error (defocus, de-center, and tilt) being decreased by alignment, the CGH's pattern errors (distortion and duty cycle errors) and substrate figure errors can be measured absolutely with twin-CGH and a lot of procedures[4−6]. However, the accuracy and resolution of the interferometric null testing method depend on the development of fabrication techniques for CGH, such as e-beam writing and laser direct writing. Especially the precision of controlling laser spot limits the maximum spatial frequency of the CGH. It has a bit of difficulty to fabricate the CGH with very large aperture. The CGH is often located in the convergence or divergence beam of light based on the phase shifted Fizeau interferometers (PSFIs) with a series of transmission spheres[7], for example, Zygo GPI or Chinese type of CXM-100, which is widely used in optical workshop. Nowadays, many kinds of fabrication devices for mask making and lithography have been used in semiconductor industry to produce the large scale integrated (LSI) circuit in China. But the size of the fabricated element is often small and the fabricated line pair (lp) is about 1 μm. Because of the limit of fabrication condition, only the possibility of a CGH used in test of an opaque aspheric component was discussed[8]. We want to study the design method of the fabricable null CGH element for testing aspheric on state of arts.

Different from the flat element, the spatial frequency of the CGH will be variable with its position along the optical axis. In other words, the designed CGH should be located at designed axial position to compensate the tested aspheric surface. Furthermore, the CGH has different apertures at different positions. We prefer small aperture of the CGH to realize the measurement of aspheric surfaces with large aperture. The CGH with small aperture is easily fabricated not at the risk of substrate figure error. This paper presents some problems concerned for technical fabrication feasibility of the CGH, appearing in design of the CGH for testing aspheric surfaces. These problems include: 1) the dependence of the aperture and necessary spatial frequency on axial position; 2) the tolerable position error of laser spot variable with radial coordinate on CGH; and 3) no zero slope of optical path difference (OPD) or phase function at any radial position except center, etc. Solutions of these problems enable one to know how to design a CGH fabricated easily within the given fabrication constraints: technical feasibility of the CGH, alignment tolerances, and existing interferometer transmission spheres.

For simplicity, a CGH element used as null corrector is often written directly on a plane by laser direct writing technology. One side of CGH plate is so called “binary optics 2” that is a surface option in ZEMAX (Zemax Development Corp.). Its phase function can be described as

$$\phi(\rho) = \sum_{i=1}^{N} a_i \rho^{2i}, \quad (1)$$

where N is the number of polynomial coefficients in series, the coordinate ρ is the radius generalized by the maximum distance from the optical axis, and a_i is the coefficient on the $2i$th power of ρ.

On the basis of a phase-shifted Fizeau interferometer, an aspheric surface can be tested with a CGH element according to the arrangement shown in Fig. 1, where the transmission sphere is necessary in Figs. 1(a) and (b), and the transmission sphere is substituted by the combination of transmission flat and a CGH in Fig. 1(c). If a series of transmission spheres exist, the setup of Fig. 1(c) is not a preferable option. The CGH element of Fig. 1(c) with small f-number has a bit of difficulty in design because it needs to play two roles: making the plane wave be converged and compensating the aspheric surface. Theoretically, the CGH element of Figs. 1(a) and (b) may be positioned anywhere between the transmission sphere and the tested aspheric surface along the optical axis. For convenience of alignment, the outside annulus of the CGH element is designed to reflect the spherical wave produced by the transmission spheres. Meanwhile, the central circle of the CGH is designed to make the rays normal into the tested aspheric surface. So the f-number of the transmission sphere should be a bit smaller than that of the tested aspheric surface.

Assume the tested aspheric surface is rotationally symmetrical and described as

$$z(h) = \frac{h^2}{R_0 + \sqrt{R_0^2 - (1+k)h^2}} + \sum_{i=2}^{m} A_{2i} h^{2i}, \quad (2)$$

where the coordinate $h = \sqrt{x^2 + y^2}$ is the distance from optical axis (z-axis), R_0 is the vertex radius, A_{2i} is the coefficient on the $2i$th power of h, and k is the conic constant.

