Modulation properties of spatial three-waveguide system using weakly coupled mode theory

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Based on the weakly coupled mode theory, the modulation properties of three-waveguide system are analyzed in general. We examine the modulation behavior for two cases that a voltage is applied on the beam-launched waveguide or non-beam-launched waveguide. The analytical intensity distributions in both cases are given. Applications of the spatial multi-waveguide coupling systems include spatial light modulators, optical switches, optical interconnection, and spatial optical signal processing.

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Many applications, such as large data streams, image manipulation, artificial vision, nerve networks, and optical calculations, need two-dimensional (2D) spatial optical signal processing. To meet those demands, we should develop integrated optical devices employing three-dimensional (3D) structures[1,2], which exhibit a higher level of integration and new functionality for both sensing and telecommunications.

The spatial multi-waveguide, waveguides on different planes, has been incorporated into many 3D integrated optical devices[3−6]. It provides these active and passive elements with the routing capabilities previously found in free space holographic interconnects. The coupled mode theory[7] presents a general analytical means for traditional planar directional couplers composed of two waveguides in the same plane. And the switching properties of the planar three-waveguide system have also been discussed[8]. However, the modulation properties of spatial multi-waveguide have not been reported yet.

In this paper, we apply the weakly coupled mode theory to analyze the modulation properties of the spatial three-waveguide system and describe the power transfer among these spatial waveguides. The analytical expressions of the guided power in each waveguide are given.

We consider a spatial three-waveguide system formed by three identical waveguides, as shown in Fig. 1. In the case of $K_{12} = K_{13} = K_{23} = K$ (K_{jl} is the coupling coefficient between waveguides j and l, which is a function of the separation distance between the waveguides and the refractive index distribution), the coupled-mode equations for the spatial three-waveguide with propagation along the z-direction can be described as

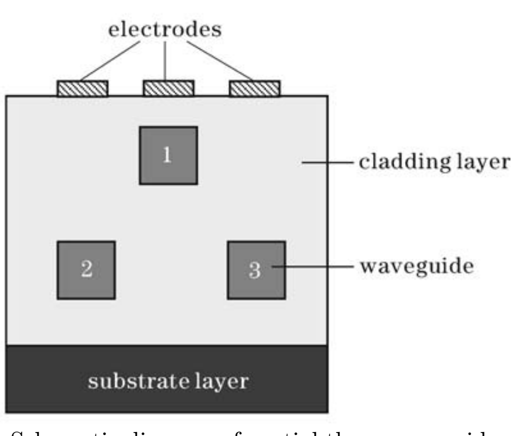

Fig. 1. Schematic diagram of spatial three-waveguide system.

$$\frac{\partial A_1}{\partial z} = iKA_2 \exp(-i\Delta\beta_1 z) + iKA_3 \exp(i\Delta\beta_3 z - i\Delta\beta_1 z), \tag{1a}$$

$$\frac{\partial A_2}{\partial z} = iKA_1 \exp(i\Delta\beta_1 z) + iKA_3 \exp(i\Delta\beta_3 z), \tag{1b}$$

$$\frac{\partial A_3}{\partial z} = iKA_1 \exp(i\Delta\beta_1 z - i\Delta\beta_3 z) + iKA_2 \exp(-i\Delta\beta_3 z), \tag{1c}$$

here A_i ($i = 1,2,3$) is the amplitude of field in each waveguide, β_i ($i = 1,2,3$) is the propagation constant of each waveguide, $\Delta\beta_1$ and $\Delta\beta_3$ are detunings of β_1 and β_3 from β_2, respectively,

$$\Delta\beta_1 = \beta_1 - \beta_2, \tag{2a}$$

$$\Delta\beta_3 = \beta_3 - \beta_2. \tag{2b}$$

For mathematical simplicity, we use the following substitutions:

$$a_1 = A_1 \exp(i\Delta\beta_1 z), \tag{3a}$$

$$a_2 = A_2, \tag{3b}$$

$$a_3 = A_3 \exp(i\Delta\beta_3 z), \tag{3c}$$

yielding

$$\frac{\partial}{\partial z}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = i\begin{bmatrix} \Delta\beta_1 & K & K \\ K & 0 & K \\ K & K & \Delta\beta_3 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}. \tag{4}$$

The matrix equation can be used to solve the weakly coupled problem of the spatial three-waveguide system and predict the intensity distribution in each waveguide.

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If the light power is injected into the waveguide 1, the initial conditions will be

$$A_1(0) = 1,\ A_2(0) = A_3(0) = 0. \quad (5)$$

We discuss the modulation properties of the spatial three-waveguide system under three cases: no applied voltage, voltage applied on the beam-launched, and non-beam-launched waveguides.

Firstly, considering the case of $\Delta\beta_1 = \Delta\beta_3 = 0$ (no applied voltage). Solving Eq. (4), and we can get the guided power in each waveguide,

$$|a_1(z)|^2 = \frac{1}{9}[5 + 4\cos(3Kz)], \quad (6)$$

$$|a_2(z)|^2 = |a_3(z)|^2 = \frac{2}{9}[1 - \cos(3Kz)]. \quad (7)$$

We define the coupling length L_c as the half of the oscillation period of the field amplitudes,

$$L_c = \frac{\pi}{3K}. \quad (8)$$

At the length of even coupling length, the power in waveguide 1 reaches the maximum value 1, and the power in waveguides 2 and 3 reaches the minimum value 0. At the length of odd coupling length, the power in waveguide 1 reaches the minimum value 1/9, and the power in waveguides 2 and 3 reaches the maximum value 4/9, respectively.

