Multi-impurity effects on the entanglement of anisotropic Heisenberg ring XXZ under a homogeneous magnetic field

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The effects of multi-impurity on the entanglement of anisotropic Heisenberg ring XXZ under a homogeneous magnetic field are studied. The impurities make the equal pairwise entanglement in a ring compete with each other so that the pairwise entanglement exhibits oscillation. If the impurities are of larger couplings, both the critical temperature and pairwise entanglement can be improved.

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Entanglement is not only the fabulous feature of quantum mechanics but also very important to the quantum information processing (QIP)[1]. In the studies of quantum entanglement, solid state system with Heisenberg model interaction is the simple and applicable candidates for the realization of quantum information. Therefore, there are many works focusing mainly on the different kinds of Heisenberg models[2−21] such as spin ring etc..

The impurities often exist in solid system and play a very obvious and important part in condensed matter physics. As a candidate of QIP, solid system with impurity is also one of our important study objects. In the previous researches, the impurity effects on the quantum entanglement have been studied in a three-spin system[22,23] and a large spin system under zero temperature[24]. However, in these works, they have just studied single impurity.

In this paper, we study the effects of multi-impurity on the pairwise thermal entanglement in a ring chain. It is found that the impurities make the equal pairwise entanglement in a ring compete with each other. If impurities are of large couplings, the critical temperature and the pairwise entanglement which coupled to the impurities can be improved. The results not only provide a standard to judge impurities but also provide a way to enhance entanglement and critical temperature.

Firstly, we investigate the multi-impurity effect when the impurities are non-nearest neighbors as shown in Fig. 1. For the case of Fig. 1(a), the Hamiltonian can be written as

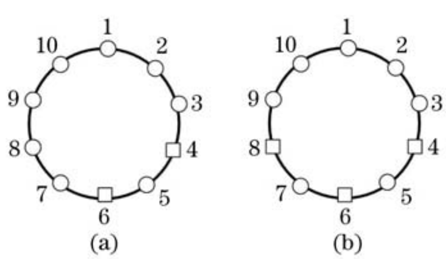

Fig. 1. Two configurations of spin ring when the impurities are non-nearest neighbors. (a) Qubit ring formed with 10 qubits, the 4th and 6th are two identical impurities; (b) qubit ring formed with 10 qubits, the 4th, 6th, and 8th are three identical impurities. Square represents impurity qubit and round stands for normal qubit.

$$H = \frac{1}{2}\sum_{i=1}^{2}[J(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J_z\sigma_i^z\sigma_{i+1}^z]$$
$$+\frac{1}{2}\sum_{i=7}^{N}[J(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J_z\sigma_i^z\sigma_{i+1}^z]$$
$$+\frac{1}{2}\sum_{i=3}^{6}[J'(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J'_z\sigma_i^z\sigma_{i+1}^z]$$
$$+\frac{1}{2}\sum_{i=1}^{N}B(\sigma_i^z + \sigma_{i+1}^z), \tag{1}$$

where $(\sigma_i^x, \sigma_i^y, \sigma_i^z)$ is the vector of Pauli matrices, J and J_z are the real coupling coefficients of arbitrary nearest neighboring two qubits. We restrict $B \geq 0$ along z direction and $N+1=1$. Choosing the parameters B, J, J_z, and T are dimensionless and assuming the coupling coefficients between normal qubit and impurity, one has the relation

$$J' = \alpha * J, \quad J'_z = \alpha * J_z, \tag{2}$$

where α characterizes the relative strength of the extra coupling between the impurity and its nearest neighboring qubits[24]. For the case of Fig. 1(b), one can write it easily following Eq. (1).

As we know, for a system in equilibrium at temperature T, the density operator is $\rho = (1/Z)\exp(-H/k_\mathrm{B}T)$, where $Z = \mathrm{Tr}[\exp(-H/k_\mathrm{B}T)]$ is the partition function and k_B is Boltzman's constant. For simplicity, we write $k_\mathrm{B} = 1$. The value of entanglement between two qubits can be measured by Concurrence C which is written as[25−28]

$$C = \max(0, 2\max\lambda_i - \sum_{i=1}^{4}\lambda_i), \tag{3}$$

where λ_i is the square root of the eigenvalues of the matrix

$$R = \rho(\sigma_1^y \otimes \sigma_2^y)\rho^*(\sigma_1^y \otimes \sigma_2^y), \tag{4}$$

where ρ is the density matrix and the symbol * stands for the complex conjugate. The Concurrence can be calculated no matter whether ρ is pure or mixed. In the following, we just take the pairwise entanglement into account. We will trace over the qubits and study the reduced density matrix of the two qubits which we are interested in.

