Analysis of eccentric photorefraction by Fourier optics

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Received May 15, 2006

Eccentric photorefraction usually is used as early eyesight diagnostic test of infants and small children. Unlike currently approved geometrical optical model of eccentric photorefractometer, the crescent formation and the light-intensity distribution in the pupil image of a myopic eye are analyzed by Fourier optics with the assumption of an isotropic scattering retina. In the case of little circular light source and rectangular slit, the simulation results of different myopic diopters are obtained by geometrical optical theory and Fourier optics respectively. It is found that the simulation results by Fourier optics are similar as those obtained by geometrical optics, and all simulations are almost corresponding to the experimental result. The result demonstrates that the new method presented here is feasible.

OCIS codes: 170.4460, 170.4470, 170.4580, 070.2580.

Eccentric photorefraction (EPR) is an important technique for measuring refractive errors. It has many advantages, such as low-cost, quick, easy and remote, which make it especially suitable for early eyesight diagnostic test of infants and small children. Since Kaakinen reported a method called EPR for the simultaneous photography of corneal and fundus reflexes using a camera and a flash unit mounted on the periphery of the lens in 1979[1], the theoretical models of EPR under different circumstances have been paid much attention. Howlland[2] used paraxial geometrical theory to get the refractive state in terms of the width of crescent, the size of pupil, the eccentricity of light source, and the distance between pupil and camera. Roorda *et al.*[3] introduced ray-tracing method based on geometrical optical theory to predict eccentric photorefraction intensity profiles for different monochromatic aberrations. Kusel *et al.*[4] used the paraxial approximation of Gaussian optics and the assumption of an isotropic scattering retina to analyze the light-intensity distribution for different eccentric photorefractor setups such as point light source, long linear light source, knife-edge aperture, and circular aperture. Chen *et al.*[5] used Zemax^{TM} and several models of human eye to analyze the three-dimensional (3D) ray trace in EPR and found that the basic physics of photorefraction could be described simply using the width and center of the dark zone of reflex images. It should be noted that these theoretical analyses are all based on geometrical optics and neglect the diffraction effect when the light passes through the slit in EPR.

In this paper, a new method is presented. Compared with foregoing works, we pay more attention to the diffraction effect due to the slit in EPR. For simplicity, the assumptions used by Kusel *et al.*[4] have been taken. In this case, aberrations of human eye are not taken into account, the light distribution scattered from retina is isotropic, and the pupil of eye is assumed to be circular. In term of the effective aperture of EPR changing with the position of the point on the pupil, EPR is not a space invariable system as usual. However, as for any single virtual point on the pupil, the light intensity distribution on charge coupled device (CCD) still attaches to the image theory. For the case, Fourier optics[6] is used for EPR firstly. The light intensity distribution in the pupil image with different myopic diopters is simulated by Fourier optics and geometrical optics respectively.

Figure 1 shows the typical optical system of EPR which is used to calculate the intensity distribution in the pupil image. The eye is assumed to be myopic. There is a point light source on the CCD camera-aperture plane. When the retina is illuminated by the source through the pupil, the isotropic backscattered light from illuminated area of retina propagates back and makes uniform irradiance on the pupil. Since the retina plane of the eye is conjugate with the far-point plane, the backscattered light from every point of the pupil will pass through the circle, which is the cross section of the illuminating light cone from the light source on the far-point plane, and form intermediate image of the illuminated retina. Then, each point on the pupil is thought as a virtual point light source. When all emitted light from different virtual point sources pass through the intermediate image, the direction of propagation depends on the location of virtual point light source. For different points on the pupil, there are different projection light cones on the CCD camera-aperture plane. Thus the effective aperture of the CCD camera lens which is the overlapping area between the CCD camera aperture and the projection cone is variable. Different diffraction effects owing to different effective apertures form different intensity patterns on the CCD, leading to the final crescent. Obviously, EPR is a very special optical image system unlike the space invariable system.

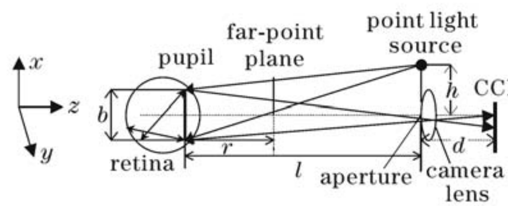

Fig. 1. Optical geometry of eccentric photorefractometer for a myopic eye.

1671-7694/2007/030146-03

In the case, the imaging formula of CCD camera lens could not be expressed by two-dimensional (2D) convolution but by superposition integral. The intensity distribution on the CCD can be described as

$$I_i(x_i, y_i) = \iint_{\infty} I_0(x_0, y_0) h_{\mathrm{I}}(x_0, y_0; x_i, y_i) \mathrm{d}x_0 \mathrm{d}y_0, \quad (1)$$

where $I_0(x_0, y_0)$ is the intensity distribution on the pupil; (x_i, y_i) is a image point on CCD; (x_0, y_0) is the conjugate object point on the pupil; $h_{\mathrm{I}}(x_0, y_0; x_i, y_i)$ is the point spread function (PSF) of CCD camera lens, and does not depend on the difference between coordinates of image and object points of the pupil but on the coordinates themselves.

