Feature analysis of the scale factor variation on a constant rate biased ring laser gyro

Shiqiao Qin (秦石乔), Zongsheng Huang (黄宗升), and Xingshu Wang (王省书)

College of Optoelectric Science and Technology, National University of Defense Technology, Changsha 410073

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Scale factor of a constant rate biased ring laser gyro (RLG) is studied both theoretically and experimentally. By analyzing experimental data, we find that there are three main terms contributing to the scale factor deviation. One of them is independent of time, the second varies linearly with time and the third varies exponentially with time. Theoretical analyses show that the first term is caused by experimental setup, the second and the third are caused by un-uniform thermal expension and cavity loss variation of the RLG.

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Scale factor is one of the key parameters describing the ring laser gyro (RLG) performance. Especially in a rate biased RLG, the scale factor deviation is the major factor affecting the RLG performance. Reducing the scale factor variation is the main way to increase the accuracy of a rate biased RLG. To overcome the scale factor variation, a rate biased RLG usually rotates clock-wise (CW) and counter-clock-wise (CCW) symmetrically[1]. A special type of rate biased RLG is constant rate biased RLG, in which the RLG rotates in one direction in a constant rate and the signal is measured every 360° rotation. With the potential of high measuring accuracy, a constant rate biased RLG is valuable for study on the effect caused by the scale factor variation in a RLG and for study on rate biased RLG.

According to Gao *et al.* and Aronowitz *et al.*[2,3], the frequency difference between the CW and CCW traveling wave of the RLG is

$$\begin{aligned}\Delta\upsilon &= \upsilon_2-\upsilon_1=\dot{\psi}=\frac{4\vec{A}}{L\lambda}\cdot\vec{\Omega}+(\sigma_2-\sigma_1)+(\rho_2I_2-\rho_1I_1)\\&\quad+(\tau_{21}I_1-\tau_{12}I_2)+\Omega_{\mathrm{l}}\sin(\psi+\chi)\\&=K_0\Omega\cos\theta+\frac{\mathrm{d}\sigma}{\mathrm{d}\upsilon}\Delta\upsilon+\frac{\mathrm{d}\rho}{\mathrm{d}\upsilon}\bar{I}\Delta\upsilon+\frac{\rho_2+\rho_1}{2}\Delta I\\&\quad+\frac{\mathrm{d}\tau}{\mathrm{d}\upsilon}\bar{I}\Delta\upsilon+\frac{\tau_{21}+\tau_{12}}{2}\Delta I+\Omega_{\mathrm{l}}\sin(\psi+\chi)\\&\approx K_0\cos\theta\Omega(1+\frac{\mathrm{d}\sigma}{\mathrm{d}\upsilon}+\frac{\mathrm{d}\rho}{\mathrm{d}\upsilon}\bar{I}+\frac{\mathrm{d}\tau}{\mathrm{d}\upsilon}\bar{I})\\&\quad+\Omega_{\mathrm{l}}\sin(\psi+\chi)+(\frac{\rho_2+\rho_1}{2}\Delta I+\frac{\tau_{21}+\tau_{12}}{2}\Delta I)\\&=K_0\cos\theta(1+\mathrm{SFC})\Omega+\Omega_{\mathrm{l}}\sin(\psi+\chi)+\Delta\upsilon_{\mathrm{d}}\\&=K\Omega+\Omega_{\mathrm{l}}\sin(\psi+\chi)+\Delta\upsilon_{\mathrm{d}}=\sqrt{(K\Omega)^2-\Omega_{\mathrm{l}}^2}+\Delta\upsilon_{\mathrm{d}}\\&=K\Omega\sqrt{1-\left(\frac{\Omega_{\mathrm{l}}/K}{\Omega}\right)^2}+\Delta\upsilon_{\mathrm{d}},\end{aligned}\tag{1}$$

where $K_0=4A/L\lambda$ is the geometrical scale factor. Scale factor correction (SFC) caused by abnormal dispersion of the gain media is a function of the RLG working frequency. SFC of the RLG is about minus hundreds ppm. $K=K_0\cos\theta(1+\mathrm{SFC})$ is the scale factor, where θ is the angle from the RLG's sensitive axis to the direction of angular velocity Ω. Ω_{l} is the threshold of lock-in. $\Delta\upsilon_{\mathrm{d}}$ is drift, which can be treated as a constant.

