Study on optical gain of one-dimensional photonic crystals with active impurity

Zhenghua Li (李正华)[1,2,4], Tinggen Shen (沈廷根)[2,3], Xuehua Song (宋雪桦)[3], Junfeng Ma (马俊峰)[2,3], Yong Sheng (盛 勇)[2,3], and Gang Wang (王 刚)[3]

[1] *Telecommunication T&R Section, Zhenjiang Watercraft College, Zhenjiang 212003*

[2] *Institute of Applied Physics, Jiangsu University, Zhenjiang 212013*

[3] *Department of Telecommunication Engineering, Jiangsu University, Zhenjiang 212013*

[4] *Department of Electronic Engineering, East China Normal University, Shanghai 200062*

Received July 11, 2007

Localized fields in the defect mode of one-dimensional photonic crystals with active impurity are studied with the help of the theory of spontaneous emission from two-level atoms embedded in photonic crystals. Numerical simulations demonstrate that the enhancement of stimulated radiation, as well as the phenomena of transmissivity larger than unity and the abnormality of group velocity close to the edges of photonic band gap, are related to the negative imaginary part of the complex effective refractive index of doped layers. This means that the complex effective refractive index has a negative imaginary part, and that the impurity state with very high quality factor and great state density will occur in the photonic forbidden band if active impurity is introduced into the defect layer properly. Therefore, the spontaneous emission can be enhanced, the amplitude of stimulated emission will be very large and it occurs most probably close to the edges of photonic band gap with the fundamental reason, the group velocity close to the edges of band gap is very small or abnormal.

OCIS codes: 130.3120, 120.7000, 160.4670, 120.4570, 160.5690, 230.4170.

More and more importance has been attached to the unique behaviors of electromagnetic wave propagating in photonic crystals, a novel and artificial photonic material with periodic structure, in recent years[1−4]. Photonic band gap, prohibiting electromagnetic waves with particular frequencies propagating in it, means that spontaneous emission can be suppressed. A solitary transmission peak can be achieved by introducing defects into a photonic band gap structure. It suggests that localized defect modes will appear in the band gap if defects are introduced into the photonic band gap structure[5].

The position and width of transmission peak can also be controlled to meet our need by doping defect properly or by adjusting an external voltage because of the medial electro-optic effect. Photonic band gap can suppress spontaneous emission effectively because the probability of spontaneous emission is directly proportional to the state density, while the probability of spontaneous emission from atoms with optical frequency of spontaneous emission falling into photonic band gap is nearly zero. However, if active impurity is introduced into photonic crystals, the impurity state with very high quality factor and great state density will occur in the photonic band gap, so the state density is increased and the corresponding spontaneous emission is enhanced. Study has shown that by introducing active medium in photonic band gap materials, there is every probability of amplifying light with high efficiency, structuring photonic crystal laser with zero threshold[6].

In this letter, the phenomena that the sum of reflectivity and transmissivity of a photonic crystal without active impurity is unity and that of a photonic crystal with active impurity is far larger than unity at certain frequency band are studied by the theory of spontaneous emission from two-level atoms[7] in detail. In fact, the interaction between radiation field and matter is universal, but the interaction can be controlled actively by adjusting the parameters of photonic crystals to enhance stimulated emission extremely. This means that the doped photonic crystals can be used in dense wavelength-division multiplexing (DWDM) optical communication systems, thereby plenty of optical amplifiers can be saved, and signal capacity and transmission distance can be increased dramatically.

The model, symbolic system, and its conclusions used here is taken from Ref. [7]. Here we consider spontaneous emission from a two-level atom embedded in a photonic crystal with an upper band, a lower band, and a photonic gap between them. The dispersion relation near the two band edges could be expressed approximately by

$$\omega_k = \begin{cases} \omega_{c1} + C_1|k - k_{10}^i|^2 & (\omega_k > \omega_{c1}) \\ \omega_{c2} - C_2|k - k_{20}^j|^2 & (\omega_k < \omega_{c2}) \end{cases}, \tag{1}$$

where ω_{c1} and ω_{c2} are the cutoff frequencies of the upper band edge and the lower band edge, respectively, and k represents both the momentum and polarization of the modes. The gap width $\omega_{12} = \omega_{c1} - \omega_{c2}$ is assumed far smaller than ω_{c1}, ω_{c2}. C_1 and C_2 are model-dependent constants. k_{10}^i and k_{20}^j are two finite collections of symmetry related points.

