

# Modified maximum likelihood registration based on information fusion

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The bias estimation of passive sensors is considered based on information fusion in multi-platform multi-sensor tracking system. The unobservable problem of bearing-only tracking in blind spot is analyzed. A modified maximum likelihood method, which uses the redundant information of multi-sensor system to calculate the target position, is investigated to estimate the biases. Monte Carlo simulation results show that the modified method eliminates the effect of unobservable problem in the blind spot and can estimate the biases more rapidly and accurately than maximum likelihood method. It is statistically efficient since the standard deviation of bias estimation errors meets the theoretical lower bounds.

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Information fusion can enhance the performance in the detection, identification and tracking in multi-platform multi-sensor system. However, certain intrinsic deficiencies of multiple sensors cause a significant problem which is usually referred to as sensor alignment or sensor registration. Sensor registration is an inherent problem in multi-sensor system and deals with the calibration of registration biases<sup>[1]</sup>. If uncorrected, the registration biases can seriously degrade the performance of global surveillance system, and large errors will lead to the ghost targets<sup>[2]</sup>.

The deterministic and time-invariant biases can be obtained by batch processing the sensor measurements from common targets. In a recent effort, an off-line registration algorithm, the maximum likelihood registration (MLR)<sup>[3]</sup>, was proposed for sensor alignment. MLR assumes that the range information between the target and passive sensors is available, i.e., the range can be calculated by triangulation location. The target is observable without sensor biases when it is not in the line of two passive sensors. Triangulation yields a unique intersection point for bearing lines<sup>[4]</sup> when the target is observable. However, the system is unobservable due to the sensor biases when the target is in the line of two passive sensors or in the neighboring areas of the line. The special area is called the blind spot. The range calculated by triangulation is imaginary when the target is in the blind spot. If the unauthentic data are used to estimate the biases, it will reduce the precision of bias estimation. Figure 1 shows that the target is unobservable in the blind spot, and the unregistered plots are divergent. The problem of unobservability for bearing-only tracking system, which is often ignored in current publications<sup>[3,4]</sup>, is analyzed in detail in this paper. The modified maximum likelihood registration method is presented. The redundant information of multi-sensor system is used to compensate the range information and estimate the biases.

Consider  $n$  ( $n \geq 3$ ) ground based passive sensors in a multi-platform multi-sensor system. Figure 2 shows the sketch map of the bearing-only measurements in the blind spot, where  $A$ ,  $B$ , and  $C$  are the passive sensors.

Without loss of generality, we take the passive sensor  $A$  as an example to estimate its biases and the target position.  $T$  and  $T'$  denote the target positions measured by  $A$  and  $B$ .  $\alpha$  and  $\phi$  are the azimuth measurements of  $A$  and  $B$ , respectively. The bearing lines lie on the two sides of the line  $AB$  and have no intersection.  $r$  is the range calculated by  $\alpha$  and  $\phi$ , and it is unauthentic when the target is in the blind spot.  $\alpha'$ ,  $\gamma'$  are the azimuth measurements of  $A$  and  $C$ , respectively.  $\alpha$ ,  $\phi$  will be replaced by  $\alpha'$ ,  $\gamma'$ , which are used to calculate the new range  $r'$ , when the target is in the blind spot.

The target position can be obtained by the bearing lines of  $A$  and  $B$  when the target is observable. However, the two bearing lines of  $A$  and  $B$  lie on the two sides of the line  $AB$  due to the sensor biases when the target is

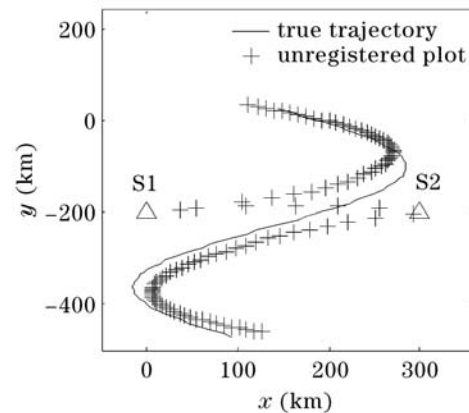


Fig. 1. True trajectory and unregistered plot.

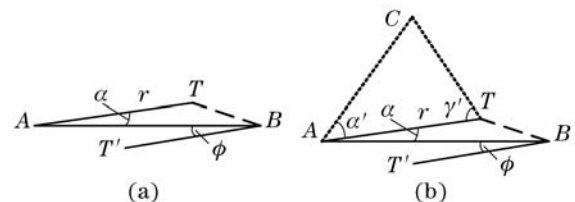


Fig. 2. Sketch maps in the blind spot.

in the blind spot. The two bearing lines have no intersection, so the range calculated by the measurements of  $A$  and  $B$  is imaginary. The precision of bias estimation will decrease if using the unauthentic range information.