At present, Ω_{l}/K is about 100 deg./h and the rotating rate is more than 100 deg./s in constant rate biased RLG. So $(\Omega_{\mathrm{l}}/K\Omega)^2\le 0.077$ ppm, which can be neglected. Thus Eq. (1) can be simplified as

$$\Delta\upsilon=K\Omega+\Delta\upsilon_{\mathrm{d}}.\tag{2}$$

The scale factor K can be measured and calibrated. The nominal value of K is K_{n}, which differs a little from K. K can be written as $K=K_{\mathrm{n}}(1+\Delta\mathrm{SFC})$, where ΔSFC is the change of SFC and it is about a few ppm.

In a constant rate biased RLG, the signal is sampled every 360° rotation. Let the rotating angular rate be Ω_{r}, the vertical component of the earth's rate be Ω_{e}, and the rotating period $T=360$ deg./Ω_{r}. Integrating Eq. (2) in one rotating period, the RLG's output can be obtained by

$$\begin{aligned}N&=\int_0^T\Delta\upsilon\mathrm{d}t=\int_0^T[K(\Omega_{\mathrm{r}}+\Omega_{\mathrm{e}})+\Delta\upsilon_{\mathrm{d}}]\mathrm{d}t\\&=K\int_0^T\Omega_{\mathrm{r}}\mathrm{d}t+K\Omega_{\mathrm{e}}T+\int_0^T\Delta\upsilon_{\mathrm{d}}\mathrm{d}t\\&=K\times360\times3600+K\Omega_{\mathrm{e}}T+\int_0^T\Delta\upsilon_{\mathrm{d}}\mathrm{d}t.\end{aligned}\tag{3}$$

Equation (3) can be written as

$$\Omega_{\mathrm{e}}+\frac{\int_0^T\Delta\upsilon_{\mathrm{d}}\mathrm{d}t}{KT}=\Omega_{\mathrm{e}}+\Delta\Omega_{\mathrm{d}}=\frac{N-K\times1296000}{KT}$$

$$\approx \frac{N - K_{\rm n}(1+\Delta{\rm SFC})\times 1296000}{K_{\rm n}T}$$

$$= \frac{N - K_{\rm n}\times 1296000}{K_{\rm n}T} - \frac{\Delta{\rm SFC}\times 1296000}{T}$$

$$= \frac{N - K_{\rm n}\times 1296000}{K_{\rm n}T} - \Delta{\rm SFC}\times\Omega_{\rm r}, \quad (4)$$

where $\Delta\Omega_{\rm d}$ is a constant written as

$$\Delta\Omega_{\rm d} = \frac{\int_0^T \Delta\upsilon_{\rm d}{\rm d}t}{KT}.$$

The unit of $\Omega_{\rm r}$ and $\Omega_{\rm e}$ is arcsec./s or deg./h, the unit of K is pulse/arcsec. or Hz/deg./h. The items in the left of Eq. (4) include the vertical component of the earth's rate $\Omega_{\rm e}$ and drift $\Delta\Omega_{\rm d}$. The items in the right include the output of a constant rate biased RLG and the correction due to scale factor variation. The form of Eq. (4) is convenient to study the effect caused by scale factor variation.

Changing Eq. (4) in an input/output mode, we obtain

$$\Omega_{\rm e} + \Delta\Omega_{\rm d} + \Delta{\rm SFC}\times\Omega_{\rm r} = \frac{N - K_{\rm n}\times 1296000}{K_{\rm n}T}. \quad (5)$$

The items in the left include the input angular rate to be measured and all kinds of errors, and the item in the right is the output of a constant rate biased RLG. Equations (4) and (5) are the two forms describing the output of a constant rate biased RLG.

The experiment setup of a constant rate biased RLG is shown in Fig. 1, which includes a rotating platform, a mechanics on which the RLG is mounted, an optoelectric synchronous trigger offering synchronous sampling signal, a data sampling circuit and a computer. The RLG's sensitive axis is in the vertical direction.

A number of experiments have been done with the setup. One typical experiment result is shown in Fig. 2. In the experiment, the mean rotating rate $\Omega_{\rm r}$ is about 243.35 deg./s, the rotating period T is about 1.48 s, the total circles is 19615 and the total time is 29018 s, the scale factor measured by the experiment $K_{\rm n} = 2.1442501069$ Hz/deg./h. The mean output of the RLG $\Omega_{\rm m} = 7.365$ deg./h. The vertical component of the earth's rate where the experiments are done $\Omega_{\rm e} = 7.1$ deg./h. Difference of the two values is caused by the non-horizontal installation of the RLG.

