Effect of the reflection of underlying surface on sky radiance distribution

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Sky radiance might be influenced by the multiple reflectance between the earth's albedo surface and the atmosphere. Based on the Lambert's law and the radiative transfer equation (RTE), a model is developed to calculate the additional sky radiance at wavelengths of 0.4—3 μm due to the reflectance contribution of the underlying surface. The iterative method is used to calculate sky radiance without the reflectance from underlying surface. The hybrid modified delta-Eddington approximation is used to compute the atmospheric reflection of the radiation from the earth's surface. An interaction factor is introduced to deal with the multiple reflectance between the atmosphere and the underlying surface. The sky radiance increment is evaluated for some different albedos of the earth's surface. The results show that the sky radiance increment rises rapidly while viewing zenith angle is near to 90°, and the larger the albedo of the earth's surface is, the more obvious this effect appears.

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The background radiance is very important for sky observation, and it may be affected from the reflectance of the earth's surface[1]. In 1971, Herman *et al.*[2] measured the sky radiance for the albedos of the earth's surface of 0.1 and 0.15, respectively. In 1980, Liou[3] described the calculation of the sky radiance according to the invariance principle, but this method was too complicated to evaluate the transmission or reflectance of the atmosphere directly. In 1989, Zibordi *et al.*[4] reckoned and measured the sky radiance at different wavelengths, but this model was restricted to the case that viewing zenith angle is less than 75°. In 2004, Boudak *et al.*[5] set up a mathematical model of solar radiation reflected by underlying surface based on the small angle modification of spherical harmonics method, but his result did not accord with Herman's measurement values. Though we can evaluate sky radiance in different ways with LOWTRAN 7 or MODTRAN software[6], the reflectance of the earth's surface was only taken into account while observing the earth's surface from the space. An approximate model is developed to evaluate the effect on the sky radiance increment as a result of the reflection from the earth's surface based on the radiative transfer equation (RTE).

Assume that the earth's surface conforms to the Lambert's law with a surface albedo r_s. Thus, the upwards diffuse radiance is given by

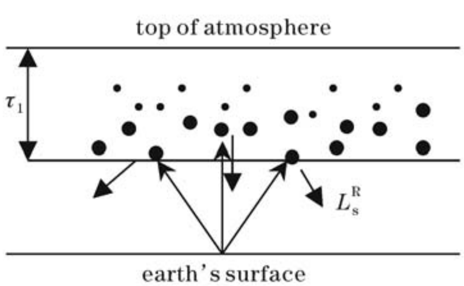

Fig. 1. Schematic diagram of reflected downward radiance by the atmosphere.

$$L(\tau_1;\mu,\phi)=L_\mathrm{s}(\mathrm{constant}), \tag{1}$$

where τ_1 is the total optical depth of the atmosphere; μ, ϕ are the cosine of viewing zenith angle and azimuth angle, respectively.

The upward isotropic radiance from the earth's surface will be downwards diffused by the atmosphere, so additional contribution to sky radiance as shown in Fig. 1 is

$$\begin{aligned}L_\mathrm{s}^\mathrm{R}(-\mu)&=\frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu,\phi;\mu',\phi')L_\mathrm{s}\mu'\mathrm{d}\mu'\mathrm{d}\phi'\\&=L_\mathrm{s}\rho(\tau_1;-\mu).\end{aligned} \tag{2}$$

Thereby, the total sky radiance including the contribution of the earth's albedo surface can be given by

$$\begin{aligned}L^*(\tau_1;-\mu,\phi)&=L(\tau_1;-\mu,\phi)+L_\mathrm{s}^\mathrm{R}(-\mu)\\&=L(\tau_1;-\mu,\phi)+L_\mathrm{s}\rho(\tau_1;-\mu),\end{aligned} \tag{3}$$

here $L(\tau_1;-\mu,\phi)$ denotes the sky radiance while the reflectance of the earth's surface is not taken into account, $\rho(\tau_1;-\mu)$ is reflectance of the atmosphere.

