Focal depth extending using rotational symmetric pupil masks

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Received May 15, 2006

Optical systems are analyzed with three kinds of rotational symmetric pupil masks: annular Gaussian ring mask, supergaussian ring mask, and quartic phase mask. In these masks, the quartic phase mask is found to be the best one to extend focal depth. Point spread function (PSF) and Strehl ratio (SR) are used to evaluate the imaging quality of the system with different defocus parameters. Without decoding needed, the focal depth of the system with quartic phase mask is four times as deep as aberration-free system. Different from the others, it suffers no obvious loss in the light throughput and lateral resolution. With twice focal depth extension, supergaussian ring mask suffers less loss in light throughput and lateral resolution than annular Gaussian ring mask.

OCIS codes: 070.0070, 110.0110, 090.1000, 120.5060.

Large focal depth enables optical system to achieve more objective information and reduces defocus-related aberrations[1]. Pupil modulation is a widely used method to solve this problem, including amplitude modulation and phase modulation. The most common way is to reduce relative aperture, which is at the expense of light throughput and lateral resolution. Aspheric surface[2−4] is used widely as another common method to get large focal depth. Wavefront coding (WFC)[5,6], the integration of optical design and digital image processing, is a novel and promising method. However, the mask used in WFC is non-rotational symmetric and difficult to manufacture and test. As a result, we will focus on rotational symmetric masks in this paper. Pupil masks with annular Gaussian ring and with supergaussian rings are discussed as amplitude masks while quartic phase mask is discussed as phase mask. Point spread function (PSF) and Strehl ratio (SR) are used to evaluate the imaging quality of these systems.

For an optical system with defocus parameter W_{20}, we can describe impulse response $P(x, y, W_{20})$ as the Fourier transformation of the pupil function $p(x', y')$

$$P(x,y,W_{20}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(x',y') \exp\left[-i\frac{2\pi}{\lambda}(xx'+yy')\right] \times \exp\left[-i2\pi W_{20}\left(x'^2+y'^2\right)\right] \mathrm{d}x'\mathrm{d}y', \tag{1}$$

where $i = \sqrt{-1}$; (x', y') and (x, y) represent the orthogonal coordinates in the pupil plane and image plane, respectively; λ is wavelength.

Considering that polar coordinate is more appropriate in rotational symmetrical optical system, let $\begin{cases} x' = r\cos\theta \\ y' = r\sin\theta \end{cases}$ and $\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \end{cases}$, where (r, θ) and (ρ, ϕ) represent the polar coordinate in the pupil plane and image plane, respectively. Note P is independent of θ or ϕ. So Eq. (1) is derived as

$$P(\rho, W_{20}) = 2\pi\int_0^{+\infty} p(r)\exp\left(-i2\pi W_{20}r^2\right) \times J_0\left(2\pi\rho r/\lambda\right) r\mathrm{d}r, \tag{2}$$

where J_0 is the zero order Bessel function of the first kind.

PSF $I(\rho, W_{20})$ can be written as

$$I\left(\rho, W_{20}\right) = P\left(\rho, W_{20}\right) P^*\left(\rho, W_{20}\right). \tag{3}$$

SR, defined as $S(W_{20}) = \frac{I(0,W_{20})}{I(0,0)}$, is a simple criterion to evaluate the imaging quality of defocus optical system. Usually, imaging quality is acceptable when SR ≥ 0.8. $I(0, 0)$ is a constant in certain system, so

$$S(W_{20}) = \kappa I(0, W_{20}), \tag{4}$$

where κ is a constant. Consequently, we focus on $I(0, W_{20})$, which is rewritten as

$$I\left(0, W_{20}\right) = \pi^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(r)p^*(r') \times \exp\left[-i2\pi W_{20}\left(r^2-r'^2\right)\right]\mathrm{d}r^2\mathrm{d}r'^2. \tag{5}$$

In order to simplify Eq. (5), we define

$$q(\zeta) = p(r), \tag{6}$$

$$(r/r_0)^2 = \zeta + 1/2 \quad (-1/2 \leq \zeta \leq 1/2), \tag{7}$$

where r_0 is the radius of the pupil. Then we can get

$$I(0, W_{20}) = \pi^2\int_0^1\left[\int_0^1 q\left(\zeta+\frac{\zeta'}{2}\right)q^*\left(\zeta-\frac{\zeta'}{2}\right) \times \exp(-i2\pi W_{20}\zeta')\mathrm{d}\zeta'\right]\mathrm{d}\zeta. \tag{8}$$