From Fig. 2 we can see a transitional process after the RLG start-up, which is similar to the exponential

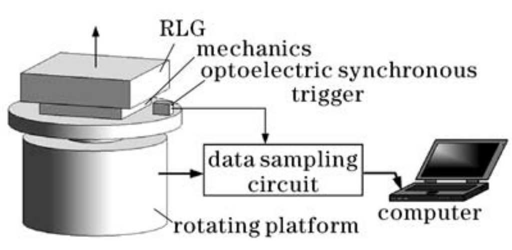

Fig. 1. Experiment setup of constant rate biased RLG.

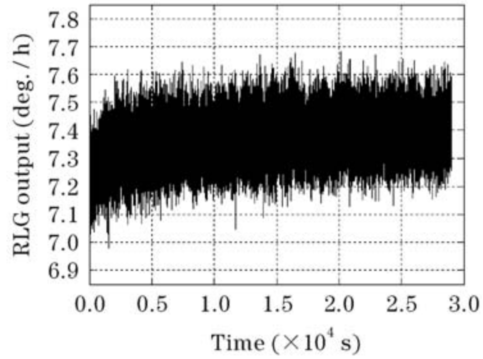

Fig. 2. Typical experiment results of constant rate biased RLG.

function. Following the transitional process the RLG's output becomes smooth. According to Eqs. (4) and (5), the transitional process indicates the scale factor variation, namely ΔSFC is in action. The RLG's output changes about 0.15 deg./h in 8 hours and the noise is about 0.4 deg./h, so it is difficult to study the effect of ΔSFC with the angular rate-time curve. To emphasize the effect of ΔSFC, we substract the mean value $\Omega_{\rm m}$ from Eq. (5) and integrate it, the angle-time curve can be obtained by

$$\int_0^{nT} (\Omega_{\rm e} + \Delta\Omega_{\rm d} + \Delta{\rm SFC}\times\Omega_{\rm r} - \Omega_{\rm m}){\rm d}T$$

$$= \int_0^{nT} (\frac{N - K_{\rm n}\times 1296000}{K_{\rm n}T} - \Omega_{\rm m}){\rm d}T, \quad (6)$$

where n is the circles. The result of Eq. (6) is shown as the solid line in Fig. 3(a). If $\Delta{\rm SFC} = 0$, the solid line in Fig. 3(a) should be a flat line passing zero. This indicates the serious effect of ΔSFC, which is a function of time. Based on the analysis of Fig. 2, the solid line in Fig. 3(a) can be fitted with

$$\Theta = b_0 + b_1 t + b_2 t^2 + b_3 {\rm e}^{-\alpha t}. \quad (7)$$

The fitting result is

$$\Theta = -156.133 - 0.01275t$$
$$+0.00000063t^2 + 152.324{\rm e}^{-0.00048t}. \quad (8)$$

The fitting result is shown as the dashed line in Fig. 3(a), which is in accordance with the experiment result. With higher order terms of t, the fitting result is almost equal to that of Eq. (8) and the coefficient of higher order term is much smaller than b_2 and almost equals to zero. Results of many experiments are in accordance with the above conclusion. The analysis indicates that Eq. (7) is an impersonal description of the scale factor variation.

b_2 in Eq. (7) shows an angular acceleration term in the output of constant rate biased RLG, which is difficult to be interpretted with the RLG theory. Let $b_2 = 0$ and fit the curve with 3 items in Eq. (7), we can get the result shown as dashed line in Fig. 3(b), where the solid line is the experiment result. Figure 3(b) shows an obvious