Secondly, $\Delta\beta_3 = 0$, $\Delta\beta_1 = \Delta\beta$ (applying voltage on the beam-launched waveguide). In this case, the solution of Eq. (4) can be written as

$$|a_1(z)|^2 = 1 - \frac{8K^2}{B^2}\sin^2(\frac{1}{2}zB), \quad (9a)$$

$$|a_2(z)|^2 = |a_3(z)|^2 = \frac{4K^2}{B^2}\sin^2(\frac{1}{2}zB), \quad (9b)$$

where

$$B = \sqrt{9K^2 - 2\Delta\beta K + \Delta\beta^2}. \quad (10)$$

Figure 2 shows the intensity in each waveguide as a function of the detuning amount when $L = L_c$. From it, we can see that the incident optical power can be completely returned back to waveguide 1 and can also be equally divided among the three waveguides as long as we applied proper detuning amount.

For the incident beam to completely return back to waveguide 1, the coupling coefficient K, the length of waveguides L, and $\Delta\beta$ should satisfy

$$L\sqrt{9K^2 - 2\Delta\beta K + \Delta\beta^2} = 2N\pi \quad (N = 0, 1, 2, \cdots). \quad (11)$$

Different from the planar three-waveguide system, the bar state of spatial three-waveguide system is on a set of concentric ellipses with 45° angle between major axis and coordinate axis. The lengths of major and minor axes of these concentric ellipses are $\sqrt{6}N$ and $\sqrt{3}N$.

Thirdly, $\Delta\beta_1 = 0$, $\Delta\beta_3 = \Delta\beta$ (applying voltage on the non beam-launched waveguide). Solving Eq. (4) under the initial conditions, and we can get the guided power in each waveguide as

$$|a_1(z)|^2 = 0.5 - \frac{2K^2}{B^2}\sin^2(\frac{1}{2}zB) + \frac{1}{2}\cos(\frac{1}{2}zB)\cos(\frac{3}{2}Kz + \frac{1}{2}\Delta\beta z) - \frac{K-\Delta\beta}{2B}\sin(\frac{1}{2}zB)\sin(\frac{3}{2}Kz + \frac{1}{2}\Delta\beta z), \quad (12a)$$

$$|a_2(z)|^2 = 0.5 - \frac{2K^2}{B^2}\sin^2(\frac{1}{2}zB) - \frac{1}{2}\cos(\frac{1}{2}zB)\cos(\frac{3}{2}Kz + \frac{1}{2}\Delta\beta z) + \frac{K-\Delta\beta}{2B}\sin(\frac{1}{2}zB)\sin(\frac{3}{2}Kz + \frac{1}{2}\Delta\beta z), \quad (12b)$$

$$|a_3(z)|^2 = \frac{4K^2}{B^2}\sin^2(\frac{1}{2}zB). \quad (12c)$$

Figure 3 shows the intensity in each waveguide as a function of the detuning amount when $L = L_c$. From the figure, it can be seen that the optical power in waveguide 3 can be completely transferred to waveguides 1 and 2.

The modulation properties of spatial three-waveguide system consisting of identical waveguides have been analyzed based on the weakly coupled mode theory. The modulation properties of spatial three-waveguide system are quite different from planar one, because the coupling

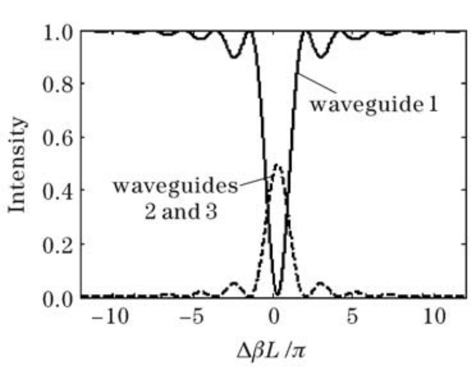

Fig. 2. Intensity versus $\Delta\beta L/\pi$ when $L = L_c$ in the case of voltage applied on the beam-launched waveguide.

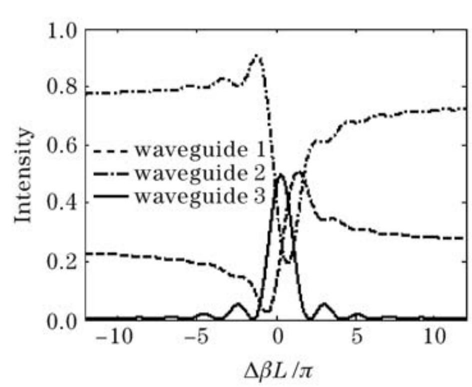

Fig. 3. Intensity versus $\Delta\beta L/\pi$ when $L = L_c$ in the case of voltage applied on the non-beam-launched waveguide.

between two arbitrary waveguides cannot be ignored. Other systems with the number of waveguides larger than three or formed by nonidentical waveguides can also been analyzed in the similar way. The spatial multi-waveguide system can be designed for the applications of 3D switches and modulators by using thermo-optic and electro-optic effects.

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