# Study on optical wave scattering from slightly Gaussian rough surface of layered medium 

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#### Abstract

The optical wave scattering from the slightly rough surface of three－layer medium is studied．The formulaes of the scattering coefficients for different polarizations are derived using the small perturbation method． A Gaussian rough surface is presented for describing rough surface of layered medium，the influence of the permittivity of layered medium，the mean layer thickness of intermediate medium，the surface roughness parameters and the incident wavelength on the bistatic scattering coefficient of HH polarization are obtained and discussed by numerical implementation．


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The study on optical wave scattering from rough sur－ faces has been the subject of intensive investigation over the past several decades for its applications in a num－ ber of important research areas such as characterization of films and optical interfaces，and the design of opti－ cal scanning instruments for use in the semiconductor industry ${ }^{[1-7]}$ ．The theory of optical wave scattering by statistically rough surfaces has been enriched by the cre－ ation of new analytical methods which include the small－ slope approximation ${ }^{[8]}$ ，the titl－invariant theory of rough surface scattering ${ }^{[9]}$ ，the local perturbation method ${ }^{[10]}$ ， the full－wave－method analysis ${ }^{[11]}$ ，the surface－field phase－ perturbation technique ${ }^{[12]}$ ，two－scale models ${ }^{[13]}$ ，and the small perturbation method（SPM）${ }^{[14,15]}$ ．

In this letter，we consider optical wave scattering from a rough interface in an arbitrary，plane－layered medium， assumed a roughness small and gentle enough for SPM to be applicable．

Consider the optical wave scattering from the rough surface $S$ between two half－spaces，which is described by the equation $z=f(\vec{r})$ ，where $\vec{r}=\{x, y\}$ ．Without re－ striction of generality we suppose that the upper homo－ geneous half－space $z>f(\vec{r})$（medium 1 ）is characterized by a constant dielectric permittivity $\varepsilon_{0}=1$ and $\mu_{0}=1$ ， and the lower stratified half－space $z<f(\vec{r})$ is character－ ized by an arbitrary＂depth－profile＂of a complex dielec－ tric permittivity $\varepsilon(z)$ with the depth of $H$ ．The random function $f(\vec{r})$ is assumed to be statistically spatially ho－ mogeneous with zero mean value $\langle f(\vec{r})\rangle=0$ ．

An incident plane monochromatic optical wave，with frequency $\omega$ and wave vector $\vec{k}_{\mathrm{i}}=\left\{k_{\mathrm{i}} \sin \theta_{\mathrm{i}}, 0,-k_{\mathrm{i}} \cos \theta_{\mathrm{i}}\right\}$ ， $k_{\mathrm{i}}=\omega / c$ ，illuminates the rough surface $S$ from medium 1 with a given angle of incidence $\theta_{\mathrm{i}}$ ，as shown in Fig． 1.

The boundary conditions for electric field $\vec{E}$ and mag－ netic field $\vec{H}$ mean the continuity of their tangent com－ ponents on the boundary $S$ ，

$$
\begin{equation*}
\vec{N} \times\left(\vec{H}^{(2)}-\vec{H}^{(1)}\right)_{S}=\vec{N} \times\left(\vec{E}^{(2)}-\vec{E}^{(1)}\right)_{S}=0 \tag{1}
\end{equation*}
$$

where $\vec{E}^{(2)}$ and $\vec{H}^{(2)}$ are the fields in the lower half－space $z<f(\vec{r}), \vec{E}^{(1)}$ and $\vec{H}^{(1)}$ are the fields in the upper half－ space $z>f(\vec{r})$ ，and $\vec{N}$ is a unit vector normal to the
surface $S$ and directed upward，

$$
\begin{equation*}
\vec{N}=(\vec{n}-\vec{s})\left(1+s^{2}\right)^{-1 / 2} \tag{2}
\end{equation*}
$$