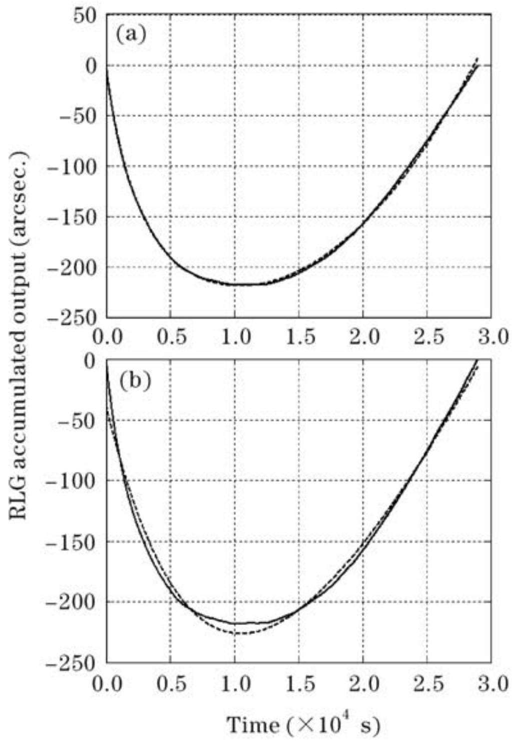

Fig. 3. Output angle-time curve (solid line) of constant rate biased RLG and the fitting curve (dashed line). (a) Curve fitted with 4 terms; (b) curve fitted with 3 terms.

deviation of the fitting result. The above analysis shows that b_2 indicates a certain physical process. Though the effect of the b_2 term is less than that of the b_1 term, the b_2 term is indispensable.

By differentiating Eq. (7), namely differentiating Eq. (6), and comparing with Eq. (5), the scale factor variation can be obtained by

$$\Delta\text{SFC} \times \Omega_\text{r} = \frac{\text{d}\Theta}{\text{d}t} = b_1 + 2b_2 t - \alpha b_3 \text{e}^{-\alpha t}. \quad (9)$$

By differentiating Eq. (8) we can get the result shown in Fig. 4(a), in which the mean value Ω_m is added to the differentiating result for comparison with Fig. 2. Obviously, Fig. 4(a) completely reflects the trend in Fig. 2. According to Eq. (4), we can get the result shown in Fig. 4(b) by substracting the scale factor change ΔSFC from the output of constant rate biased RLG, namely substracting the differentiating result of Eq. (8) from the output shown in Fig. 2. Obviously, the performance of constant rate biased RLG is improved.

Equation (9) can be written as

$$\Delta\text{SFC} = b_1/\Omega_\text{r} + (2b_2/\Omega_\text{r})t - (\alpha b_3/\Omega_\text{r})\text{e}^{-\alpha t}$$
$$= \Delta\text{SFC}_0 + \Delta\text{SFC}_1 t + \Delta\text{SFC}_2 \text{e}^{-\alpha t}, \quad (10)$$

where the first term is a normal term, the second is an angular acceleration term proportional to time, and the third is an exponential term. According to the above analysis, Eq. (10) is an impersonal description of the scale factor variation in a constant rate biased RLG. Thus the three terms reflect some error natures in constant rate biased RLG. This will be analyzed in detail hereafter.

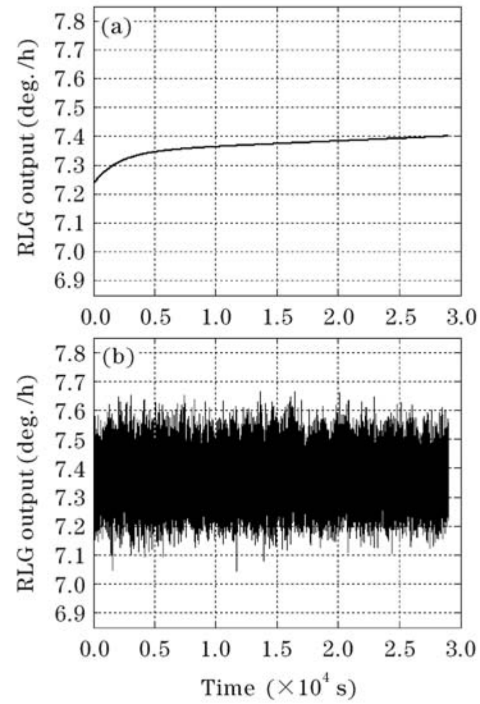

Fig. 4. (a) Curve of scale factor change; (b) correction result of RLG's output.

The normal term ΔSFC_0 is deviation of the nominal value K_n from the true value K. The nominal value K_n is measured by experiment under certain conditions, so it unavoidably differs a little from the true value K. According to Eq. (8), $\Delta\text{SFC}_0 = -2.15 \times 10^{-8} = -0.0215$ ppm.