The upper level $|1\rangle$ of a two-level atom is coupled by electromagnetic modes to the lower level $|0\rangle$ with the resonant frequency ω_1 between them,

$$H = \hbar\omega_1 |1\rangle \langle 1| + \sum_k \hbar\omega_k b_k^+ b_k + i\hbar \sum_k g_k(b_k^+ |0\rangle \langle 1| - b_k |1\rangle \langle 0|), \tag{2}$$

1671-7694/2007/120696-04

where $b_k^+(b_k)$ is the creation (annihilation) operator for the kth reservoir mode with frequency ω_k, $g_k^{(j)} = (\omega_j d_j/\hbar)(\hbar/2\varepsilon_0\omega_k V_0)^{1/2}\mathbf{e}_k\cdot\mathbf{u}_j$ is the coupling coefficient between the atomic transition $|1\rangle \to |0\rangle$, d_j and $\mathbf{u}_j$ are the magnitude and unit vector of the atomic dipole moment of transition $|1\rangle \to |0\rangle$, V_0 is the quantization volume, $\mathbf{e}_k$ is the transverse unit vector for the reservoir mode, and ε_0 is the dielectric constant.

The state vector of the system at arbitrary time t may be written as

$$|\psi(t)\rangle = A_1(t)\mathrm{e}^{-i\omega_1 t}|1,\{0\}\rangle + \sum_k B_k(t)\mathrm{e}^{-i\omega_k t}|0,\{1_k\}\rangle, \tag{3}$$

where the state vector $|1,\{0\}\rangle$ describes the atom in its excited state $|1\rangle$ with no photons in all reservoir modes, and the state vector $|0,\{1_k\}\rangle$ describes the atom in its ground state $|0\rangle$ and a single photon in kth mode with frequency ω_k.

The evolution of amplitudes $A_1(t)$ and $B_k(t)$ can be obtained from the Schrödinger equation $i\hbar(\partial/\partial t)|\psi(t)\rangle = H|\psi(t)\rangle$,

$$\frac{\partial}{\partial t}A_1(t) = -\sum_k g_k\mathrm{e}^{i(\omega_1-\omega_k)t}B_k(t),$$

$$\frac{\partial}{\partial t}B_k(t) = g_k\mathrm{e}^{-i(\omega_1-\omega_k)t}A_1(t). \tag{4}$$

And the following expression can be obtained from Eq. (4),

$$\frac{\partial}{\partial t}A_1(t) = -\sum_k g_k^2\int_0^t \mathrm{e}^{i(\omega_1-\omega_k)(t-t')}A_1(t')\mathrm{d}t', \tag{5}$$

with $A_1(t)$ obtained by performing the inverse Laplace transform.

The amplitude of the radiation field at a particular space point $\mathbf{r}$ can be calculated from $B_k(t)$ via $A_1(t)$,

$$E(r,t) = \sum_k\sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V_0}}\mathrm{e}^{-i(\omega t-k\cdot r)}B_k(t)e_k. \tag{6}$$

In fact, the main parts of the emission field $E(r,t)$ can be written as

$$E(r,t) = E_\mathrm{l} + E_\mathrm{p} + E_\mathrm{d}, \tag{7}$$

where $E_\mathrm{l}(r,t)$ describes the localized field part in the radiation field, $E_\mathrm{p}(r,t)$ represents the propagating field, and $E_\mathrm{d}(r,t)$ represents a typical diffusion field.