$\vec{n}$ is a unit vector of normal to the horizontal plane $S_{p}(z=0), \vec{s}(\vec{r})=\nabla_{r} f(\vec{r})$ is a vector field of surface slopes，$\nabla_{r}=\{\partial / \partial x, \partial / \partial y\}$ ．Assuming that deviations and slopes of surface $S$ with respect to $S_{p}$ are small enough，we can expand the boundary conditions（1）in powers of $f(\vec{r})$ and $\vec{s}(\vec{r})$ ，and retain the first－order terms only ${ }^{[3]}$ ，

$$
\left\{\begin{array}{rl}
(\vec{n} \times \Delta \vec{H})_{S_{p}} & =(\vec{s} \times \Delta \vec{H})_{S_{p}}-f\left(\vec{n} \times \frac{\partial \Delta \vec{H}}{\partial z}\right)_{S_{p}}  \tag{3}\\
(\vec{n} \times \Delta \vec{E})_{S_{p}} & =(\vec{s} \times \Delta \vec{E})_{S_{p}}-f\left(\vec{n} \times \frac{\partial \Delta \vec{E}}{\partial z}\right)_{S_{p}}
\end{array} .\right.
$$

Here $\Delta \vec{H}=\vec{H}^{(2)}-\vec{H}^{(1)}$ and $\Delta \vec{E}=\vec{E}^{(2)}-\vec{E}^{(1)}$ ．We can represent the solution of the diffraction problem in the form of

$$
\begin{equation*}
\vec{H}^{(1,2)}=\vec{H}_{0}^{(1,2)}+\vec{H}_{1}^{(1,2)}, \quad \vec{E}^{(1,2)}=\vec{E}_{0}^{(1,2)}+\vec{E}_{1}^{(1,2)} \tag{4}
\end{equation*}
$$

$\vec{E}_{0}^{(1)}, \vec{H}_{0}^{(1)}\left(\vec{E}_{0}^{(2)}, \vec{H}_{0}^{(2)}\right)$ are the unperturbed wave fields in the upper $z>0$（lower $z<0$ ）half－space corresponding to the specular reflection（refraction）at the plane boundary $S_{p}$ ．They satisfy the uniform boundary conditions

$$
\begin{equation*}
\vec{n} \times\left(\vec{H}_{0}^{(2)}-\vec{H}_{0}^{(1)}\right)_{S_{p}}=\vec{N} \times\left(\vec{E}_{0}^{(2)}-\vec{E}_{0}^{(1)}\right)_{S_{p}}=0 . \tag{5}
\end{equation*}
$$

And $\vec{E}_{1}^{(1)}, \vec{H}_{1}^{(1)}\left(\vec{E}_{1}^{(2)}, \vec{H}_{1}^{(2)}\right)$ are the corrections to the first order of $f(\vec{r})$ and $\vec{s}(\vec{r})$ ，i．e．，scattered fields in the upper （lower）half－space，respectively．They satisfy the nonuni－ form boundary conditions


Fig．1．Geometry of scattering from the Gaussian rough sur－ face of layered medium．
$\left\{\begin{array}{l}\left(\vec{n} \times \Delta \vec{H}_{1}\right)_{S_{p}}=\left(\vec{s} \times \Delta \vec{H}_{0}\right)_{S_{p}}-f\left(\vec{n} \times \frac{\partial \Delta \vec{H}_{0}}{\partial z_{0}}\right)_{S_{p}}=\vec{J}^{H} \\ \left(\vec{n} \times \Delta \vec{E}_{1}\right)_{S_{p}}=\left(\vec{s} \times \Delta \vec{E}_{0}\right)_{S_{p}}-f\left(\vec{n} \times \frac{\partial \Delta E_{0}}{\partial z}\right)_{S_{p}}=\vec{J}^{E},\end{array}\right.$
where $\Delta \vec{H}_{0}=\vec{H}_{0}^{(2)}-\vec{H}_{0}^{(1)}, \Delta \vec{H}_{1}=\vec{H}_{1}^{(2)}-\vec{H}_{1}^{(1)}, \Delta \vec{E}_{0}=$ $\vec{E}_{0}^{(2)}-\vec{E}_{0}^{(1)}$, and $\Delta \vec{E}_{1}=\vec{E}_{1}^{(2)}-\vec{E}_{1}^{(1)}$. It is seen from Eq. (6) that surface roughness leads to the appearance of the effective electric and magnetic surface currents $\vec{J}^{H}$ and $\vec{J}^{E}$ on the mean plane $z=0$ which generate the scattered fields. Currents $\vec{J}^{H}$ and $\vec{J}^{E}$ differ from the usually used electric and magnetic currents only by sign and the factor $4 \pi / c$.
We consider the incident monochromatic horizontally polarized (TE) plane optical wave, which propagates in medium 1 , in the direction of wave vector $\vec{k}_{i}$,

