

Faraday rotation in a resonant five-level system via electromagnetically induced transparency

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We study the Faraday rotation of polarization of a probe field in a cold, coherently driven five-level system with an M-type configuration. By means of a method of multiple scales we derive two coupled nonlinear envelope equations, which govern the evolution of two circularly polarized components of the probe field. It is shown that due to the quantum interference effect induced by two control fields, one can obtain a large rotation angle with a very low absorption of the probe field. In addition, an efficient control over the polarization state of the probe field in the system can also be easily realized.

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The Faraday effect, i.e., the rotation of polarization experienced by light propagating inside a medium along an applied magnetic field, is a well-known phenomenon and has been widely studied since the dawn of modern physics^[1]. In recent years, considerable attention has been paid to the nonlinear Faraday effect in resonant atomic and molecular systems, which is of much interest in both fundamental physics and practical applications in optical information processing and engineering^[2]. Due to the resonance between optical field and matter the nonlinear Faraday effect is potentially strong. However, the resonance leads also to a significant optical absorption, and hence such system is usually less efficient and produces only small polarization rotation.

A method to solve this problem is to use the effect of electromagnetically induced transparency (EIT)^[3], which has attracted increased interest because EIT effect can provide giant Kerr nonlinearity with largely-suppressed optical absorption even that a probe field is tuned on to a very strong one-photon transition. The wave propagation in EIT-based optical media displays also many other striking features such as significant reduction of group velocity^[4]. Based on the resonantly enhanced nonlinearity, the low optical absorption and the ultraslow propagation property, it has been shown recently that it is possible to produce a new type of optical solitons, i.e., ultraslow optical solitons^[5–10] in highly resonant optical media.

In recent years, it has been shown that efficient control over the polarization states and large nonlinear Faraday rotation of probe fields can be realized by using cold EIT-based system with various atomic level configurations^[11–16]. Some related experiments have also been carried out for measuring the probe transmission and the rotation angle^[17–19].

In the present study, we consider a laser-cooled, coherently driven five-level atomic system with an M-type configuration, which can be easily realized in Zeeman-split alkali atomic gases by applying an external magnetic field. The atomic coherence is induced by EIT effect contributed by two strong control fields that manipulate the optical property of the system, including the significant changes of linear and nonlinear dispersions and absorp-

tion of weak probe field. A large Faraday rotation of the probe field can be obtained and an efficient control over the linear polarization states of the probe field can be easily realized. Moreover, a circular dichroism significant in non-symmetric configurations is avoided in the system due to the symmetry of our scheme. This results may facilitate potential applications such as optical switching, logic gates^[20], and storage^[22].

We consider a lifetime-broadened five-level atomic system which interacts with a weak pulsed probe field of central frequency $\omega_p/(2\pi)$ and two strong, continuous-wave (CW) control fields of frequencies $\omega_{c1}/(2\pi)$ and $\omega_{c2}/(2\pi)$, respectively (see Fig. 1). Such level configuration can be realized in Zeeman-split alkali atoms (e.g., a ⁸⁷Rb gas) by applying an external magnetic field B . The atoms are trapped in a cell with the temperature lowed to $\leq 0.5 \mu\text{K}$ to cancel Doppler broadening. The electric-field vector of the system can be written as $\mathbf{E} = (\hat{\mathbf{x}}\mathcal{E}_{px} + \hat{\mathbf{y}}\mathcal{E}_{py}) \exp[i(k_p z - \omega_p t)] + \hat{\mathbf{e}}_{c1}\mathcal{E}_{c1} \exp[i(k_{c1}r - \omega_{c1}t)] + \hat{\mathbf{e}}_{c2}\mathcal{E}_{c2} \exp[i(k_{c2}r - \omega_{c2}t)] + \text{c.c.}$, where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{e}}_{c1}$ and $\hat{\mathbf{e}}_{c2}$ are the unit vectors denoting the polarizing directions of the probe and control fields, with corresponding envelopes \mathcal{E}_p , \mathcal{E}_{c1} , and \mathcal{E}_{c2} , respectively. Based on the relations $\hat{\mathbf{x}} = (\hat{\mathbf{e}}_+ + \hat{\mathbf{e}}_-)/\sqrt{2}$ and $\hat{\mathbf{y}} = (\hat{\mathbf{e}}_+ - \hat{\mathbf{e}}_-)/(i\sqrt{2})$, where $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ are the unit vectors denoting the right-(σ^+)