Note that $W_g(x,u) = \int_{-\infty}^{+\infty} g\left(x+\frac{x'}{2}\right)g^*\left(x-\frac{x'}{2}\right) \times \exp\left(-i2\pi x'u\right)\mathrm{d}x'$ is in the form of Wigner distribution function (WDF). We obtain $I(0, W_{20}) =$

$\pi^2 \int_{-\infty}^{+\infty} W_q(\zeta, W_{20}) \mathrm{d}\zeta$. Substitute it into Eq. (4), SR can then be described as

$$S(W_{20}) = \kappa \int_{-\infty}^{+\infty} W_q(\zeta, W_{20}) \mathrm{d}\zeta. \tag{9}$$

According to the analysis above, properties of the optical system can be described by pupil function. Consequently, the system can be modulated if a mask is added in the pupil plane. Generally, pupil mask can be divided into two classes, amplitude mask and phase mask.

Firstly, let us take a look at an aberration-free optical system: $p(r) = \mathrm{circ}(r/r_0)$. Considering Eq. (7), Eq. (6) becomes $q(\zeta) = \mathrm{rect}(\zeta)$. With substitution into Eqs. (3) and (9), the one-dimensional (1D) PSF and SR are described in Figs. 1 and 2. From Fig. 1, we found that the peak intensity of PSF is about 0.8 when $W_{20} = 0.25$ which is consistent with SR distribution in Fig. 2, i.e., the focal depth of the aberration-free system is 0.5. The peak intensity of PSF becomes lower and lower when defocus parameter gets larger. The focus intensity PSF can be approximated by a Gaussian function, which is an efficient and accurate algorithm used to estimate the position of an object to be tracked[7]. However, when $|W_{20}| > 0.25$, the intensity PSF can hardly be fitted into a Gaussian function any more, which means the imaging quality is unacceptable.

Focal depth can be extended by applying supergaussian ring mask in the pupil plane[8]. The pupil function is given by $P(r) = \mathrm{circ}(r/r_0) \exp\left[-\frac{(r/r_0)}{\Omega}\right]^{2\alpha}$, where α and Ω are parameters of mask, which determine the amplitude and width of the pupil function, respectively. Equation (6) then becomes $q(\zeta) = \mathrm{rect}(\zeta) \exp\left[-\frac{(\zeta+1/2)^\alpha}{\Omega^{2\alpha}}\right]$. When $\alpha = 1$, the amplitude transmittance becomes an annular Gaussian ring.

Pupil functions with annular Gaussian ring and supergaussian rings are both taken into consideration. Let $\alpha = 1$, $\Omega = 0.4$ and $\alpha = 5$, $\Omega = 0.7$ for each situation, and the amplitude of the pupil function is described in Fig. 3. Compared with the clear pupil, the amplitude decreases, which means the light throughput is reduced. When $W_{20} = 0.5$, the peak intensity of PSF still reaches to 0.8, that is to say, the focal depth reaches to 1, twice of the aberration-free system's focal depth. This conclusion can also be obtained from the SR distribution curve shown in Fig. 2. The intensity PSF is described in Fig. 4. Obviously, the peak intensity gets higher and the shape still can be approximated by Gaussian function even if defocus parameter exceeds focal depth. However, compared with Fig. 1, both of their PSFs have been broadened, which will decrease the lateral resolution.

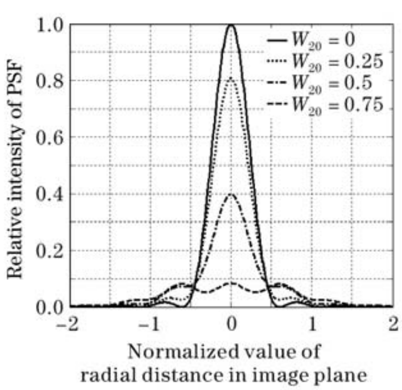

Fig. 1. 1D PSF of aberration-free optical systems with different defocus parameters.