$$
\begin{equation*}
\vec{E}_{\text {in }}^{(1)}=\vec{P}_{0} \sqrt{\frac{\mu_{0}}{k_{z}}} \mathrm{e}^{i \vec{k}_{\mathrm{i}} \cdot \vec{R}}=\vec{P}_{0} \sqrt{\frac{\mu_{0}}{k_{z}}} \mathrm{e}^{i \vec{k} \cdot \vec{r}-k_{z} z} \tag{7}
\end{equation*}
$$

where $\vec{R}=\{\vec{r}, z\}$ is a three-dimensional (3D) spatial radius vector, $\vec{P}_{0}=\vec{e}_{y}=(0,1,0)$ is a unit polarization vector, $\vec{k}=\{k, 0,0\}$ is a two-dimensional (2D) projection of incident wave vector $\vec{k}_{\mathrm{i}}=\left\{\vec{k},-k_{z}\right\}$ on the plane $z=0$, and $k_{z}=\sqrt{k_{1}^{2} \mu_{0} \varepsilon_{0}-k^{2}}=k_{\mathrm{i}} \cos \theta_{\mathrm{i}}$.
Let us represent the scattered fields in the form of an expansion over plane waves, i.e.,

$$
\left\{\begin{array}{l}
\vec{E}_{1}^{(1)}(\vec{r}, z)=\iint \sqrt{\frac{\mu_{0}}{q_{z}}} \tilde{\vec{E}}_{1}^{(1)}(\vec{q}) \mathrm{e}^{i\left(\vec{q} \cdot \vec{r}+q_{z} z\right)} \mathrm{d} \vec{q}  \tag{8}\\
\vec{H}_{1}^{(1)}(\vec{r}, z)=\iint \sqrt{\frac{\varepsilon_{0}}{q_{z}}} \tilde{\vec{H}}_{1}^{(1)}(\vec{q}) \mathrm{e}^{i\left(\vec{q} \cdot \vec{r}+q_{z} z\right)} \mathrm{d} \vec{q}
\end{array} .\right.
$$

Here, $\tilde{\vec{E}}_{1}^{(1)}(\vec{q}), \tilde{\vec{H}}_{1}^{(1)}(\vec{q})$ are the amplitudes of scattered plane waves propagating in the upper half-space in the positive direction of $z$-axis, $q_{z}=\sqrt{k_{\mathrm{i}}^{2}-q^{2}}=$ $\sqrt{k_{\mathrm{s}}^{2}-q^{2}}=k_{\mathrm{i}} \cos \theta_{\mathrm{s}}=k_{\mathrm{S}} \cos \theta_{\mathrm{s}}$ is the vertical component of the upward scattered field wave vector $\vec{k}_{\mathrm{s}}=\left\{\vec{q}, q_{z}\right\}$.
The solution of Maxwell's equations for scattered fields with boundary conditions (6) has the form of ${ }^{[6]}$