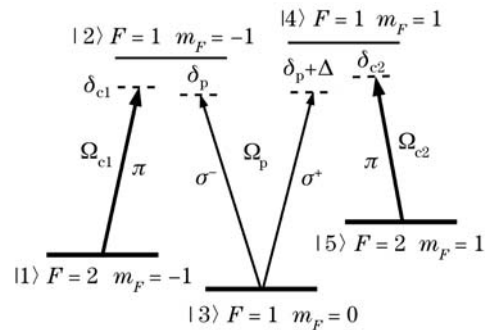


Fig. 1. Energy level diagram and excitation scheme of a symmetric lifetime-broadened five-level atomic system interacting with a weak, pulsed probe field of central frequency $\omega_p/(2\pi)$ and two strong, CW control fields of frequencies $\omega_{c1}/(2\pi)$ and $\omega_{c2}/(2\pi)$, respectively.

and left- (σ^-) circular polarizations, we can take the probe pulse as a superposition of the σ^+ and σ^- polarized components, i.e., we have $\mathbf{E}_p = (\hat{\epsilon}_+ \mathcal{E}_{p+} + \hat{\epsilon}_- \mathcal{E}_{p-}) \exp[i(k_p z - \omega_p t)] + \text{c.c.}$ with $\mathcal{E}_{p\pm} = (\mathcal{E}_{px} \mp i\mathcal{E}_{py})/\sqrt{2}$. Thus, the σ^- (σ^+) component of the probe pulse drives the transition $|2\rangle \leftrightarrow |3\rangle$ ($|3\rangle \leftrightarrow |4\rangle$), while the control field \mathbf{E}_{c1} (\mathbf{E}_{c2}) drives the transition $|1\rangle \leftrightarrow |2\rangle$ ($|4\rangle \leftrightarrow |5\rangle$). Thus, both σ^- and σ^+ components of the probe field form respectively an EIT Λ -configuration, and there is no absorption at resonance. The equations of motion for the slowly varying atomic probability amplitude A_j take the following form

$$\begin{aligned} \left(\frac{\partial}{\partial t} + id_1\right) A_1(t) &= -i\Omega_{c1}^* A_2(t), \\ \left(\frac{\partial}{\partial t} + id_2\right) A_2(t) &= -i\Omega_{c1} A_1(t) - i\Omega_{p1} A_3(t), \\ \left(\frac{\partial}{\partial t} + id_4\right) A_4(t) &= -i\Omega_{p2} A_3(t) - i\Omega_{c2} A_5(t), \\ \left(\frac{\partial}{\partial t} + id_5\right) A_5(t) &= -i\Omega_{c2}^* A_4(t), \end{aligned} \quad (1)$$

together with conservation equation $\sum_{j=1}^5 |A_j|^2 = 1$. Here, $\Omega_{p1} = -(\mathbf{p}_{23} \cdot \hat{\epsilon}_- \mathcal{E}_{p-})/\hbar$, $\Omega_{p2} = -(\mathbf{p}_{43} \cdot \hat{\epsilon}_+ \mathcal{E}_{p+})/\hbar$, $\Omega_{c1} = -(\mathbf{p}_{21} \cdot \hat{\epsilon}_{c1} \mathcal{E}_{c1})/\hbar$ and $\Omega_{c2} = -(\mathbf{p}_{45} \cdot \hat{\epsilon}_{c2} \mathcal{E}_{c2})/\hbar$ are the Rabi frequencies where \mathbf{p}_{ij} is the electric dipole matrix element associated with the transition from $|j\rangle$ and $|i\rangle$. In Eq. (1) we define $d_1 = (\delta_p - \delta_{c1}) - i\Gamma_1/2$, $d_2 = \delta_p - i\Gamma_2/2$, $d_3 = -i\Gamma_3/2$, $d_4 = (\delta_p + \Delta) - i\Gamma_4/2$, and $d_5 = (\delta_p + \Delta - \delta_{c2}) - i\Gamma_5/2$ with Γ_j being the decay rate of the state $|j\rangle$. $\Delta = (2\mu_B/\hbar)gB$ is the Zeeman shift of the sublevels in the upper level with μ_B being the Bohr magneton and g the gyromagnetic factor.