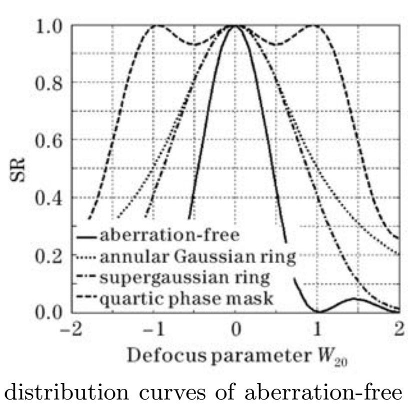

Fig. 2. SR distribution curves of aberration-free optical system and systems with three kinds of masks.

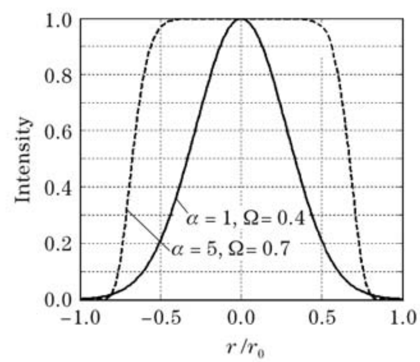

Fig. 3. Amplitudes of the pupil functions with annular Gaussian ring mask and supergaussian ring mask.

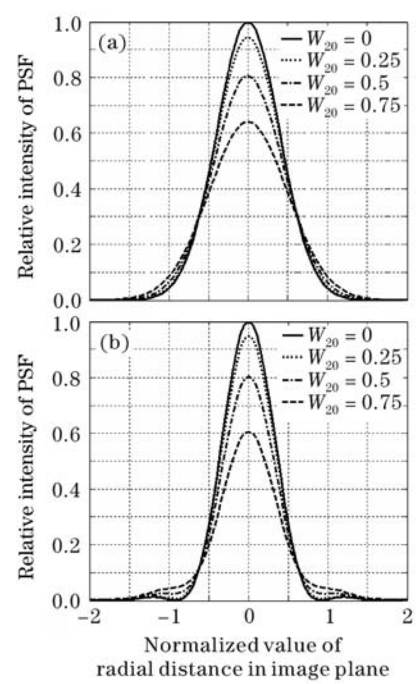

Fig. 4. 1D PSF distributions of the system with (a) annular Gaussian ring mask, $\alpha = 1$, $\Omega = 0.4$; and (b) supergaussian ring mask, $\alpha = 5$, $\Omega = 0.7$.

The system with supergaussian ring mask has a better performance than the one with annular Gaussian ring mask. Its width of amplitude of pupil function is much larger, which means larger light throughput; its width of PSF is smaller, which means higher lateral resolution. Thereby, both of them can extend the focal depth. Larger light throughput and higher lateral resolution will be achieved by using supergaussian ring mask other than annular Gaussian ring mask.

Quartic phase mask is introduced to extend the focal depth[9], with which the pupil function is given by $p(r) = \mathrm{circ}(r/r_0)\exp\left\{-i\pi\alpha\left[(r/r_0)^4-(r/r_0)^2+1/4\right]\right\}$. Considering Eq. (7), in this case, Eq. (6) becomes $q(\zeta) = \mathrm{rect}(\zeta)\exp(-i\pi\alpha\zeta^2)$.

Assuming $\alpha = 4.55$ and with substitution into Eqs. (3) and (9), 1D PSF is obtained, as shown in Fig. 5. SR distribution curve is shown in Fig. 2. From Fig. 5, it is obvious that the focal depth has been greatly extended. The peak intensity of PSF is greater than 0.8 even if $W_{20} = 1$. That is to say, the focal depth is extended for more than four times while the focal depth of system using amplitude mask is extended for twice. Light throughput will never be affected as the mask is a phase mask. Furthermore, we can learn from Fig. 5 that the influence on the width of PSF is so small that it can be ignored, i.e., the lateral resolution can maintain the same level with the aberration-free system. However, the side lobe of the PSF increases a little when defocus parameter is small. This performance will influence the imaging quality.

Although the focus imaging quality will be influenced a little, the focal depth of the system with quartic phase mask can be extended into more than four times of the aberration-free system with no deterioration on light throughput.