Now, we review the difference between an ideal ring chain and an open chain. For an ideal ring chain, every qubit is of the same position with the others so that any pairwise qubits are of the same amount of entanglement. But for an ideal open chain, pairwise entanglement is related to the position of the qubits and exhibit oscillations due to the breaking of the symmetries[24].

Because here we study multi-impurity, we cannot obtain analytic expression of the system. We will directly numerically calculate and plot entanglement. Figure 2 is the pairwise entanglement as a function of α, corresponding to Figs. 1(a) and (b), respectively. For both of the two cases, in the regions far away from impurities, the entanglements, for example C_{12} and C_{101}, are slightly affected by the various values of α. Within the impurities regions, it is observed that there is the almost same threshold value of α, after which a qubit and its nearest impurity start entangling such as C_{34}, C_{45} etc.. Figure 3 clearly shows the pairwise entanglement versus site i. If α is small, as shown in Fig. 3(a), the case equals to cutting at the 4th and 8th, thus the 9-10-1-2-3 chain is similar to the open chain[24] while the part 4-5-6-7-8 chain has no entanglement because of the weak couplings. If $\alpha > 1$ such as $\alpha = 2$ shown in Fig. 3(b),

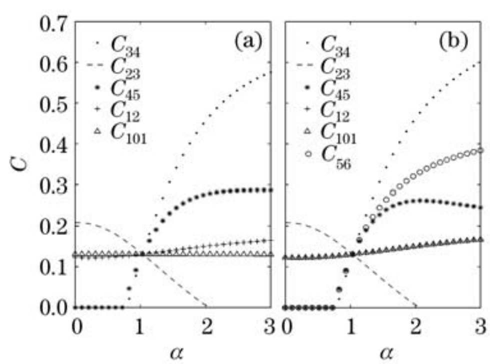

Fig. 2. The nearest neighboring Concurrences as function of α for (a) the two-impurity model (the 4th and 6th are two identical impurities) and (b) the three-impurity model (the 4th, 6th, and 8th are three identical impurities). $T = 1$, $B = 0.4$, $J = 1$, $J_z = 0.65$.

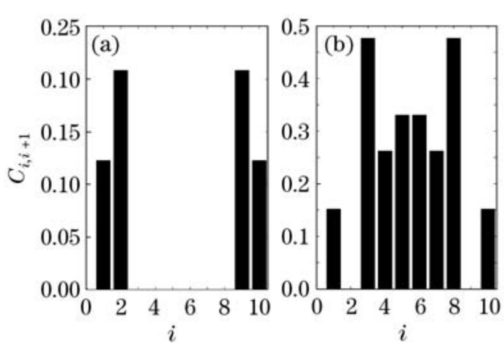

Fig. 3. The nearest neighboring Concurrences as function of i for the three-impurity model (the 4th, 6th, and 8th are three identical impurities). (a) $\alpha = 0.1$, (b) $\alpha = 2$. $T = 1$, $B = 0.4$, $J = 1$, $J_z = 0.65$.

the chain still can be cut into two parts because $J' > J$. Within the pure regions, entanglement will compete while in the part containing impurity, pairwise entanglement still competes each other.

Figure 4 shows the influence of temperature and the values of α on the entanglement in the three-impurity model. From it, we can judge again that the second and the third qubits are pure qubits while the third and the forth contain one impurity. Usually, it is difficult to adjust the coupling coefficients, which means we will meet with difficulty if directly using the behavior of Fig. 3 to judge which one is impurity. But it still can be done by measuring the Concurrence changed with temperature (Refs. [29,30] proposed that Concurrence can be measured), because changing the temperature is very easy. On the other hand, α can effectively enhance the Concurrence and critical temperature if $\alpha > 1$ as shown in Figs. 3 and 4 clearly. By introducing impurities with large coupling, one can also improve critical temperature and entanglement. Therefore, this paper not only provides a standard to judge impurity but also exhibits a way to enhance entanglement and critical temperature.