The equations of motion for $\Omega_{pn}(z, t)$ ($n = 1, 2$) can be obtained by Maxwell equation under a slowly-varying envelope approximation, which read

$$i \left(\frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_{pn} - \kappa_{3l} A_{2n} A_3^* = 0, \quad (l = 2n), \quad (2)$$

where $\kappa_{32} = \mathcal{N}_a |\mathbf{p}_{32} \cdot \hat{\epsilon}_-|^2 \omega_p / (2\hbar\epsilon_0 c)$ and $\kappa_{34} = \mathcal{N}_a |\mathbf{p}_{34} \cdot \hat{\epsilon}_+|^2 \omega_p / (2\hbar\epsilon_0 c)$ with \mathcal{N}_a being the atomic density, ϵ_0 the vacuum dielectric constant and c the light speed in vacuum.

Before solving the nonlinearly coupled Eqs. (1) and (2), we examine the linear properties of the system. These linear properties are the main contributors to pulsed spreading and attenuation, and they also lead to linear Faraday effect. To achieve this, we assume that the probe field is weak so that the atomic ground state $|3\rangle$ is not depleted, i.e., $A_3 \approx 1$. In this case one can make a linear analysis on Eqs. (1) and (2). Through taking Ω_{pn} and A_j ($j = 1, 2, 4, 5$) as being proportional to $\exp[i(k(\omega)z - \omega t)]$, one can easily get the linear dispersion relation of the system, which displays two branches, i.e., $k(\omega) = k_{1,2}(\omega)$ with $k_1(\omega) = \omega/c + \kappa_{32}(\omega - d_1)/D_1(\omega)$ and $k_2(\omega) = \omega/c + \kappa_{34}(\omega - d_5)/D_2(\omega)$, corresponding respectively to σ^- and σ^+ components of the probe field. Here we define $D_1(\omega) = |\Omega_{c1}|^2 - (\omega - d_1)(\omega - d_2)$ and $D_2(\omega) = |\Omega_{c2}|^2 - (\omega - d_4)(\omega - d_5)$.

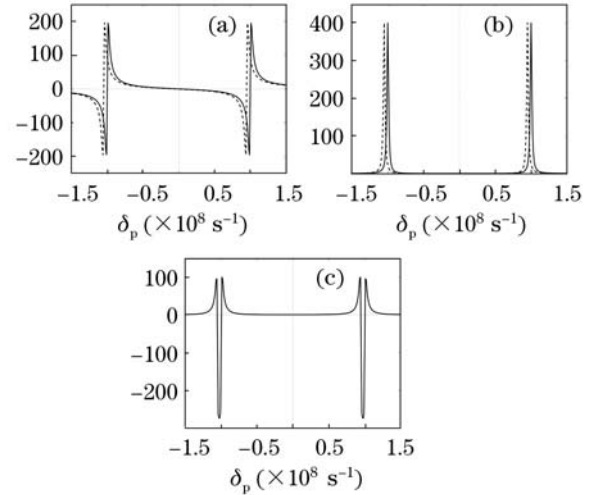


Fig. 2. (a) Dispersion spectra for Ω_{p1} (solid line) and Ω_{p2} (dashed line). The parameters are taken as $\Gamma_2 \simeq \Gamma_4 = 0.5 \times 10^7 \text{ s}^{-1}$, $\Gamma_1 \simeq \Gamma_3 \simeq \Gamma_5 = 0.5 \times 10^3 \text{ s}^{-1}$, $\Omega_{c1} = \Omega_{c2} = 1.0 \times 10^8 \text{ s}^{-1}$, $\Delta = 0.5 \times 10^7 \text{ s}^{-1}$, and $\delta_{c1} = \delta_{c2} = 0$. (b) Absorption spectra for Ω_{p1} (solid line) and Ω_{p2} (dashed line) under the same conditions. (c) Difference dispersion between Ω_{p1} and Ω_{p2} , which corresponds to Faraday rotation caused by linear effect.