Considering the clear aperture of the tested aspheric surface is $2h_{\max}$ and the CGH element is positioned at

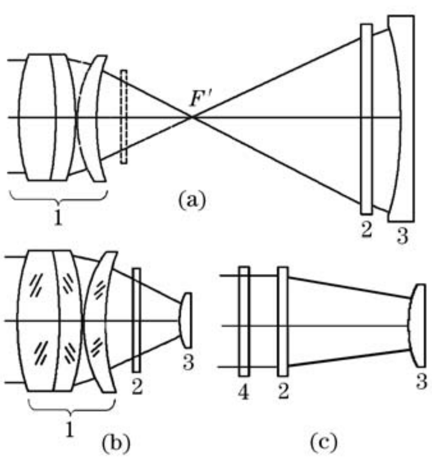

Fig. 1. Diagrams of testing the aspheric surface with CGH. 1, transmission sphere; 2, phase CGH; 3, tested aspheric surface; 4, transmission flat.

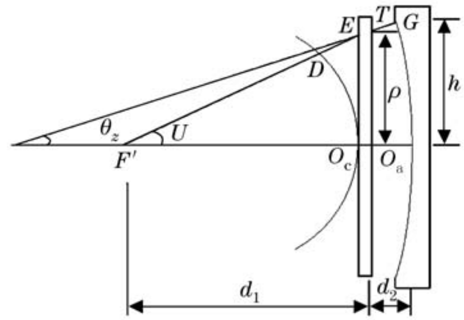

Fig. 2. Configuration parameters for calculation of phase function of CGH.

the distance of d_2 from it, then the partial configuration from the focus F' of the transmission sphere to the tested aspheric surface of Fig. 1(a) is redrawn in Fig. 2. When the ray from F' of the transmission sphere with half cone angle of U enters the CGH at the point E with the height of ρ, the emergence ray TG will enter the tested aspheric surface normally with the height of h. So the ray TG is also the normal of the tested aspheric surface at point G with the angle of θ_z from optical axis.

If one spherical wave from the left transmission sphere (not shown in Fig. 2) is tangent with the CGH at the centre $O_{\rm c}$ and intersect at point D with the ray $F'E$, then according to the basic fundamental of classic optics[9], the optical path from point D to G should equal that from point $O_{\rm c}$ to $O_{\rm a}$ ($O_{\rm a}$ is the vertex point of the tested aspheric surface). So the phase function of the CGH may be given as

$$\phi(\rho) = 2\pi \cdot w(\rho)/\lambda, \quad (3)$$

$$w(\rho) = w(\rho = 0) + d_2 - (d_2 + z)/\cos\theta_z + d_1(1 - 1/\cos U), \quad (4)$$

provided that the CGH is regarded as thin diffraction element, where

$$\theta_z = \cos^{-1}(\partial z/\partial h) - \pi/2, \quad (5)$$

$$\rho = (h/\mathrm{tg}\theta_z - z - d_2)\mathrm{tg}\theta_z = h - (z + d_2)\mathrm{tg}\theta_z, \quad (6)$$

$$U = \mathrm{tg}^{-1}(\rho/d_1). \quad (7)$$

When the value of h is variable from 0 to $h_{\max}$, the value of ρ will vary from 0 to $\rho_{\max}$ correspondingly. The values of d_1 and $\rho_{\max}$ will be determined by

$$d_1 = \rho_{\max}\mathrm{tg}U_{\max}, \quad (8)$$

$$\rho_{\max} = h_{\max} - (z|_{h=h_{\max}} + d_2)\mathrm{tg}\theta_{z\max}, \quad (9)$$

$$\theta_{z\max} = \cos^{-1}\left[\frac{\partial z}{\partial h}\right]\bigg|_{h=h_{\max}} - \frac{\pi}{2}. \quad (10)$$

By the way, the optical power of the CGH can be given by

$$\varphi'_{\rm C} = \frac{(d_1 + R_0 + d_2)}{d_1(R_0 + d_2)}. \quad (11)$$

Now the spatial frequency function of the CGH is obtained by the calculation of gradient of phase function $\phi(\rho)$,

$$\gamma(\rho) = \frac{1}{2\pi}\nabla\phi(\rho) = \frac{\phi(\rho + \Delta\rho) - \phi(\rho)}{2\pi\Delta\rho}. \tag{12}$$