There is one localized field related to the upper band edge when an atom is set at $\mathbf{r}$ in a photonic crystal,

$$E_{\mathrm{l}1}^{(1)}(r,t) = E_{\mathrm{l}1}(0)\,\mathrm{e}^{-i\left(\omega_1-b^{(1)}\right)t-r/L_1}, \tag{8a}$$

and the localized field related to the lower band edge is,

$$E_{\mathrm{l}2}^{(1)}(r,t) = E_{\mathrm{l}2}(0)\,\mathrm{e}^{-i\left(\omega_1-b^{(1)}\right)t-r/L_2}. \tag{8b}$$

The relative position of energy band and energy level can be changed by changing the surrounding environment of photonic crystals or exert an influence on photonic crystals to change the position of the upper level. The number of pure imaginary roots may increase from zero to one, two or three. Therefore, the number of localized modes occurring in the band gap, being one, two, and three or larger can be controlled because each pure imaginary root will correspond to a localized mode.

The structure of the model examined in this letter is a multilayer system arranged alternatively by dielectric layers A and B with the relative permittivities ε_a and ε_b and the thicknesses a and b, respectively, therefore the period of the model d equals $a+b$, and the structure is $\cdots$A B A B A B A B A$\cdots$. Here the z direction is from left to right and electromagnetic waves enter into the system along the z direction.

When a beam with angular frequency ω is incident upon the one-dimensional (1D) periodic dielectric structured materials stated above, the wave vector of optical wave propagating in the medium is

$$k^2(z) = (\omega^2/c^2)\varepsilon(z) = (\omega^2/c^2)n(z)^2. \tag{9}$$

Basing on the definition of group velocity[8] and the expression of $\mathrm{d}k(\omega)/\mathrm{d}\omega$ described in the density of states $\rho(\omega)$[9],

$$v_\mathrm{g} = \left[\frac{\mathrm{d}\omega n(\omega)/c}{\mathrm{d}\omega}\right]^{-1} = \frac{c}{n(\omega)+\omega\frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega}}, \tag{10}$$

$$\frac{\mathrm{d}k(\omega)}{\mathrm{d}\omega} = (6\pi^2)^{1/3}\rho(\omega)\left[\int_0^\omega \rho(\omega')\mathrm{d}\omega'\right]^{2/3}, \tag{11}$$

one can figure out that the group velocity of electromagnetic wave traveling in a photonic crystal with a finite periodic layered structure is directly proportional to the inverse of state density.

Transmissivity is defined as

$$T = |t|^2, \tag{12}$$

where $t = 2\eta_0/(m_{11}\eta_0 + m_{12}\eta_0\eta_{N+1} + m_{21} + m_{22}\eta_{N+1})$ represents transmission coefficient expressed by optical transfer matrix[10].

The relation between the transmissivity T of 1D photonic crystal and the radiation frequency ω of incident light can be obtained by programming according to Eq. (12), thereby the band gap structure of 1D photonic crystal can be obtained.

The complex effective refractive index[11] of the doped defect layer stated above is

$$n_\mathrm{d} = n_\mathrm{eff}(\omega) = n(\omega) + is(\omega), \tag{13}$$

where $n(\omega)$, the real part of the complex effective refractive index, describes the dispersive properties of doped medium, and $s(\omega)$, the imaginary part of the complex effective refractive index, describes the characteristics that light is attenuated and evanescent inside photonic band gap.

According to the theoretical model given above, we have simulated the structures numerically in this letter. We made numerical analysis by programming with the parameters: dielectric layer A is made from ZnS, with $\varepsilon_\mathrm{a} = 5.5225$, thickness $a = 740$ nm; dielectric layer B is made from MgF_2, with $\varepsilon_b = 1.9044$, thickness $b = 1260$ nm. Spatial period $d = a + b = 2000$ nm, and optical thicknesses of the two dielectric layers are equal ($n_a a = n_b b$, $n_a = \sqrt{\varepsilon_a}$, $n_b = \sqrt{\varepsilon_b}$). Selecting different values of ω, with fundamental frequency $\omega_0 = \frac{c\pi}{n_a a + n_b b} = 271$ THz, the thickness of the doped layer with defect without active impurity (the imaginary part of the complex effective refractive index of the doped layer, $s = 0$) is $d = 2026$ nm, and the refractive index is $n_d = 2$. By calculating the transmissivity T we can obtain the transmission spectra from photonic crystals with different doped structures, as shown in Figs. 1—3.