Figures 2(a) and (b) show the dispersion spectra and absorption spectra of Ω_{p1} and Ω_{p2} under a particular set of parameters (given below Eq. (6)). We see that near the central frequency of the probe field (i.e., $\omega = 0$), both k_1 and k_2 modes are transparent, which is due to the quantum coherence effect induced by two control fields. In linear theory, the two branches of the dispersion relation are relatively independent. The difference between $\text{Re}[k_1(\omega)]$ and $\text{Re}[k_2(\omega)]$ as shown in Fig. 2(c) signifies a difference in phase velocities of the two circular components and, as a result, the plane of polarization rotates. However, such rotation is caused only by linear effect, in this paper we will discuss the rotation caused by both linear and nonlinear effects.

If Taylor expanding the linear dispersion relation around the central frequency of the probe field ω_p , we obtain $k_n(\omega) = K_{0n} + K_{1n}\omega + \mathcal{O}(\omega^2)$ ($n = 1, 2$) with $K_{jn} = (\partial^j k_n(\omega)/\partial \omega^j)|_{\omega=0}$. Here $K_{01} = \phi_1 + i\alpha_1/2$ ($K_{02} = \phi_2 + i\alpha_2/2$), with ϕ_1 and α_1 (ϕ_2 and α_2) being respectively the phase shift per unit length and absorption coefficient of the σ_- (σ_+) component of the probe field, and $1/K_{11}$ ($1/K_{12}$) being the corresponding group velocity.

We are interested in the nonlinear effect of the system. To this aim we apply the method of multiple scales to derive two nonlinear coupled equations that describe the evolution and interaction of the envelopes of two polarized components of the probe field. We make the following asymptotic expansion $A_j = \sum_{l=0}^{\infty} \mu^l A_j^{(l)}$ ($j = 1$ to 5) and $\Omega_{pn} = \sum_{l=1}^{\infty} \mu^l \Omega_{pn}^{(l)}$ ($n = 1, 2$), where μ is a small parameter characterizing the small population depletion of the ground state. From the conservation equation one has $A_3^{(0)} = 1$ and $A_3^{(1)} = A_1^{(0)} = A_2^{(0)} = A_4^{(0)} = A_5^{(0)} = 0$. To obtain a divergence-free solution in high-order approximations, all quantities on the right hand side of asymptotic expansion must be considered as functions of the

multi-scale variables $z_l = \mu^l z$ ($l = 0$ and 2) and $t_l = \mu^l t$ ($l = 0$ and 2). Substituting these expansions into Eqs. (1) and (2), we obtain a chain of linear, but inhomogeneous equations on $A_j^{(l)}$ and $\Omega_{pn}^{(l)}$, which can be solved order by order.

To the leading order, $O(\mu)$, the solution of Eqs. (1) and (2) is just that obtained in the linear regime described above. The polarized components of the probe field take the form as $\Omega_{pn}^{(1)} = F_n \exp(i\theta_n)$ where $\theta_n = k_n(\omega)z_0 - \omega t_0$ and F_n is a yet to be determined envelope function depending on the slow variables t_2 and z_2 .

To the next order, $O(\mu^3)$, a solvability condition yields the nonlinearly coupled envelope equation

$$i \left(\frac{\partial F_n}{\partial z_2} + \frac{1}{V_{gn}} \frac{\partial F_n}{\partial t_2} \right) - (W_{nn}|F_n|^2 + W_{nm}|F_m|^2)F_n = 0$$

$$(m, n = 1, 2; m \neq n), \quad (3)$$

where $V_{gn} = 1/K_{1n}$ is the group velocity of the respective component and

$$W_{nm} = \frac{\kappa_{3l}(\omega - d_{4n-3})(|\omega - d_{4m-3}|^2 + |\Omega_{cm}|^2)}{D_n|D_m|^2}$$

$$(l = 2n) \quad (4)$$

is nonlinear coefficients characterizing the self-phase (W_{nn}) and cross-phase (W_{nm} , $n \neq m$) modulations of the σ_- and σ_+ polarized components. Returning to original variables, Eq. (3) can be written as

$$i \frac{\partial U_n}{\partial z} + (-1)^{n-1} i \delta \frac{\partial U_n}{\partial \tau} - (W_{nn}|U_n|^2 + W_{nm}|U_m|^2)U_n = 0, \quad (5)$$

where $U_n = \Omega_{pn} \exp(-iK_{0n}z)$ (when taking $\omega = 0$), $\tau = t - z/V_g$, and $\delta = (1/V_{g1} - 1/V_{g2})/2$, with $V_g = 2V_{g1}V_{g2}/(V_{g1} + V_{g2})$. The coefficient δ is a quantity characterizing the group-velocity match of two envelopes. Note that the signs of δ and W_{nm} depend on the signs of detunings, which is easy to control and give rich dynamics for the interaction of two polarized components.