According to Eq. (3), we should evaluate the upward diffuse radiance of the earth's surface L_s at first. Now let $f(\mu,\phi;\mu',\phi')$ represent the bidirectional reflectance distribution function (BRDF) of underlying surface. If the diffuse reflectance at the earth's surface conforms to the Lambert's law, its BRDF is

$$f(\mu,\phi;\mu',\phi')=\frac{r_\mathrm{s}}{\pi}, \tag{4}$$

where r_s is the albedo of the earth's surface. As shown in Fig. 2, the diffused fluxes include the solar direct flux, the fluxes scattered singly by the atmosphere, and the double or more order scattered fluxes. Downward fluxes at the earth's surface include three components, as following: 1) The solar flux transmitted directly, $\mu_0\pi F_0\mathrm{e}^{-\tau_1/\mu_0}$, πF_0 is the solar flux when the sun is

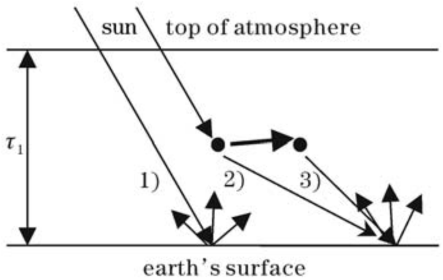

Fig. 2. Composite fluxes downwards at the earth's surface.

incident perpendicularly on the top of the atmosphere, μ_0 is the cosine of the solar zenith angle. 2) The solar flux scattered singly by the atmosphere, $\int_0^{2\pi}\int_0^1 L_1(\tau_1;-\mu,\phi)\mu\mathrm{d}\mu\mathrm{d}\phi$. 3) The solar flux double and quasi-multiple scattered by the atmosphere, $\int_0^{2\pi}\int_0^1 (L_2(\tau_1;-\mu,\phi)+L_3(\tau_1;-\mu,\phi))\mu\mathrm{d}\mu\mathrm{d}\phi$. If once interaction between the earth's surface and the atmosphere is only taken into account, upward diffuse radiance at $\tau=\tau_1$ is therefore

$$\begin{aligned} L_\mathrm{s} &= \mu_0\pi F_0\mathrm{e}^{-\tau_1/\mu_0} f(\mu,\ \phi;-\mu_0,\phi_0) \\ &+\int_0^{2\pi}\int_0^1 L_1(\tau_1;\Omega')f(\mu,\ \phi;\mu',\phi')\,\mu'\mathrm{d}\mu'\mathrm{d}\phi, \\ &+\int_0^{2\pi}\int_0^1 L_2(\tau_1;\Omega')f(\mu,\ \phi;\mu',\phi')\,\mu'\mathrm{d}\mu'\mathrm{d}\phi \\ &+\int_0^{2\pi}\int_0^1 L_3(\tau_1;\Omega')f(\mu,\ \phi;\mu',\phi')\,\mu'\mathrm{d}\mu'\mathrm{d}\phi, \end{aligned} \quad (5)$$

where ϕ_0 is the solar azimuth angle, the negative symbol within $f(\mu,\ \phi;-\mu_0,\phi_0)$ represents a downward direction of the solar radiation. $L_1(\tau;\Omega)$, $L_2(\tau;\Omega)$ and $L_3(\tau;\Omega)$ are respectively the downward single scattering, double scattering and quasi-multiple scattering component at arbitrary optical depth τ and direction angle Ω, their values can be calculated with the iterative method from the RTE in the approximation of the plane-parallel atmosphere, as

$$\begin{aligned} L_n(\tau,-\mu,\phi) &= \frac{1}{\mu}\int_0^{2\pi}\int_{-1}^1 J_{n-1}(\tau;\mu',\phi') \\ &\times p(\mu',\phi';-\mu,\phi)\mu'\mathrm{d}\mu'\mathrm{d}\phi', \end{aligned} \quad (6)$$

where $J_n()$, $p()$ are respectively the nth source function and the one-term Henny-Greenstain scattering phase function.

Equation (5) comes into existence while the earth's surface is slightly rough. Multiple interactions may occur under other ways, thereupon we introduce an interaction factor $\beta(r_\mathrm{s})$ to keep the format of Eq. (5) unchanged. When multiple interactions occur, more diffuse radiance will be scattered back to the surface. An empirical model based on experiments of this factor, which depends only on the surface bihemispherical reflectance is given by

$$\beta(r_\mathrm{s}) = \begin{cases} 2.0-\frac{0.15}{r_\mathrm{s}} & \text{if } r_\mathrm{s}>0.15 \\ 1.0 & \text{otherwise} \end{cases} . \quad (7)$$

From Eq. (7), we can see that if the surface albedo is less than 0.15, multiple interactions between the earth's surface and the atmosphere can be negligible. The interaction factor $\beta(r_\mathrm{s})$ is proportional to the surface reflectance. Thus we need multiply the surface BRDF by the interaction factor in order to account for the multiple interactions while keeping the single interaction formula.