Now we study the effect of the nearest neighboring impurities, as shown in Fig. 5, on entanglement. According to Fig. 5(a), the Hamiltonian is

$$H = \frac{1}{2}\sum_{i=1}^{3}[J(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J_z\sigma_i^z\sigma_{i+1}^z]$$

$$+\frac{1}{2}\sum_{i=7}^{N}[J(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J_z\sigma_i^z\sigma_{i+1}^z]$$

$$+\frac{1}{2}\sum_{i=4,6}[J'(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J'_z\sigma_i^z\sigma_{i+1}^z]$$

$$+\frac{1}{2}\sum_{i=5}[J''(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + J''_z\sigma_i^z\sigma_{i+1}^z]$$

$$+\frac{1}{2}\sum_{i=1}^{N}B(\sigma_i^z + \sigma_{i+1}^z) \quad (5)$$

with

$$J'' = \beta * J, \quad J''_z = \beta * J_z, \quad (6)$$

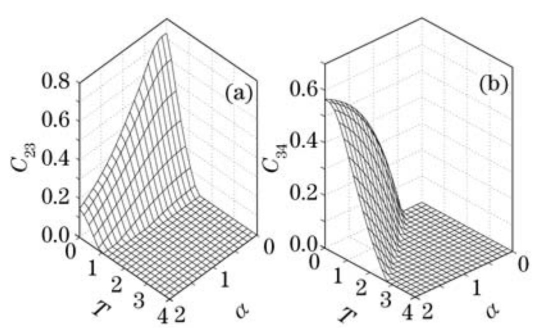

Fig. 4. The nearest neighboring Concurrences (a) C_{23} and (b) C_{34} versus α and T for the three-impurity model (the 4th, 6th, and 8th are three identical impurities). $B = 0.4$, $J = 1$, $J_z = 0.65$.

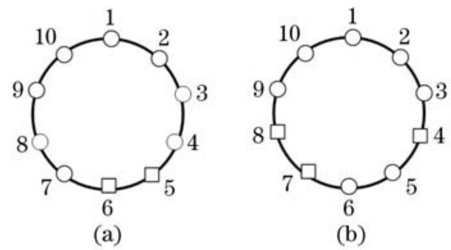

Fig. 5. Two configurations of spin ring with the nearest neighboring impurity. (a) Qubit ring formed with 10 qubits, the 5th and 6th are two identical impurities; (b) qubit ring formed with 10 qubits, the 4th, 7th, and 8th are three identical impurities.

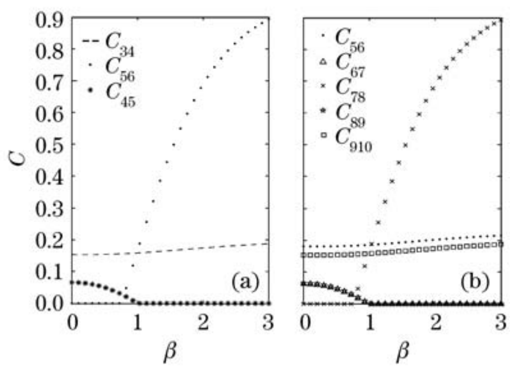

Fig. 6. The nearest neighboring Concurrences versus β for (a) the two-nearest-impurity model (the 5th and 6th are two identical impurities) and (b) the three-nearest-impurity model (the 4th, 7th, and 8th are three identical impurities). $B = 0.4$, $J = 1$, $J_z = 0.65$, $\alpha = 0.8$.

where β characterizes the relative strength of the extra coupling between the two nearest neighboring impurities and J', J'_z still have the relation of Eq. (2). Similarly, one can write the Hamiltonian corresponding to Fig. 5(b). Figure 6 plots the pairwise Concurrence near the two-nearest-impurity qubits area as a function of β. From it, we can see easily that the nearest neighboring impurities coupling only affect the nearest two-impurity and the others which couple with the impurities. For example, in Fig. 6(a), the nearest neighbor impurity C_{56} has a threshold value of β, affected by β heavily while C_{45} also decreases as a result of the competition between neighbor qubits. For the case of Fig. 5(b), although we have more impurities, the nearest neighbor coupling only affects entanglement of themselves C_{78} and that coupling with the impurities C_{67}, C_{89}; and all the others pairwise entanglement almost cannot be affected.

In conclusion, for a Heisenberg XXZ ring under a homogeneous magnetic field, we study the entanglement in two-impurity and three-impurity under the two cases of non-nearest-impurity and nearest-impurity. We find that the introduction of impurities make the originally equal pairwise entanglement compete with each other. For the weak and strong α, we can cut the ring chain into different open chains and then use the open chain property to explain the competition. For the case with the nearest neighbor qubits, the change of the relative coupling β can only affect the qubits which couple to the impurities. If introducing impurity with large α and β, the pairwise entanglement, which couples with the impurities directly, can be enhanced and the critical temperature also will be improved.

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