To make the following physical discussion more transparent and numerical calculation more convenient, Eq. (5) is written as the following dimensionless form

$$i \frac{\partial u_n}{\partial s} + (-1)^{n-1} i g_\delta \frac{\partial u_n}{\partial \sigma} - (g_{nn}|u_n|^2 + g_{nm}|u_m|^2)u_n = 0, \quad (6)$$

where we have scaled the variables by using $s = z/L_{NL}$, $\sigma = \tau/\tau_0$, $u_n = U_n/U_0$, $g_\delta = \text{sign}(\delta)(L_{NL}/L_\delta)$, and $g_{nm} = (W_{nm}/|W_{22}|)$. Here, $L_\delta = \tau_0/|\delta|$ is group velocity mismatch length and $L_{NL} = 1/(|W_{22}|U_0^2)$ is nonlinear length with U_0 being typical Rabi frequency of the probe field.

We consider a set of realistic parameters to demonstrate the control over the polarization of the probe pulse. Our system can be experimentally realized by choosing a vapor cell of cold alkali atoms, with $\Gamma_2 \simeq$

$\Gamma_4 = \Gamma = 0.5 \times 10^7 \text{ s}^{-1}$, $\Gamma_1 \simeq \Gamma_3 \simeq \Gamma_5 = 10^{-4}\Gamma$, and $\kappa_{32} \simeq \kappa_{34} = 1.0 \times 10^9 \text{ cm}^{-1}\cdot\text{s}^{-1}$. We take $\Omega_{c1} = \Omega_{c2} = 1.0 \times 10^8 \text{ s}^{-1}$, $\delta_p = 0.8 \times 10^8 \text{ s}^{-1}$, $\Delta = 0.5 \times 10^7 \text{ s}^{-1}$, and $\delta_{c1} = \delta_{c2} = 0$. With these parameters, we obtain $K_{01} = -22.15 + 1.23i \text{ cm}^{-1}$, $K_{02} = -30.45 + 2.33i \text{ cm}^{-1}$, $K_{11} = (1.25 - 0.14i) \times 10^{-6} \text{ cm}^{-1}\cdot\text{s}$, $K_{12} = (2.20 - 0.34i) \times 10^{-6} \text{ cm}^{-1}\cdot\text{s}$, $W_{11} = (-2.80 + 0.15) \times 10^{-14} \text{ cm}^{-1}\cdot\text{s}^2$, $W_{12} = (-4.93 + 0.27) \times 10^{-14} \text{ cm}^{-1}\cdot\text{s}^2$, $W_{21} = (-3.84 + 0.29) \times 10^{-14} \text{ cm}^{-1}\cdot\text{s}^2$, and $W_{22} = (-6.77 + 0.53) \times 10^{-14} \text{ cm}^{-1}\cdot\text{s}^2$. The group velocities of the two polarized components are respectively $\text{Re}(V_{g1}) = 2.6 \times 10^{-5} \text{ c}$ and $\text{Re}(V_{g2}) = 1.5 \times 10^{-5} \text{ c}$, which means that the probe pulse indeed propagates with ultraslow group velocities in both polarized components in comparison with the light speed in vacuum. Note that the imaginary parts of these quantities are much less than their relevant real parts. The physical reason resulting in so small imaginary parts is due to quantum destructive interference induced by two CW control fields. With the choice of our parameters, the group velocity mismatch length $L_\delta = 10.6 \text{ cm}$, pulse duration of the probe field $\tau_0 = 5.0 \times 10^{-6} \text{ s}$, and the nonlinear length $L_{NL} = 0.2 \times 10^{-1} \text{ cm}$ are given with $U_0 = 3.0 \times 10^7 \text{ s}^{-1}$. Due to the condition that $L_\delta \gg L_{NL}$, which can be easily realized under the requirement $\tau_0 U_0^2 \gg |\delta/W_{22}|$, we obtain $g_\delta \ll 1$ and the terms associated with the group-velocity match can be safely neglected. Thus Eq. (6) can be reduced into $i\partial u_n/\partial s - (g_{nn}|u_n|^2 + g_{nm}|u_m|^2)u_n = 0$.