In Fig. 1, we introduces a defect layer without active impurities ($s = 0$) into the symmetry system of a 1D photonic crystal, which results in the interaction between primitive cells in the system and the change of the position of energy band. The number of defect modes introduced into the energy band is the same as that of pure imaginary roots.

Equation (8) shows that the localized field decays exponentially with the distance increasing, the sum of reflectivity and transmissivity in a photonic crystal without active impurity equals unity, and the properties of localized field depend on the relative position of defect level and energy band edge.

If a layer of defect is made of active impurity (e.g., Er), the imaginary part of the complex effective refractive index of the doped layer is negative. From Figs. 2 and 3, we can find obviously that the enhancement of stimulated radiation occurs at certain frequencies and the transmissivity is far larger than unity. We also notice that the defect modes with transmissivity larger than unity most likely occurs at the band gap edges, because the group velocity close to the region is usually rather small or near to zero[4,12]. From Eqs. (10) and (11), the group velocity of electromagnetic wave traveling in a photonic crystal

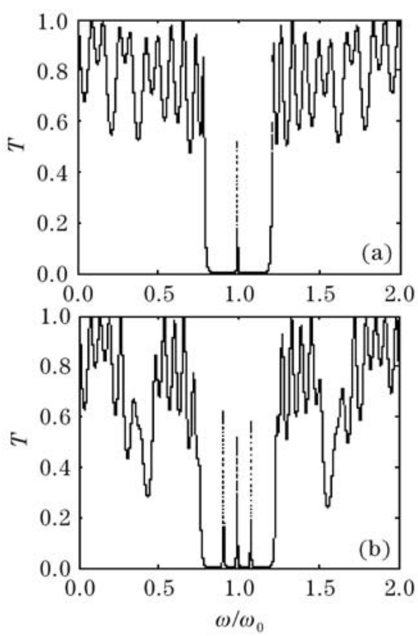

Fig. 1. Transmission spectra from one photonic crystal. (a) One doped layer D with five compound lattices AB on each side, $n_d = 2$, ABABABABABDABABABABAB, (b) with the same compound lattices AB but with three doped layers D, $n_d = 2$, ABABABABDABDABDABABABAB.

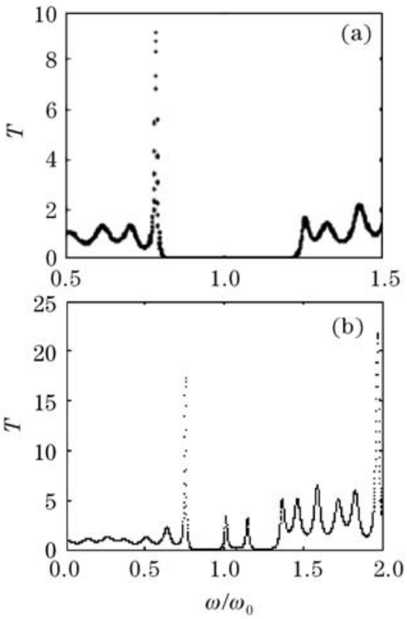

Fig. 2. Transmission spectra from photonic crystals. (a) With one doped layer D, $n_d = 2.2 - 0.1i$, ABABABABD-ABABABAB, (b) with two doped layers D $n_d = 2.2 - 0.1i$, ABABABABDABDABABAB.