The redundant information of multi-sensor system can eliminate the influence of unobservability and improve the location precision. When the target enters the blind spot, the unauthentic range is replaced by the new range which is calculated by the bearing measurements of  $A$  and  $C$ . Then the new range and the azimuth  $\alpha$  are used to calculate the target position. Since the new range is nearer to the true range than the unauthentic range, the target position calculated by the new range is reliable. Then the target position is used to estimate the sensor biases, which will improve the precision of bias estimation. This is the basic idea of the modified method.

The measurement model with biases is adopted as

$$z_i(k) = h_i(x(k)) + \beta_i + w_i(k), \quad (1)$$

where  $i = 1, \dots, n$ ,  $n$  is the number of sensors;  $k = 1, \dots, N$ ;  $z_i(k)$  is the measurements;  $x(k)$  is the target state;  $h_i(x(k))$  is a nonlinear measurement function;  $\beta_i$  is assumed to be time-invariant, independent of  $x(k)$ ;  $w_i(k)$  is the zero-mean Gaussian random noise with covariance  $R_i$ . Without loss of generality, let  $x_i(k)$  is a trigonometric function of two azimuths, and it is denoted as  $\text{tri}(\theta_A, \theta_B)$ . The modified state estimation is given as

$$\begin{aligned} x_i(k) &= \text{tri}(\theta_A, \theta_B), & x \notin M, \\ x_i(k) &= \text{tri}(\theta_A, \theta_C), & x \in M, \end{aligned} \quad (2)$$

where  $M$  is the blind spot;  $\theta_A$ ,  $\theta_B$ , and  $\theta_C$  are the bearing measurements of passive sensors  $A$ ,  $B$ , and  $C$ , respectively.

Given the measurements  $\{z(k); k = 1, \dots, N\}$ , the biases and the target state are estimated by maximizing the joint likelihood function<sup>[3]</sup>:

$$p(Z|X, \beta) = \arg \max_{\beta} \left\{ \prod_{k=1}^N \max_{x(k)} [p(z(k) | x(k), \beta)] \right\}. \quad (3)$$

The inner maximization in Eq. (3) is

$$\begin{aligned} & p(z_1, z_2, \dots, z_n | x, \beta) \\ & \approx K \exp \left\{ -\frac{1}{2} (x - \hat{x}^T) \left[ \sum_{i=1}^n R_{xi}^{-1} \right] (x - \hat{x}) \right. \\ & \quad \left. - \frac{1}{2} \left( \left[ \sum_{i=1}^n x_i^T R_{xi}^{-1} x_i \right] - \left[ \sum_{i=1}^n R_{xi}^{-1} x_i \right]^T \cdot \hat{x} \right) \right\}, \quad (4) \end{aligned}$$

where

$$\hat{x} = \left[ \sum_{i=1}^n R_{xi}^{-1} \right]^{-1} \left[ \sum_{i=1}^n R_{xi}^{-1} x_i \right], \quad (5)$$

$$R_{xi} = H_i^T R_{zi} H_i, \quad (6)$$

where  $H_i$  is the Jacobian of  $h_i(x(k))$ .

The outer maximization in Eq. (3) is

$$\begin{aligned} & p(z(1), \dots, z(N) | \hat{X}, \beta) \\ & = \bar{K} \exp \left\{ -\frac{1}{2} ((\beta - \hat{\beta})^T \left[ \sum_{k=1}^N \Omega^T(k) \Psi^{-1}(k) \Omega(k) \right] \right. \\ & \quad \left. \times (\beta - \hat{\beta}) + C \right\}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \hat{\beta} &= \left[ \sum_{k=1}^N \Omega^T(k) \Psi^{-1}(k) \Omega(k) \right]^{-1} \\ & \quad \times \left[ \sum_{k=1}^N \Omega^T(k) \Psi^{-1}(k) \bar{X}_0(k) \right], \end{aligned} \quad (8)$$

$$\Omega(k) = \text{diag}[H_1^{-R}(k), \dots, H_n^{-R}(k)], \quad (9)$$

$$\begin{aligned} \Psi^{-1}(k) &= \text{diag}[R_{x1}^{-1}(k), \dots, R_{xn}^{-1}(k)] \\ & \quad - \left[ \left\{ R_{xi}^{-1}(k) \left[ \sum_{l=1}^n R_{xl}^{-1}(k) \right]^{-1} R_{xj}^{-1}(k) \right\}_{ij} \right], \end{aligned} \quad (10)$$

$$\bar{X}_0(k) = X_0(k) + Q(k)\beta_0, \quad (11)$$

where the superscript ‘-R’ denotes a right inverse of matrix.

The modified maximum likelihood algorithm is described as follows.