$$
\left\{\begin{align*}
\left(\vec{P} \tilde{\vec{E}}_{1}^{(1)}\right)= & \frac{1}{2} k_{\mathrm{i}} \sqrt{\frac{\mu_{0}}{q_{z}}}\left(1+R_{\mathrm{h}}^{\mathrm{s}}\right)\left(\vec{P} \tilde{\vec{J}}^{H}\right)  \tag{9}\\
& +\frac{1}{2} \sqrt{\frac{q_{z}}{\mu_{0}}}\left(1-R_{\mathrm{h}}^{\mathrm{s}}\right)\left[\vec{P} \cdot\left(\vec{n} \times \tilde{\vec{J}}^{E}\right)\right] \\
\left(\vec{P} \tilde{\vec{H}}_{1}^{(1)}\right)= & -\frac{1}{2} k_{\mathrm{i}} \sqrt{\frac{\varepsilon_{0}}{q_{z}}}\left(1+R_{\mathrm{v}}^{\mathrm{s}}\right)\left(\vec{P} \tilde{\vec{J}}^{E}\right) \\
& +\frac{1}{2} \sqrt{\frac{q_{z}}{\varepsilon_{0}}}\left(1-R_{\mathrm{v}}^{\mathrm{s}}\right)\left[\vec{P} \cdot\left(\vec{n} \times \tilde{\vec{J}}^{H}\right)\right]
\end{align*}\right.
$$

where $\vec{P}$ is a horizontal unit vector orthogonal to the plane of scattering, $R_{\mathrm{h}}^{\mathrm{s}}$ and $R_{\mathrm{v}}^{\mathrm{s}}$ are the reflection coefficients of a horizontally or vertically polarized plane wave with wave vector $k_{\mathrm{s}}^{(1)}$ from a layered medium with a perfectly plane boundary $z=0$, and $\tilde{\tilde{J}^{E, H}}$ is the Fourier transform of the surface current,

$$
\begin{equation*}
\tilde{\vec{J}}^{E, H}=\tilde{\vec{J}}^{E, H}(\vec{q})=\frac{1}{(2 \pi)^{2}} \iint \vec{J}^{E, H}(\vec{r}) \mathrm{e}^{-i \vec{q} \cdot \vec{r}} \mathrm{~d} \vec{r} . \tag{10}
\end{equation*}
$$

Using the definitions of surface currents $\vec{J}^{E, H}$ and representing the differences $\Delta \vec{H}_{0}, \Delta \vec{E}_{0}$ of zero-order fields
through corresponding reflection coefficients $R_{\mathrm{h}}^{\mathrm{i}}$ and $R_{\mathrm{v}}^{\mathrm{i}}$ for an incident wave, we obtain scattering amplitudes for a horizontally polarized incident wave

$$
\left\{\begin{array}{l}
\vec{P} \tilde{\vec{E}}_{1}^{(1)}=S_{\mathrm{hh}}=\frac{i k_{i}^{2}}{2 \sqrt{q_{z_{2}}} z_{z}} \tilde{f}(\vec{q}-\vec{k}) F_{\mathrm{hh}}(\vec{q}, \vec{k})  \tag{11}\\
\vec{P} \tilde{\vec{H}}_{1}^{(1)}=S_{\mathrm{vh}}=\frac{i k_{i}^{2}}{2 \sqrt{q_{z} k_{z}}} \tilde{f}(\vec{q}-\vec{k}) F_{\mathrm{vh}}(\vec{q}, \vec{k})
\end{array},\right.
$$

where

$$
\begin{equation*}
\tilde{f}(\vec{q}-\vec{k})=\frac{1}{(2 \pi)^{2}} \iint f(\vec{r}) \mathrm{e}^{-i(\vec{q}-\vec{k}) \cdot \vec{r}} \mathrm{~d} \vec{r} \tag{12}
\end{equation*}
$$