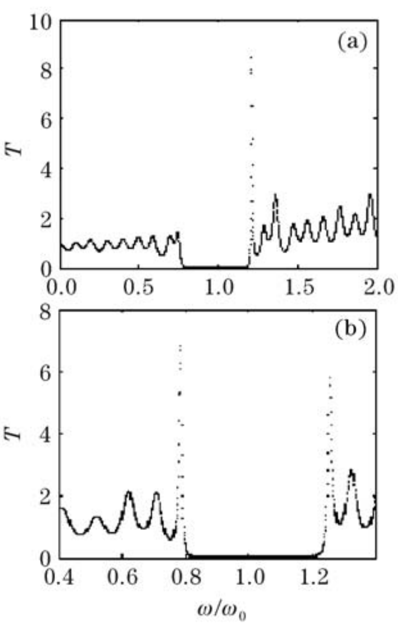

Fig. 3. Transmission spectra from photonic crystals. (a) With one doped layer D, $n_d = 2.0 - 0.1i$, ABABABD-ABABABABAB, (b) with one doped layer D, $n_d = 2.2 - 0.215i$, ABABABABDABABABAB.

is directly proportional to the inverse of state density. It is illustrated that the state density of photon close to the edge of band gap is very large, which enhances the gain dramatically. Generally spontaneous emission with frequencies within band gaps is suppressed to a certain extent, but it will be enhanced dramatically if active impurity is introduced into photonic crystals. Whether the transmitted peak occurs at the edge of upper energy gap or lower energy gap depends on the refractive index and the thickness of the doped layer.

On the other hand, near the edge of band gap, because the curve of the real part of $n(\omega)$ has an extraordinary steep positive slope[4,12], the group velocity v_g drops according to Eq. (10) and the corresponding state density of photon is enhanced dramatically. When the defect is made of active impurity, optical gain is increased obviously and light is highly localized, which means that the light traveling through defect layer with active impurity must absorb energy from pumping resources, that is to say, light is amplified instead of being absorbed. Taking advantage of the structure, one can design lasers with high gain, and control the transmission process of light, and amplify weak optical signals etc..

Comparing Fig. 2(a) with Fig. 3(b), if we increase the imaginary part of complex effective refractive index of the doped layer, there are two peaks of defect mode larger than unity at the edges of the upper and lower of band gaps simultaneously, which results in the split of the stimulated level and the change of the mode from single-level to double-level. When the doped layers are increased, for example, there is one more defect layer in Fig. 2(b) than that in Fig. 2(a), the interaction of defect modes influences the localization of the field, which leads to split of energy level. There are two transmission peaks corresponding to two defect modes. The amplitude of transmission peak with defect mode at the edge of the upper band gap is far larger than those of the two transmission peaks in the band gap. This is because the group velocity close to the edge of band gap drops dramatically. Therefore it is further proved that there is an inherent relation between the enhancement of stimulated radiation and the abnormity in the group velocity at the edge of photonic band gap.

In this letter, the localized fields in the defect mode of 1D photonic crystals with active impurity as well as its properties are studied with the help of the theory of spontaneous emission from two-level atoms embedded in photonic crystals, which provide theoretical proof for studying on the inherent laws of spontaneous radiation from the doped medium. When an atom is set into a photonic crystal and its frequency of spontaneous emission falls into a photonic forbidden band properly, spontaneous emission is suppressed because the state density of photon is nearly zero. On the other hand, spontaneous emission can be enhanced with photonic crystals as long as we increase the state density of photon. If active impurity is introduced into photonic crystals, the impurity state with very high quality factor and great state density will occur in the photonic forbidden band. Therefore, the amplification of spontaneous emission can be realized and occur most probably close to the edge of photonic band gap with the fundamental reason, the group velocity close to the edge of band gap is very small or abnormal. Using these characteristics of this kind of structure, one can design band edge lasers with high gain, control the propagating process of light, and amplify weak optical signals etc.. And one can also employ it in the DWDM communication systems to save plenty of optical amplifiers and to boost fiber information-carrying capacity and transmission distance extremely.

This work was supported by the Natural Science Foundation of Jiangsu Province under Grant No. BK2004059.
Z. Li's e-mail address is liharder@vip.163.com.

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