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Optimal wavelet filter bank design for image fusion

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A novel optimization-based method for designing wavelet filter banks in image fusion is proposed. The filter bank design is formulated as a nonlinear optimization problem. The objective function of the optimization problem consists of both the performance metrics of the image fusion, such as the root mean square error (RMSE), and those of individual filters. The optimization problem is solved using simulating annealing.

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Image fusion approaches based on the wavelet multi-resolution representations are now widely used. The basic idea of these approaches is to perform a wavelet multi-resolution transform (WMRT) on the source images, and to construct a composite multi-resolution representation by using an appropriate fusion rule. The fused image is then obtained by taking the inverse wavelet multi-resolution transform (IWMRT). Wavelet filter bank is a key issue in multi-resolution transform (MRT). Existing filter-bank design methods focus on biorthogonal filter banks with perfect reconstruction[1]. However, in image fusion, information of the fused image is the incomplete representation of source images. Not all small reconstruction errors introduced by filter banks lead to worse fusion quality. In this letter, an optimal wavelet filter bank for image fusion is proposed.

The generic image fusion scheme is shown in Fig. 1, from which we can see the importance of MRT. As the key issue of MRT, a generic two-channel finite impulse response (FIR) filter bank[1] is shown in Fig. 2, where $H_0(z)$ and $H_1(z)$ represent the low-pass and high-pass filters in the analysis bank, respectively, $G_0(z)$ and $G_1(z)$ are their corresponding synthesis filters, respectively. The filter bank consists of an analysis stage and a synthesis stage. For the purpose of aliasing free quality, there is a relationship between analysis filters and synthesis filters. The synthesis filters can be calculated by

$$G_0(z) = H_1(-z), \qquad G_1(z) = -H_0(-z). \tag{1}$$

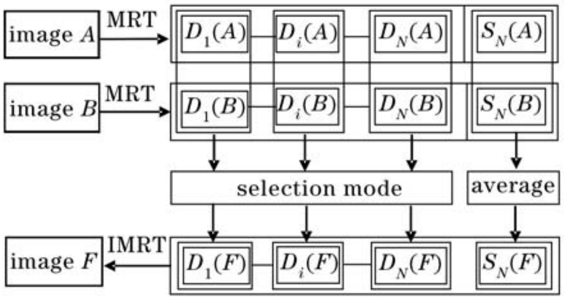

Fig. 1. Generic image fusion scheme.

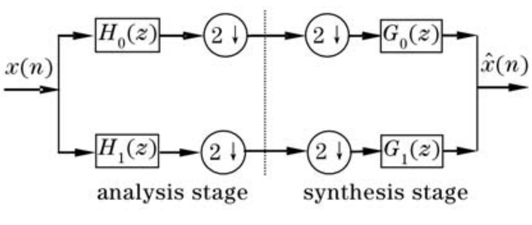

Fig. 2. A two-channel filter bank.

The two analysis filters $H_0(z)$ and $H_1(z)$ should also satisfy the following conditions[1]: the sum of the lengths must be a multiple of 4; both of them are FIR filters.

Individual filters should include regularity properties in wavelet theory. Regularity is that the iterated low-pass filter converges to a continuous function[2].

In our method, the goal is to find the filter bank that produces the best image fusion quality. We use a metric, the root mean square error (RMSE), to measure the performance of image fusion[3]:

$$\mathrm{RMSE} = \sqrt{\frac{1}{M \cdot N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(R(i,j) - F(i,j)\right)^2}, \tag{2}$$

where F is the fused image with the size of $M \times N$, and R is a standard reference image with the same size.

It is difficult to get good filters by minimizing RMSE. So a second objective, which consists of stop-band and pass-band energies of individual filters, is introduced. The stop-band energy $E_\mathrm{s}(h_0)$ with stop-band cut-off frequency ω_s is

$$\begin{aligned}
E_\mathrm{s}(h_0) &= \int_{\omega_\mathrm{s}}^{\pi} H_0^2(\omega)\,\mathrm{d}\omega \\
&= h_0^2\left((N_0+1)/2\right)(\pi - \omega_\mathrm{s}) \\
&\quad -4h_0\left((N_0+1)/2\right) \sum_{n=1}^{(N_0-1)/2} h_0(n) \frac{\sin\left(\frac{N_0+1}{2} - n\right)\omega_\mathrm{s}}{\frac{N_0+1}{2} - n} \\
&\quad +2 \sum_{n=1}^{(N_0-1)/2} h_0(n)^2 \left[(\pi - \omega_\mathrm{s}) - \frac{\sin(N_0+1-2n)\,\omega_\mathrm{s}}{N_0+1-2n}\right] \\
&\quad -2 \sum_{n=1}^{(N_0-1)/2} \sum_{m=1,m\neq n}^{(N_0-1)/2} h_0(n)\,h_0(m) \\
&\qquad \times \left[\frac{\sin(n-m)\,\omega_\mathrm{s}}{n-m} + \frac{\sin(N_0+1-n-m)\,\omega_\mathrm{s}}{N_0+1-n-m}\right].
\end{aligned} \tag{3}$$