and

$$
\left\{\begin{align*}
F_{\mathrm{hh}}= & \mu_{0}\left(1+R_{\mathrm{h}}^{\mathrm{i}}\right)\left(1+R_{\mathrm{h}}^{\mathrm{s}}\right)  \tag{13}\\
& \times\left[\frac{\mu-\mu_{0}}{\mu} \varepsilon_{0} \sin \theta_{\mathrm{s}} \sin \theta_{\mathrm{i}}+\left(\varepsilon-\varepsilon_{0}\right) \cos \phi\right] \\
& -\varepsilon_{0}\left(1+R_{\mathrm{h}}^{\mathrm{h}}\right)\left(1+R_{\mathrm{h}}^{\mathrm{s}}\right)\left(\mu-\mu_{0}\right) \cos \theta_{\mathrm{s}} \cos \theta_{\mathrm{i}} \cos \phi \\
F_{\mathrm{vh}}= & \left\{\mu_{0}\left(1-R_{\mathrm{v}}^{\mathrm{s}}\right)\left(1+R_{\mathrm{h}}^{\mathrm{i}}\right)\left(\varepsilon-\varepsilon_{0}\right) \cos \theta_{\mathrm{s}}\right. \\
& \left.-\varepsilon_{0}\left(1-R_{\mathrm{h}}^{\mathrm{i}}\right)\left(1+R_{\mathrm{v}}^{\mathrm{s}}\right)\left(\mu-\mu_{0}\right) \cos \theta_{\mathrm{i}}\right\} \sin \phi
\end{align*}\right.
$$

In these equations $\varepsilon$ and $\mu$ are the limiting values of $\varepsilon(z \rightarrow-0)$ and $\mu(z \rightarrow-0)$, and their variations on the vertical scale of $f$ are neglected. The scattering amplitudes $S_{\mathrm{vv}}, S_{\mathrm{hv}}$ for a vertically polarized incident wave can be obtained without any additional derivations simply by changing notations in previous equations: $\vec{E} \rightarrow \vec{H}$, $\vec{H} \rightarrow-\vec{E}, \mathrm{~h} \rightarrow \mathrm{v}, \mu \rightarrow \varepsilon$.
Specific incoherent scattering cross sections of up-going waves are related with the corresponding scattering amplitudes by a simple equation ${ }^{[5,14]}$,

$$
\begin{align*}
\sigma_{\alpha \beta}^{0}\left(\vec{k}_{\mathrm{s}}, \vec{k}_{\mathrm{i}}\right) & \left.=\left.\lim _{S_{p} \rightarrow \infty} \frac{16 \pi^{3}}{S_{p}}\left|q_{z} k_{z}\right|\langle | S_{\alpha \beta}\right|^{2}\right\rangle \\
& =\pi k_{\mathrm{i}}^{4}\left|F_{\alpha \beta}\right|^{2} W_{f}(\vec{q}-\vec{k}) \\
& =\pi k_{\mathrm{i}}^{4}|\varepsilon-1|^{2}\left|f_{\alpha \beta}\right|^{2} W_{f}(\vec{q}-\vec{k}), \tag{14}
\end{align*}
$$

where $\alpha$ and $\beta$ characterize the polarization state of scattered and incident wave, and $W_{f}(\vec{q}-\vec{k})$ is a spatial power spectrum of surface roughness, $\vec{q}-\vec{k}=k_{\mathrm{s}} \sin \theta_{\mathrm{s}} \cos \phi-$ $k_{\mathrm{i}} \sin \theta_{\mathrm{i}}$.
For the simplest case of scattering by a rough boundary of non-magnetic layered medium ( $\mu=1$ ), the full set of factors $f_{\alpha \beta}$ in an explicit form for different polarization states of incident and scattered waves are