- 1) Initialize the bias vector  $\hat{\beta} = 0$ .
- 2) Calculate the target position using the bearing measurements of  $A$  and  $B$ .
- 3) If the target is not in the blind spot, go to step 4. If it enters the blind spot, the redundant information will be used to replace the unauthentic range. The new range  $r'$  is gained by Eq. (2). The target position  $T_{xy}$  is calculated from the azimuths  $\alpha$  and  $r'$ .  $T_{xy}$  is regarded as the measurement position of  $A$  and  $B$ .
- 4) Estimate the bias vector by Eq. (8).
- 5) Judge whether the bias vector is convergent. If it is not convergent, go to step 2. The convergent condition is

$$\|\hat{\beta}_k - \hat{\beta}_{k-1}\| \leq \varepsilon, \quad \varepsilon > 0, \quad (12)$$

where  $\|\cdot\|$  is the Euclidean vector norm, and  $\varepsilon$  is the stopping criterion threshold. The lower the threshold is, the more accurate the estimation is. The value of  $\varepsilon$  can be given according to the precision requirements in practice.

- 6) Calculate the target state by Eq. (5).

The target trajectory used in the simulated example is modeled as<sup>[3]</sup>:  $x(k) = 130 + 150 \sin(0.06k)$ ,  $y(k) = 30 - 5k$ , the unit of coordinates is kilometer and  $k = 1, 2, \dots, 100$  ( $N = 100$ ). The sensor coordinates in kilometer are (300, -200), (80, 75), (0, -200). The bias vector is  $\beta_b = [-4^\circ, 7^\circ, -7^\circ]$ . The measurement

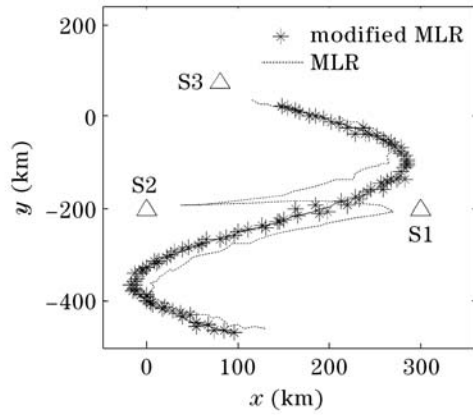


Fig. 3. Trajectory of the modified MLR and MLR.

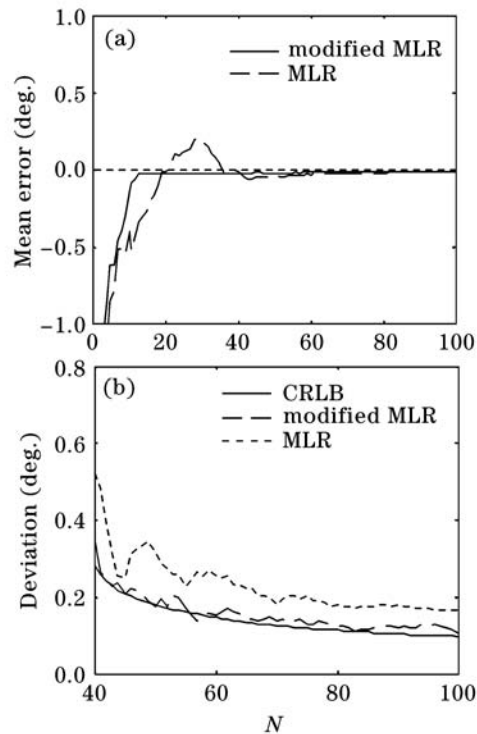


Fig. 4. Comparisons between the modified MLR and MLR. (a) Mean error; (b) standard deviation error.

noise is zero-mean Gaussian with covariance  $R = \text{diag}[(0.5^\circ)^2, (1^\circ)^2, (1^\circ)^2]$ . The stopping criterion threshold  $\varepsilon = \|\beta_b\| \times 0.001 = 0.0107$ .

The trajectory comparison between the modified method and MLR is shown in Fig. 3. It is shown that the redundant information solves the unobservable problem in the blind spot.

Figure 4(a) shows the mean of sensor bias estimation error. The modified method can estimate the registration biases more rapidly and accurately than MLR. The results with 100 Monte Carlo runs in Fig. 4(b) show that the modified method is statistically efficient, i.e., the standard deviation of bias estimation error meets the theoretical lower bounds (square root of Cramer Rao lower bound,  $\sqrt{\text{CRLB}}$ ).

In this paper, the modified MLR is investigated. The redundant information of multi-platform multi-sensor system is used to remedy the range in the blind spot, which ultimately resolves the unobservable problem of passive bearing location. The experimental results show that the modified method can efficiently estimate the passive sensor registration biases. The modified method is not limited by target tracks. It is also adapted for other similar situations when the target passes through the blind spot of passive sensors in multi-sensor system. It also provides a scheme to distribute and dispose the sensors in multi-platform multi-sensor system.

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