By Eqs. (2)—(12), the CGH element can be designed. The phase function calculated with the above equations is the target value whether the diffraction order is ±1st or ±2nd. On the basis of Seidel aberration theory[10], for the given clear aperture of the CGH, if the 1st or 2nd diffraction order is used respectively to design the CGH and meet the same target phase function, then the coefficients of Eq. (1) for the two designs will have the relationship described by

$$a_i(\text{1st}) = 2a_i(\text{2nd}). \tag{13}$$

Of course, the 1st diffraction order light is preferred to design the CGH for its high diffraction efficiency. However, the CGH of producing the zero diffraction order light equals the parallel glass flat.

As the examples, we want to test an aspheric surface with clear aperture of 100.32 mm, which is one of elements used in a total station instrument. The tested aspheric surface is a parabola with the fourth and sixth powers of the radial coordinate to describe the asphericity. It is defined according to Eq. (2) by vertex radius R_0 of −158.47 mm, conic constant k of −1.0, coefficient A_4 of 2.32×10^{-8}, and coefficient A_6 of 2.54×10^{-12}. So the transmission sphere with f-number less than 1.5 should be selected to test the aspheric surface. In the design of CGH, actual f-number of the transmission sphere is 1.0. According to the above equations, the spatial frequency of the CGH will change with the position parameter d_2 and the maximum spatial frequency will occur at the boundary of the CGH. By calculation, the curves of the maximum spatial frequency and necessary aperture varying with the position parameter of d_2 are shown in Fig. 3. Based on the results of Fig. 3, if it is positioned near the tested concave aspheric surface with large aperture, the CGH is also necessary to have very large aperture, but the spatial frequency decreases below the value of 240 lp/mm. In general, we hope the aperture of the CGH to be small for insuring little of substrate figure error. So the cross of the two curves may be optimal.

When we choose the distance of d_2 at the cross of Fig. 3, i.e. about 36 mm, the maximum spatial frequency of the CGH at the half aperture of 38 mm is 283 lp/mm.

In general, we should fabricate the mask for the CGH at first, and then make the phase type CGH by way of e-beam exposure in silica substrate according to the mask. In this situation, if we want the wavefront difference less than a tenth wavelength ($\lambda/10$) between that produced by the CGH and the tested aspheric surface, then the tolerable position error of laser spot variable with ρ can be calculated by the derivative of Eq. (4),

$$\Delta\rho = \frac{\partial w(\rho)}{\partial\rho} \cdot \Delta w(\rho). \tag{14}$$

The curve of the tolerable position error of laser spot $\Delta\rho$ versus ρ is shown in Fig. 4. Because the spatial frequency of the boundary zone is much bigger than that of central zone, the position error in boundary zone is about 1.0 μm. By the results shown in Fig. 4, if we want to test the aspheric surface with the clear aperture of 100.32 mm, then position error near boundary ($\rho = 38$ mm) should be less than 1.0 μm, but the error near the center could get to 200 μm. It is already possible now to fabricate the CGH mask with the position error of 1 μm. The fabrication period needs about 8 hours for the CGH mask with the size of 75 mm. We have started the project of fabricating the CGH mask and the phase type CGH for the future testing experiment.

The CGH element used to test the aspheric surface can add phase to entrance spherical wave according to the phase function of Eq. (1). Because of phase value existing ambiguity of 2π, the results of phase function of the CGH for testing the same aspheric surface may be more than one. For the fore-type of aspheric surface described above, two different phase functions of the CGH are calculated by ZEMAX-EE with the same distance of d_2 (20 mm), and the results are shown in Figs. 5(a) and (b), respectively. The phase value of Fig. 5(a) has one maximum extreme value at the aperture of 37.02 mm. Under this condition, there is a zero slope of phase and all diffraction orders are overlapped at the radial position of 37.02 mm. The CGH does not have smooth circular pattern and the corresponding annular area of the aspheric surface may not be tested efficiently. However, the CGH with the phase function described as Fig. 5(b) is the better choice than that as Fig. 5(a).

In order to avoid the zero slope of the phase within the clear aperture of the CGH, the derivative of $\phi(\rho)$ must not equal zero, i.e. $\partial\phi(\rho)/\partial\rho \neq 0$. If two terms of Eq.