$$
\left\{\begin{array}{l}
f_{\mathrm{hh}}=\left[1+R_{\mathrm{h}}\left(\theta_{\mathrm{i}}\right)\right]\left[1+R_{\mathrm{h}}\left(\theta_{\mathrm{s}}\right)\right] \cos \phi  \tag{15}\\
f_{\mathrm{vh}}=-\left[1+R_{\mathrm{h}}\left(\theta_{\mathrm{i}}\right)\right]\left[1-R_{\mathrm{v}}\left(\theta_{\mathrm{s}}\right)\right] \cos \theta_{\mathrm{s}} \sin \phi \\
f_{\mathrm{vv}}=\frac{1}{\varepsilon}\left[1+R_{\mathrm{v}}\left(\theta_{\mathrm{i}}\right)\right]\left[1+R_{\mathrm{v}}\left(\theta_{\mathrm{s}}\right)\right] \sin \theta_{\mathrm{i}} \sin \theta_{\mathrm{s}} \\
-\left[1-R_{\mathrm{v}}\left(\theta_{\mathrm{s}}\right)\right]\left[1-R_{\mathrm{v}}\left(\theta_{\mathrm{s}}\right)\right] \cos \theta_{\mathrm{i}} \cos \theta_{\mathrm{s}} \cos \phi \\
f_{\mathrm{hv}}=\left[1-R_{\mathrm{v}}\left(\theta_{\mathrm{i}}\right)\right]\left[1+R_{\mathrm{h}}\left(\theta_{\mathrm{s}}\right)\right] \cos \theta_{\mathrm{i}} \sin \phi
\end{array},\right.
$$

where $R_{\mathrm{h}}$ and $R_{\mathrm{v}}$ are the specular reflection coefficients for horizontal and vertical polarization waves from the lower medium ( $z<0$ ) with a planar surface $S_{p}(z=0)$ into the upper half-space respectively, $\phi$ is the azimuthal
angle of scattering.
Equation (15) describes only the diffused part of scattered energy and does not include the specular reflected field, which is dominant at specular direction $\left(\theta_{\mathrm{s}}=\theta_{\mathrm{i}}\right.$, $\phi=0)$. In the general case of an arbitrarily stratified medium, $R_{\mathrm{h}}$ and $R_{\mathrm{v}}$ can be represented as

$$
\begin{equation*}
R_{\mathrm{h}}=\frac{R_{0 \mathrm{~h}}+R_{\mathrm{h}}^{\prime}}{1+R_{0 \mathrm{~h}} R_{\mathrm{h}}^{\prime}}, \quad R_{\mathrm{v}}=\frac{R_{0 \mathrm{v}}+R_{\mathrm{v}}^{\prime}}{1+R_{0 \mathrm{v}} R_{\mathrm{v}}^{\prime}} \tag{16}
\end{equation*}
$$

where $R_{0 \mathrm{~h}}$ and $R_{0 \mathrm{v}}$ are the Fresnel reflection coefficients from the planar surface between two homogeneous me$\operatorname{dia}\left(\varepsilon_{0}=1\right.$ and $\left.\varepsilon\right)$, and $R^{\prime}$ is the reflection coefficient from the sub-surface layers ( $R^{\prime}=0$ for a uniform $\varepsilon(z)=$ constant homogeneous half-space $z<0$ ).

Now consider the simplest case of a layered structure (Fig. 1), namely a homogeneous layer of mean thickness $H$, with complex dielectric permittivity $\varepsilon=\varepsilon^{\prime}+i \varepsilon^{\prime \prime}$, overlying a homogeneous half-space (substrate) with a complex dielectric permittivity constant $\varepsilon_{1}=\varepsilon_{1}^{\prime}+i \varepsilon_{1}{ }^{\prime \prime}$. The reflection coefficients $R_{\mathrm{h}}$ and $R_{\mathrm{v}}$ from this structure is given by Eq. (16), where $R^{\prime}$ can be written as

$$
\begin{align*}
& R_{\mathrm{h}}^{\prime}(\theta)=R_{1 \mathrm{~h}}(\theta) \exp \left(2 i k H \sqrt{\varepsilon-\sin ^{2} \theta}\right) \\
& R_{\mathrm{v}}^{\prime}(\theta)=R_{1 \mathrm{v}}(\theta) \exp \left(2 i k H \sqrt{\varepsilon-\sin ^{2} \theta}\right) \tag{17}
\end{align*}
$$