The second term with the coefficient ΔSFC_1 is an angular acceleration term proportional to time. According to the RLG theory[4,5], no physical factor produces such an error, so it is caused by external factor. Analysis shows that it is due to the installation angle variation caused by the thermal deformation. According to Eq. (1), the scale factor K includes $\cos\theta$, where θ is unavoidable and it changes minutely when the RLG heats or the environmental temperature changes. The value of θ is relative to the mechanical structure, material of the mechanism, especially the change of temperature, thus it is relative to time. SFC by the change of θ is $-\sin\theta \text{d}\theta$. When θ is small and $\text{d}\theta$ is much smaller, for example $\text{d}\theta \ll 1''$, $\text{d}\theta$ varies linearly with time under the ordinary temperature condition, namely $\text{d}\theta \propto t$. Thus the SFC changes linearly with time. According to Eq. (8), $\Delta\text{SFC}_1 = 2.139 \times 10^{-6}$ ppm/s $= 0.0077$ ppm/h. Let $\theta = 1°$, and the change of θ in an hour can be attained by $\text{d}\theta = \Delta\text{SFC}_1/(-\sin\theta) = -0.09''$, which is very small. When θ is larger, $\text{d}\theta$ becomes larger and it is a complicated function of time. When θ is near 90°, experiments show that $\text{d}\theta$ can change 10 arcsec./h, which perfectly verifies the above suggestion.

The third term with the coefficients ΔSFC_2 and α is an exponential term. Analysis shows that it is caused by the thermal effect when RLG starts up. According to Ref. [3],

$$\text{SFC} = \text{SFC}_\text{n} - \frac{aG}{1 + xP_\text{o}} + (\sqrt{1 - (\frac{\Omega_\text{l}/K}{\Omega})^2} - 1), \quad (11)$$

where SFC_n is the normal scale factor correction, namely the first term mentioned above. The last term in Eq. (11) is the SFC due to lock-in of RLG, which is the same as that in Eq. (1). The second term due to the working media in RLG is relative to gain and loss. It can be written as

$$\mathrm{SFC_m} = -\frac{aG}{1 + xP_o}, \quad P_o = P_c(G/\Gamma - 1), \tag{12}$$

where G is gain, Γ is loss, P_o is the output power, α, x, and P_c are experiential parameters. By differentiating Eq. (12) the effect of G and Γ can be attained, and the ratio of them is

$$\left(\frac{\partial}{\partial \Gamma}\mathrm{SFC_m}\right) \Big/ \left(\frac{\partial}{\partial G}\mathrm{SFC_m}\right) = \frac{xP_c(G/\Gamma)^2}{1 - xP_c}. \tag{13}$$

For most practical gyros the G/Γ ratio is about 2 or greater. According to Ref. [3], xP_c is 0.56 and for most practical gyros it is about several tenths. Equation (13) shows that the effect of Γ is much larger than that of G. And the variation of gain in RLG is minute because of the discharge current control. Thus the variation of SFC_m is mainly arised from Γ. According to experiment data offered by Aronowitz[3], SFC_m and Γ change exponentially with time. This indicates that in the process of RLG start-up to stabilization, thermal deformation caused by temperature variation results in exponential change of Γ. SFC_m reflects the third term of the above experiment, namely

$$\mathrm{SFC_m} = \Delta\mathrm{SFC_2}\mathrm{e}^{-\alpha t}. \tag{14}$$

According to the data in Eq. (8), $\Delta SFC_2 = 1.23 \times 10^{-7} = 0.123$ ppm, $\alpha = 4.8 \times 10^{-4}$ (1/s). $1/\alpha = 2083$ s is the thermal delay time after RLG start-up.

Through theoretical and experimental study on constant rate biased RLG, the rules and formulas of scale factor variation are concluded. Experiments show that the SFC is composed of three terms, one is the normal term, the second is the angular acceleration term and the third is the exponential term. Theoretical analysis shows that the normal term is a constant deviation of scale factor, the angular acceleration term is due to the installation angle variation caused by the thermal deformation. The exponential term is due to the loss variation of the RLG caused by the internal thermal effect. The results in this paper such as Eq. (11) are in better accordance with practical application than that of Aronowitz[3], so they are more convenient to practical application and real-time correction. Further more, the analysis includes the effect of angular acceleration due to the installation angle variation. These show great value to the application of RLG and to the study of rate biased RLG.

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