As described in Eq. (2), the another factor affecting sky radiance increment is the atmospheric reflectance, i.e., the ratio of downward flux reflected by the atmosphere from the earth's diffuse radiance to incident flux. According to the Helmholtz's principle of reciprocity[7], we have

$$\begin{cases} T(\mu,\phi;\mu',\phi') = T(\mu',\phi';\mu,\phi) \\ R(\mu,\phi;\mu',\phi') = R(\mu',\phi';\mu,\phi) \end{cases} , \quad (8)$$

where $T(\mu,\phi;\mu',\phi')$, $R(\mu,\phi;\mu',\phi')$ are transmission and reflection function, respectively.

Thus, we obtain the atmospheric reflectance from the earth's surface to space

$$\rho(\tau_1;-\mu) = \frac{F^+(\tau=0)}{\mu\pi F_0}, \quad (9)$$

where $F^+(\tau=0)$ represents the upward reflected flux.

Although Liou set up an equation group, which was constituted with four nonlinear differential integral eqautions to evaluate the reflectance of the atmosphere based on the invariance principle[3], it was very difficult to calculate the reflectance of the atmosphere. Therefore reflectance of the atmosphere was usually calculated by making use of various approximate methods from the RTE, for example double-adding method, two-stream approximation, modified Eddtingdon approximation, discrete-ordinate method (DISORT)[8], etc.. Though DISORT is deemed a precise method to deal with the multiple scattering problem of RTE, it is too complicated to use. Despite of the simpleness of Liou's two-stream approximation, the results have usually biggish error. Meador *et al.* developed a mode named the hybrid modified delta-Eddington approximation[9,10] to deal with multiple scattering accurately and quickly. Thus, this method is used to calculate the atmospheric reflection here.

Equation (9) was used to compute the atmospheric reflection for some different optical depth. Figure 3

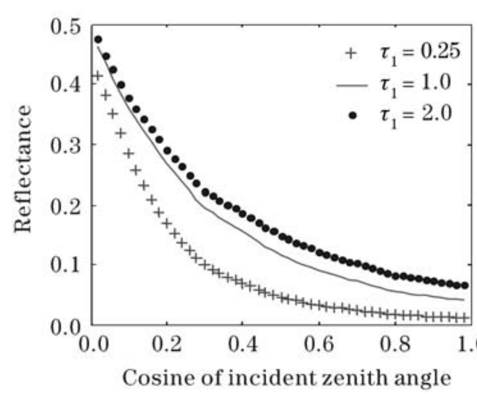

Fig. 3. Reflection of upward diffuse radiation by the atmosphere.

shows atmospheric reflectance as a function of the cosine of incident zenith under conditions: $\varpi_0 = 0.8$, $g = 0.75$, $\tau_1 = 0.25$, 1.0, 2.0, respectively. Note that the reflectance descents as the cosine of the incident angle increases, but the atmospheric reflectance rises gradually as the optical depth increases.

In the following data analysis, all calculations are implemented by setting $F_0 = 1$ for simplicity, and thus downward radiance increment due to the reflection of the underlying surface is denoted as a relative sky radiance. Some results are given in Figs. 4 and 5 under conditions: $\varpi_0 = 1.0$, $g = 0.65$, $\tau_1 = 0.44$, $\theta_0 = 33°$. Relevant comparison is made in Fig. 4 and our analytic model provides good agreement with Tanaka's values[11]. Figure 5 shows the viewing zenith angle distribution of the relative sky radiance for three different surface albedos of 0.15, 0.25, and 0.35, the distribution of the sky radiance increment has a marked limb-brightening effect, and influence of reflectance of underlying surface on sky radiance becomes stronger while the earth's albedo increases.