with the following expressions for the Fresnel reflection coefficient $R_{1}(\theta)$ from the surface $z=-H$ of two media with dielectric permittivities $\varepsilon$ and $\varepsilon_{1}$ :

$$
\begin{align*}
& R_{1 \mathrm{~h}}(\theta)=\frac{\sqrt{\varepsilon-\sin ^{2} \theta}-\sqrt{\varepsilon_{1}-\sin ^{2} \theta}}{\sqrt{\varepsilon-\sin ^{2} \theta}+\sqrt{\varepsilon_{1}-\sin ^{2} \theta}} \\
& R_{1 \mathrm{v}}(\theta)=\frac{\varepsilon_{1} \sqrt{\varepsilon-\sin ^{2} \theta}-\varepsilon \sqrt{\varepsilon_{1}-\sin ^{2} \theta}}{\varepsilon_{1} \sqrt{\varepsilon-\sin ^{2} \theta}+\varepsilon \sqrt{\varepsilon_{1}-\sin ^{2} \theta}} \tag{18}
\end{align*}
$$

Here, Gaussian rough surface is selected as rough surface of layered medium model and for Gaussian rough surface, $W_{f}$ in Eq. (14) is given by

$$
\begin{equation*}
W_{f}(K)=\frac{\delta^{2} l}{2 \sqrt{\pi}} \exp \left(-K^{2} l^{2} / 4\right) \tag{19}
\end{equation*}
$$

where $\delta$ and $l$ are the height standard deviation and correlation length of the surface, respectively. Substituting Eq. (19) into Eq. (14), specific incoherent scattering cross sections of up-going waves from Gaussian rough surface of layered medium is obtained according to the SPM,

$$
\begin{equation*}
\sigma_{\alpha \beta}^{0}\left(\vec{k}_{\mathrm{s}}, \vec{k}_{\mathrm{i}}\right)=\frac{\pi}{2 \sqrt{\pi}} k_{\mathrm{i}}^{4}|\varepsilon-1|^{2}\left|f_{\alpha \beta}\right|^{2} \delta^{2} l\left(-K^{2} l^{2} / 4\right) \tag{20}
\end{equation*}
$$

In this way, we can obtain the scattering coefficient of Gaussian rough surface of layered medium as

$$
\begin{equation*}
\sigma=10 \log _{10} \sigma_{\alpha \beta}^{0}\left(\vec{k}_{\mathrm{s}}, \vec{k}_{\mathrm{i}}\right) \tag{21}
\end{equation*}
$$

We use Eq. (21) to study the optical wave scattering from Gaussian rough surface of layered medium mainly.

The two conditions of SPM are

$$
\begin{equation*}
k \delta<0.3, \quad \sqrt{2} \delta / l<0.3 \tag{22}
\end{equation*}
$$

In the following calculations, emphasis is put on examining the light scattering behavior for HH polarization in the $x-z$ plane (shown in Fig. 1), hence $\phi=0, k_{\mathrm{i}}=k_{\mathrm{s}}$, and $k=k_{\mathrm{i}} \sin \theta_{\mathrm{s}}-k_{\mathrm{i}} \sin \theta_{\mathrm{i}}$. In calculating of Eq. (21), the incident wavelength $\lambda$ is chosen as $1.06 \mu \mathrm{~m}$ and the incident angle is $30^{\circ}$. We first compare the angular distribution of $\sigma$ from the slightly rough surface of layered medium with different $\varepsilon_{1}$, and the other related parameters $\varepsilon=1.6+0.01 i, H=2 \lambda, \delta=0.2 / k$, and $l=10 / k$. It is observed that the scattering pattern corresponding to different $\varepsilon_{1}$ in Fig. 2 are almost same, which indicates that the influence of $\varepsilon_{1}$ on the bistatic scattering coefficient is small.