The phase distribution of backscattered light from the retina on the pupil is a 2D stochastic process, so is the intensity distribution. That means the light field on the pupil is incoherent, and the PSF can be written as square of the amplitude of the coherent PSF as

$$h_{\mathrm{I}}(x_0, y_0; x_i, y_i) = |h_{\mathrm{c}}(x_0, y_0; x_i, y_i)|^2 . \quad (2)$$

Here the coherent PSF is the Fourier transform of the exit-pupil function of the CCD camera lens,

$$h_{\mathrm{C}}(x_0, y_0; x_i, y_i) = K \iint_{\infty} P(x_l, y_l)$$

$$\exp\{-j\frac{2\pi}{\lambda d}[(x_i - Mx_0)x_l + (y_i - My_0)y_l]\}\mathrm{d}x_l\mathrm{d}y_l, \quad (3)$$

where K is a complex constant, M is magnification of the CCD camera lens, λ is the wavelength of the operation light, d is the distance between the CCD and its exit-pupil, x_l and y_l are the exit-pupil coordinates, and $P(x_l, y_l)$ is its exit-pupil function.

Since the pupil is assumed to be circular, the intermediate image of the retina and the projection area are also circular. The eccentricity of the point light source (h) is far shorter than the distance (l) between the pupil and the CCD camera-aperture plane. For simplicity, assuming that the CCD camera lens is a thin lens, the exit-pupil function is the same as the entrance-pupil function and equals the overlapping area between the CCD camera aperture and the projection circle (see Fig. 2). Because the direction of propagation of the backscattered light from any point on the pupil is changing, the exit-pupil function is variable.

The radius R_{f} and central coordinates $(x_{\mathrm{f}0}, y_{\mathrm{f}0})$ of the projection circle on the far-point plane should be

$$R_{\mathrm{f}} = \frac{b}{2}\frac{l-r}{l}, \quad x_{\mathrm{f}0} = h\frac{r}{l}, \quad y_{\mathrm{f}0} = 0, \quad (4)$$

where b represents the diameter of the pupil, r is the distance between the pupil and the far-point plane, as shown in Fig. 1. The area on the camera lens illuminated by a point of pupil is also a circle with central coordinates (x_{l0}, y_{l0}) and radius R_l,

$$x_{l0} = x_0(1 + \frac{l}{r}) - h, \quad y_{l0} = y_0\frac{r-l}{r}, \quad R_l = \frac{l-r}{2r}b. \quad (5)$$

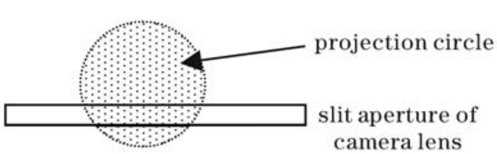

Fig. 2. Effective exit-pupil of camera lens.

Assuming that the light intensity on the pupil is uniform, all simulations are done with a pupil diameter of 6 mm, the CCD camera lens aperture of 27 mm, and focal length of 135 mm. The distance between the CCD camera lens and the eye is 1 m, the eccentric distance is 6 mm, the wavelength of monochromatic light emitted by the source is assumed to be 555 nm, the diameter of circular light source is 1 mm, and the CCD camera entrance pupil is a 2 × 27 (mm) rectangular slit whose length is parallel to y_l-coordinate. Firstly, calculate the central coordinates and radius of the projection circle from a point of the pupil on the CCD camera plane by Eq. (5) (or Eq. (4)). Secondly, calculate the overlapping area of the CCD camera aperture and the projection circle. Thirdly, calculate the coherent PSF of CCD camera for this point of the pupil by Eq. (3) and the incoherent PSF by Eq. (2). Finally, calculate the intensity distribution of the pupil image by Eq. (1). The calculations are iterative and done by computer.

The simulation results by Fourier optics are shown in Fig. 3. It is found that the width of the crescent increases and the shape of the crescent takes a change as the myopic diopter increases. The intensity distribution