Figure 6 shows the surface reflection contribution to the downward sky radiance under conditions: $\varpi_0 = 1.0$, $g = 0.65$, $r_s = 0.25$, $\theta_0 = 33°$, $\theta = 45°$, $\phi - \phi_0 = 180°$. It obviously shows that influence of underlying surface on sky radiance becomes enhanced while the atmospheric total optical depth increases, but there is a maximum for the sky radiation increment at a certain atmospheric optical depth.

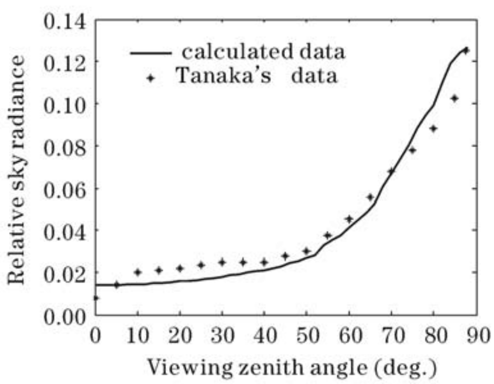

Fig. 4. Comparison of additional sky radiance.

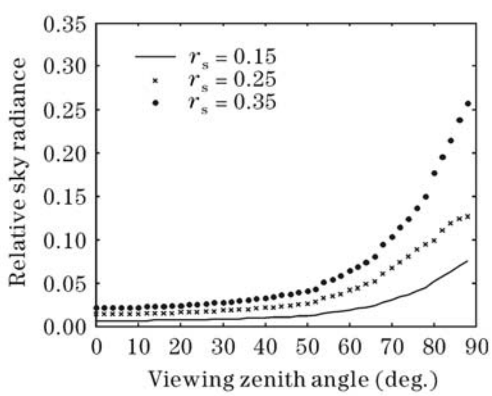

Fig. 5. Distribution of relative sky radiance under different albedos of underlying surface.

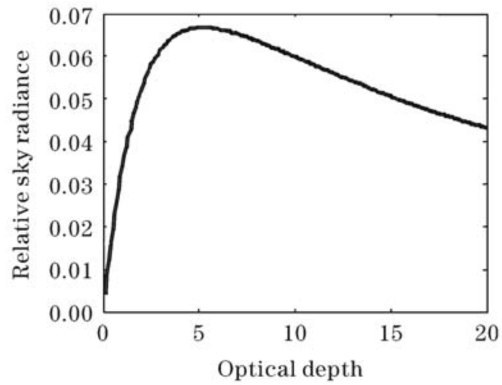

Fig. 6. Distribution of relative sky radiance along with the atmospheric optical depth.

In conclusion, with the precondition of the Lambert surface, the viewing zenith angle distribution of the sky radiance increment is evaluated and analyzed by virtue of the RTE and the hybrid modified delta-Eddington approximation. The results show that influence on the sky radiance becomes enhanced as the albedo of underlying surface rises, and it will have a limb-brightening effect while the line of sight is near to horizontal direction. This is a result of the longer path of radiative transmission and stronger multiple interactions while viewing zenith angle is close to 90°.

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References

1. L. Guo, Y. Wang, and Z. Wu, Chin. Opt. Lett. **2,** 431 (2004).
2. B. M. Herman, S. R. Browning, and R. J. Current, J. Atmos. Sci. **8,** 419 (1971).
3. K. N. Liou, *Introduction to Atmospheric Radiation* (Academic Press, New York, 1980).
4. G. Zibordi and K. J. Voss, Remote Sense Environment **27,** 343 (1989).
5. V. P. Boudak, A. V. Kozelsky, and E. N. Savisky, Proc. SPIE **5396,** 221 (2004).
6. A. Berk, L. S. Bernstein, and D. C. Roberston, Technology Report GL-TR-89-0122, Air Force Geophysics Laboratory (1994).
7. S. Chandrasekhar, *Radiative Transfer* (Dover, New York, 1960).
8. K. Stamnes, S. C. Tsay, W. Wiscombe, and K. Jayaweera, J. Appl. Opt. **27,** 2502 (1988).
9. W. E. Meador and W. R. Weaver, J. Atmos. Sci. **37,** 630 (1980).
10. W. J. Wiscombe and G. W. Grams, J. Atmos. Sci. **33,** 2440 (1976).
11. M. Tanaka, J. Met. Soc. Japan **49,** 321 (1971).