The reduced equations admit solutions $u_n = \exp(-i\phi_n s)$ with $\phi_n = g_{nn} + g_{nm}$. The phase shift ϕ_n is contributed by both self-Kerr and cross-Kerr nonlinearities. The two output polarized components of the probe field take the form $\Omega_{pn} = U_0 \exp\{i[K_{0n} - (g_{nn} + g_{nm})/L_{NL}]L\}$, where L is length of atomic cell. For characterizing the Faraday effect in the system, we introduce the following parameters associated with the output state of the probe field: T for fractional transmission, ψ for the rotation angle of the polarization, and S for the degree of circular polarization, which results from circular dichroism of the atomic gas. Since the imaginary parts of the coefficient g_{nm} are very small, we can neglect them in leading-order approximation. Based on such consideration we obtain

$$T = \frac{1}{2} (e^{-\alpha_1 L} + e^{-\alpha_2 L}), \quad (7a)$$

$$\psi = \frac{L}{2} \left(\phi_1 - \phi_2 - \frac{g_{11} + g_{12} - g_{21} - g_{22}}{L_{NL}} \right), \quad (7b)$$

$$S = \frac{e^{-\alpha_1 L} - e^{-\alpha_2 L}}{e^{-\alpha_1 L} + e^{-\alpha_2 L}}, \quad (7c)$$

where ψ is contributed by both linear and nonlinear responses, while T and S are only contributed by the linear response. With the choice of our parameters, we obtain $T = 0.7$, $\psi = -0.9 \text{ rad}$ ($\simeq 51.6^\circ$), and $S = 0.1$ when we take $L = 0.1 \text{ cm}$. We also note that a circular dichroism being significant in non-symmetric level configurations is avoided due to the symmetry of our system.

For an injected linearly polarized probe field, its (elliptic) polarization state in the system can be efficiently controlled by adjusting the external magnitude. We have made a calculation on T , ψ , and S as functions of Δ

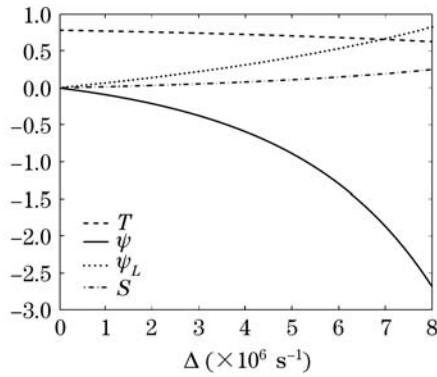


Fig. 3. Curves of T (dashed line), ψ (solid line), ψ_L (dotted line), and S (dot-dashed line) versus Δ with $L = 0.1$ cm. The nonlinear effect enhanced by EIT exerts a significant modification on ψ_L , which can be characterized by $\psi - \psi_L$.

(proportion to B). Figure 3 is the T , ψ , and S versus Δ for $L = 0.1$ cm. In order to make a comparison, we also plot the curve of ψ_L , which characterizes the rotation caused only by linear effect, defined by $\psi_L = (\phi_1 - \phi_2)L/2$. The nonlinear effect enhanced by EIT exerts a significant modification on ψ_L , which can be characterized by $\psi - \psi_L$. From it we can see that with increasing Δ , a larger Faraday rotation is obtained. However, the absorption and circular dichroism also increase.

In conclusion, we have investigated the Faraday effect in a cold, coherently driven five-level system with an M-type configuration. From the equations of motion describing the evolution of atomic amplitudes and the probe field, we have derived two nonlinearly coupled envelope equations governing the dynamics of two polarized components of the probe field by means of the method of multiple-scales. We have given the slow-light solutions of the nonlinear envelope equations and discussed the Faraday effect caused by the linear and nonlinear response of the system. The system proposed can be used to obtain a large Faraday rotation angle and to implement an efficient control over the polarization state of the probe field in the system. The results we obtained may be used to construct optical switching and logic gates and hence facilitate practical applications in future optical information processing and engineering.

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