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The pass-band energy $E_{\mathrm{p}}(h_0)$ with pass-band cut-off frequency ω_{p} is

$$E_{\mathrm{p}}(h_0) = \int_0^{\omega_{\mathrm{p}}} (H_0(\omega) - 1)^2 \mathrm{d}\omega$$

$$= (h_0((N_0+1)/2) - 1)^2 \omega_{\mathrm{p}}$$

$$+4\left(h_0 \frac{N_0+1}{2} - 1\right) \sum_{n=1}^{(N_0-1)/2} h_0(n) \frac{\sin\left(\frac{N_0+1}{2} - n\right)\omega_{\mathrm{p}}}{\frac{N_0+1}{2} - n}$$

$$+2 \sum_{n=1}^{(N_0-1)/2} h_0(n)^2 \left[\omega_{\mathrm{p}} - \frac{\sin(N_0+1-2n)\omega_{\mathrm{p}}}{N_0+1-2n}\right]$$

$$+2 \sum_{n=1}^{(N_0-1)/2} \sum_{m=1, m\neq n}^{(N_0-1)/2} h_0(n) h_0(m)$$

$$\times \left[\frac{\sin(n-m)\omega_{\mathrm{p}}}{n-m} + \frac{\sin(N_0+1-n-m)\omega_{\mathrm{p}}}{N_0+1-n-m}\right]. \quad (4)$$

The objective is formulated to minimize the total stop-band and pass-band energies of both analysis filters, i.e.,

$$\min_{h_0, h_1} E(h_0, h_1), \quad (5)$$

where $E(h_0, h_1) = E_{\mathrm{s}}(h_0) + E_{\mathrm{s}}(h_1) + E_{\mathrm{p}}(h_0) + E_{\mathrm{p}}(h_1)$ is the total energy of both filters.

The overall objective is a combination of RMSE and $E(h_0, h_1)$,

$$\min_{h_0, h_1} w(\mathrm{RMSE}(A, R, F)) + (1-w) E(h_0, h_1), \quad (6)$$

where $0 \leq w \leq 1$ is a constant weight; A is the image fusion algorithm, for which we can select several image fusion algorithms[4,5]. Note that when $w = 1$, the objective becomes just minimized RMSE.

In the experiments, we applied the optimization-based method to design 9/7 wavelet biorthogonal filter banks for image fusion and obtained promising results. The performance of the filter banks was evaluated using the generic image fusion method. The optimization problem was solved using simulating annealing[6].

The amplitude responses of Antonini's 9/7 filter bank[2] and the new filter bank designed by our method are shown in Fig. 3. An image fusion example is shown in Fig. 4. Input 'clock' images in Figs. 4(a) and (b) are fused using Antonini's 9/7 filter bank with generic fusion scheme[3] into the fused image in Fig. 4(c). Image fusion result using the proposed filter bank is shown in Fig. 4(d). Table 1 shows that the new filter bank is more effective than Antonini's filter bank for image fusion. We selected $w = 1$, 0.8, 0.2, 0.1 and 0.02, the best result was obtained when $w = 0.02$. The optimal wavelet filter coefficients are

$$h_0 = [0.03533; -0.01; -0.11; -0.061724; -0.059086;$$
$$-0.061724; -0.11; -0.01; 0.03533]. \quad (7)$$

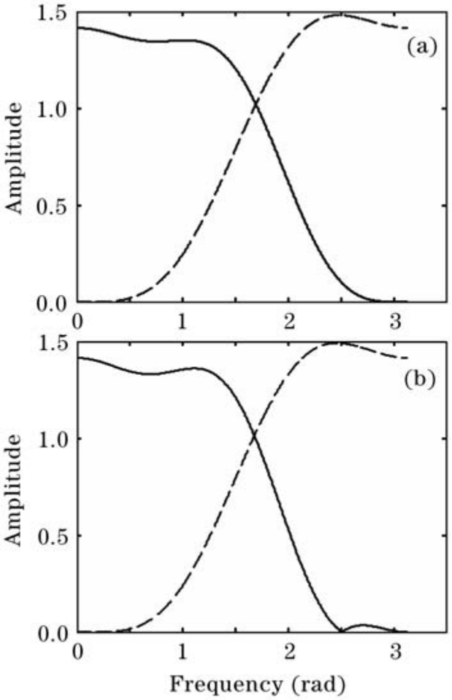

Fig. 3. (a) Amplitude responses of the pair of analysis filters in Antonini's 9/7 filter bank[2] and (b) those in the new 9/7 filter bank obtained by the proposed method.

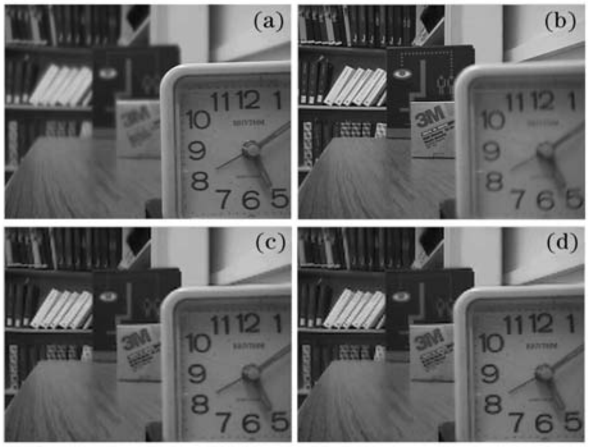

Fig. 4. Fusion results of 'clock' images. (a), (b) Input multi-focus images; (c) fused image using the Antonini's 9/7 filter bank; (d) fused image using the optimal filter bank.

Table 1. Fusion Results of 'Clock' Images

Filter	RMSE	Entropy
9/7 Wavelet	12.3364	4.9880
Optimal 9/7 Wavelet	11.2813	4.9965

Furthermore, in experiments, we found that the entropy of the image was increased. The evaluation measure entropy is defined as

$$\mathrm{EN} = -\sum_i^H p_i \ln p_i, \quad (8)$$

where p_i is the probability when the pixel number of gray level is i. The measurement indicates how much information of an image is contained.

In this paper, the design of two-band wavelet filter banks for image fusion is studied. In experiments, the fusion result using optimal filter banks outperforms the result using traditional 9/7 filter banks. Our design method can also be applied to the design of other types of filter banks, such as multi-rate and multi-band filter banks for various applications.

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