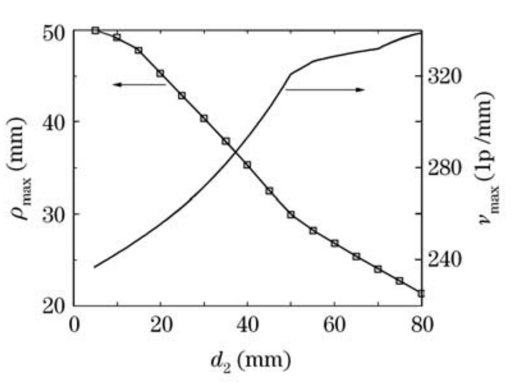

Fig. 3. Maximum spatial frequency ν_{max} and necessary aperture ρ_{max} versus the position of CGH.

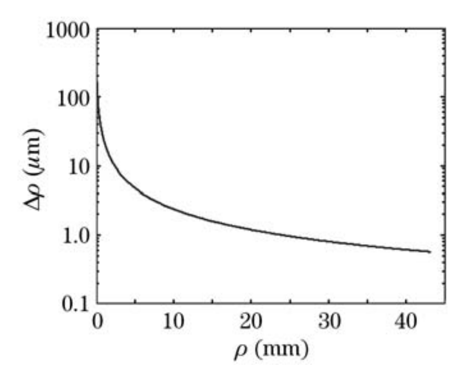

Fig. 4. Tolerable fringe position error variable with value of ρ while the wavefront difference of a CGH is less than $\lambda/10$.

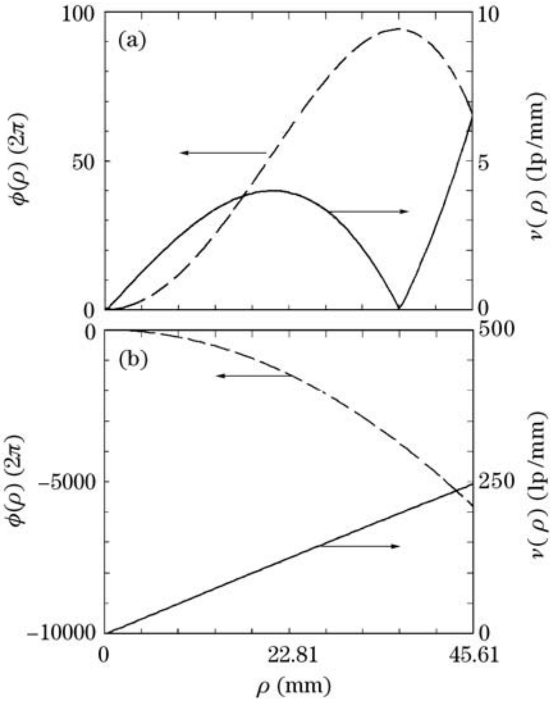

Fig. 5. Design of CGH with zero slope of phase (a) without and (b) with the constraint of Eq. (16).

(1) are chosen, the constraint for the coefficients a_1 and a_2 should be considered as

$$\frac{a_1}{a_2} > 0. \quad (15)$$

If three terms of Eq. (1) are chosen, the constraint becomes

$$a_2^2 - 6a_3a_1 < 0. \quad (16)$$

The results of phase functions in Fig. 5 are all made up of three terms of Eq. (1). But only the results of Fig. 5(b) are designed with the constraint described by Eq. (16).

By the above discussion, we have obtained a set of equations to design a phase type of CGH for testing aspheric surfaces. Based on these equations, we can determine the phase function, the radial spatial frequency distribution, the proper position at optical axis of the CGH element (transmission), and the maximum OPD error relative to the accuracy of controlling of laser spot. The designed CGH should have small aperture as well as low spatial frequency by selection of suitable position of d_2 and should avoid zero slope of phase. For a given aspheric surface example, we design the CGH with the d_2 value of 36 mm and the maximum spatial frequency of 283 lp/mm. We make the design with no zero slope of phase by use of constraint of $a_2^2 - 6a_3a_1 < 0$. These results ensure the possibility of test of an aspheric surface efficiently.

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