Using a spoke as an object, the simulated images of different systems mentioned above are described in Fig. 6. There are four lines in Fig. 6, which describe images of aberration-free system, system with annular Gaussian ring mask ($\Omega = 0.4$), system with supergaussian ring mask ($\alpha = 5$, $\Omega = 0.7$), and system with quartic phase mask ($\alpha = 4.55$), respectively. And the four columns in Fig. 6 represent four defocus situations of $W_{20} = 0$, 0.25, 0.5, and 0.75, respectively. Consistent with the above analysis, aberration-free system can get a sharp image

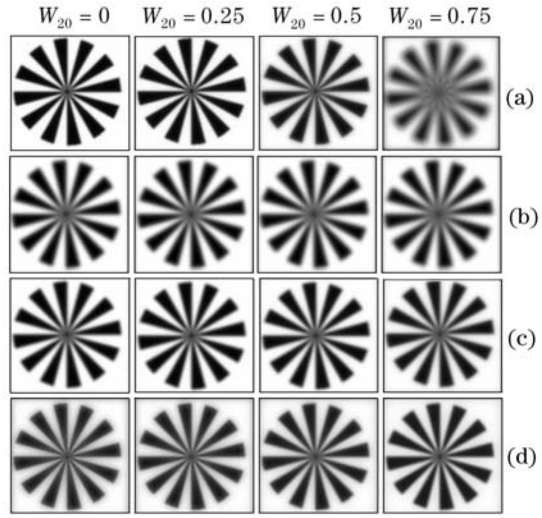

Fig. 6. Images of different systems with different defocus parameters. (a) Aberration-free system; (b) system with annular Gaussian ring mask, $\Omega = 0.4$; (c) system with supergaussian ring mask, $\alpha = 5$, $\Omega = 0.7$; (d) system with quartic phase mask, $\alpha = 4.55$.

when W_{20} is within 0.25, i.e. SR is larger than 0.8; and the image is more blurred when the defocus parameter becomes large. Compared with the aberration-free system, the images of all other three systems have a much better quality when $W_{20} = 0.75$. That is to say, systems with pupil modulated mask can extend focal depth to some extent, although the focus imaging quality is influenced inevitably. The imaging quality of the system with quartic mask is better than the one with supergaussian ring mask, which is better than the one with annular Gaussian ring mask when $W_{20} = 0.75$.

PSF and SR are studied here to evaluate the imaging quality of defocus optical system. With regard to light throughput and lateral resolution, quartic mask performs better than supergaussian ring mask, and the latter is better than annular Gaussian ring when the focal depth is extended to the same extent.

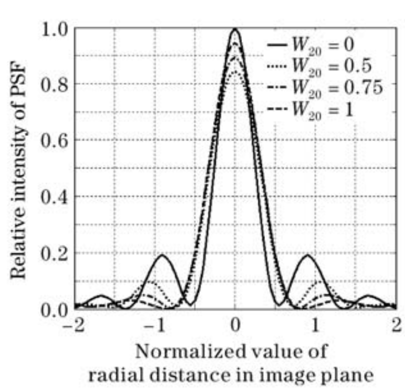

Fig. 5. 1D PSF distributions of optical system with quartic phase mask with different defocus parameters ($\alpha = 4.55$).

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References

1. E. R. Dowski, Jr., W. T. Cathey, and S. C. Bradburn, Opt. Rev. **3,** 429 (1996).
2. X. Dong and C. Du, Acta Opt. Sin. (in Chinese) **24,** 869 (2004).
3. Y. Li, Acta Opt. Sin. (in Chinese) **24,** 865 (2004).
4. J. Pan, Acta Opt. Sin. (in Chinese) **23,** 1475 (2003).
5. E. R. Dowski, Jr. and W. T. Cathey, Appl. Opt. **34,** 1859 (1995).
6. W. Zhang, Y. Chen, T. Zhao, Z. Ye, and F. Yu, Chin. Opt. Lett. **4,** 515 (2006).
7. X. Liu, X. Cai, and C. P. Grover, Proc. SPIE **5174,** 51 (2003).
8. J. Ojeda-Castañeda, J. C. Escalera, and M. J. Yzuel, Opt. Commun. **114,** 189 (1995).
9. D. Zalvidea and E. E. Sicre, Appl. Opt. **37,** 3623 (1998).