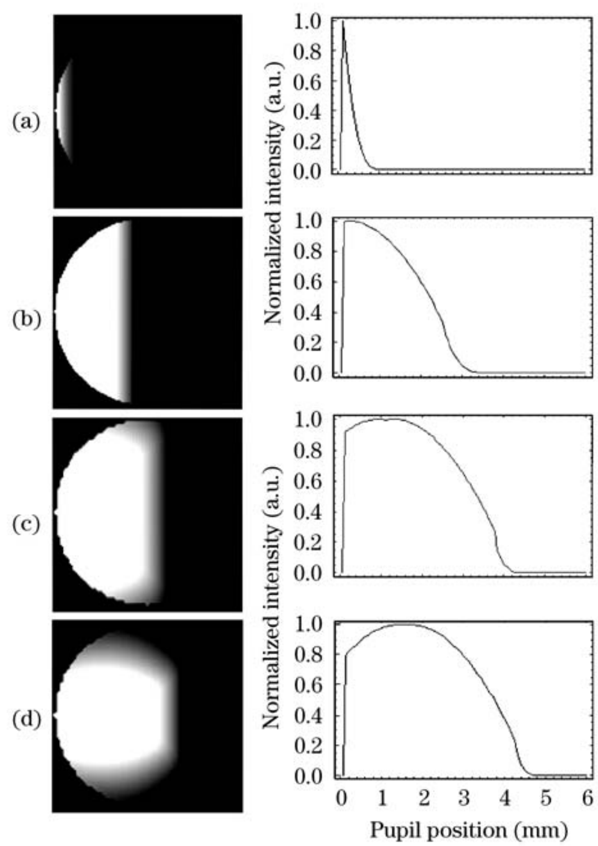

Fig. 3. Crescents and normalized intensities on the meridian of the images simulated by Fourier optics at different myopic diopters. (a) −2.0D; (b) −3.0D; (c) −4.0D; (d) −5.0D.

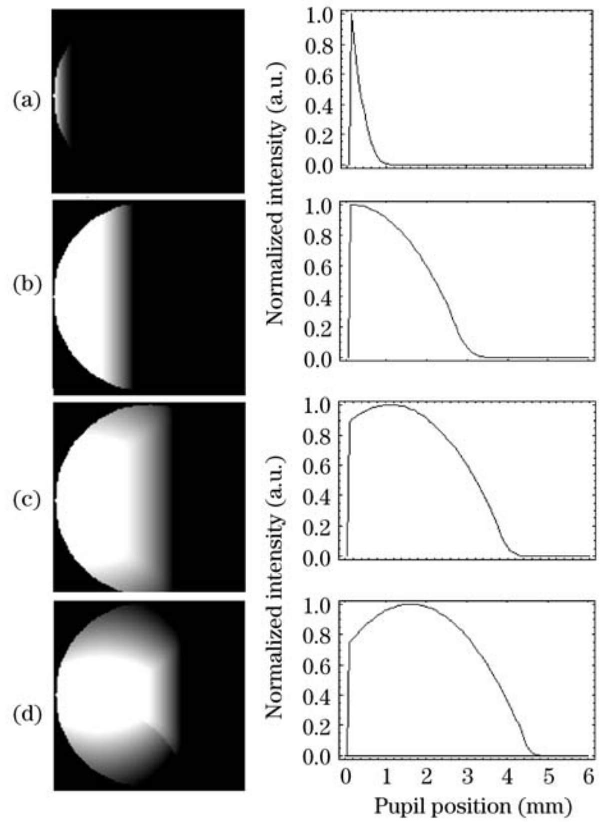

Fig. 4. Crescents and normalized intensities on the meridian of the images simulated by geometric optics at different myopic diopters. (a) −2.0D; (b) −3.0D; (c) −4.0D; (d) −5.0D.

of the crescent is symmetrical. In vertical orientation of the meridian, the intensity is almost uniform in Figs. 3(a) and (b), but has a maximum along the symmetric axis in Figs. 3(c) and (d). When the refractive error increases, the slope intensity profile across the center from EPR images becomes less sensitive. In EPR, the center location and width of the dark zone are very important parameters to measure the refractive state of the eye. Compared with those images (see Fig. 4) obtained by geometrical optical method[7], the widths of dark zones are similar, while the slope profiles of dark zones show difference due to the diffraction effect on the boundary of the slit. When the effective pupil of CCD camera lens is very small, diffraction effect becomes significant and could not be neglected. When the light source, the aberrations of the eye, the light distribution scattered from retina, and the shape of pupil are taken into account, the light intensity distribution of the pupil image would be simulated more accurately. Figure 5 shows the experimental result of a volunteer with −5.0 myopic diopters. In comparison, the measured width of dark zone is almost corresponding to the theoretical value obtained by geometrical optics and Fourier optics. However, the shape of crescent is different from the simulation owing to the wavelength band and bigger size of the light source, the shape of pupil, and the background light from the reflection of the pupil.

Fig. 5. Measured normalized intensity from a volunteer with −5.0 myopic diopters.

We have presented a new theoretical analysis of EPR using Fourier optics, described the forming of the crescent, and simulated the intensity distribution of the pupil image. Compared with geometric optics analysis, the results from this investigation show that the effective CCD camera aperture depends on the location of the points on the pupil, and the diffraction effect is dominant when the effective CCD camera aperture is very small. For the simulations using Fourier optics and geometrical optics, it is found that the widths of dark zones are similar, while slope profiles of dark zones show differences. The experiment result has a shape of crescent different from the simulation, but the measured width of dark zone is almost equal to the theoretical value. Since the boundary determines the width of dark zone with which the refractive errors are calculated directly, the analysis using Fourier optics is suitable for EPR.

This work was supported by the National Basic Research Program of China (No. 2005CB724304), the Optical Science and Technology Program of Science and Technology Committee of Shanghai (No. 046105014), and the Youth Program of Zhaoqing University (No. 0303). The authors' e-mail addresses are liuyong7612@sina.com (Y. Liu) and jbchenk@online.sh.cn (J. Chen).

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