In Fig. 3, the dependence of the scattering coefficient $\sigma$ on $\varepsilon$ is plotted. It is shown that as the real part of $\varepsilon$ increases, the distribution of $\sigma$ exhibits more gentle oscillation for the same imaginary part, while the value of $\sigma$ is visually larger in the whole scattering angle region, except for $60^{\circ} \leq \theta_{\mathrm{s}} \leq 75^{\circ}$. This phenomenon implies that $\sigma$ is sensitive to the varying of $\varepsilon$. Figure 3 also shows the comparison of the angular scattering distribution of scattering coefficient with different imaginary part of $\varepsilon$. It can be seen that the discrepancy of the scattering pattern is small, especially for the small and moderate incident angles, and this is mainly due to the different absorption for imaginary part of $\varepsilon$.

To further explore the important scattering characteristic of the mutilayered surface model, we consider the calculated behavior for different depth of $H$, and the influence of $H$ on the bistatic scattering coefficient is depicted in Fig. 4, where $\varepsilon=1.6+0.01 i, \varepsilon_{1}=80+30 i$, $\delta=0.2 / k, l=10 / k$. It is shown that the scattering coefficient of the Gaussian surface with three-layer permittivity will increase with decreasing $H$, and the scattering pattern also shows small oscillation. It should be noted that if $H=0$, our simulation result will reduce to the case of the conventional SPM for the light incident upon the homogeneous medium.


Fig. 2. Scattering distribution of $\sigma$ for different $\varepsilon_{1}$.


Fig. 3. Scattering distribution of $\sigma$ for different $\varepsilon$.

The effect of surface roughness parameters ( $\delta$ and $l$ ) on the scattering coefficient of the slightly rough surface with three layers is also examined. Figure 5 illustrates the distribution of $\sigma$ as a function of $\delta$ with different scattering angles, where $\varepsilon=1.6+0.01 i, \varepsilon_{1}=80+30 i$, $H=2 \lambda, l=10 / k$. It is observed that $\sigma$ increases with increasing $\delta$ for any incident angle. Another point worth noting is the scattering pattern $\sigma$ does not always change that for any given value of $\delta$ within the limitation of SPM.

We also calculated the scattering coefficient for different scattering angles when $l$ is varied, as shown in Fig. $6(\delta=0.2 / k)$. It is found that the influence of $l$ on the backscattering coefficient is significant, that is, in the directions close to specular scattering direction, the value of $\sigma$ is the same, while in other scattering directions, the larger value of $l$ is, the smaller $\sigma$ is, especially for the large scattering angles away from the specular direction. Figure 7 depicts the influence of incident wavelength $\lambda$ on the bistatic scattering coefficient under the condition of $\varepsilon=1.6+0.01 i, \varepsilon_{1}=80+30 i$, $H=2 \lambda, \delta=3 \times 10^{-8} \mathrm{~m}, l=3 \times 10^{-6} \mathrm{~m}, \theta_{\mathrm{i}}=30^{\circ}$, for $\theta_{\mathrm{s}}=50^{\circ}$ and $60^{\circ}$, respectively. It is observed that


Fig. 4. Scattering distribution of $\sigma$ for different $H$.


Fig. 5. Scattering distribution of $\sigma$ for different $\delta$.


Fig. 6. Scattering distribution of $\sigma$ for different $l$.


Fig. 7. Dependence of $\sigma$ on $\lambda$ for different scattering angles.
with smaller $\lambda(<2 \mu \mathrm{~m}), \sigma$ will increase rapidly with increasing the incident wavelength, while $\sigma$ exhibits a small oscillation for the large incident wavelength for different scattering angles.
In conclusion, the optical wave scattering from the slightly rough surface of three-layer medium is investigated, the influence of the dielectric permittivity of layered medium, the mean thickness of intermediate medium, surface roughness parameters, and the incident wavelength on the scattering coefficient is discussed.
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