Use of wavelet in specifying optics

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Received September 28, 2006

Using power spectral density (PSD) function to specify large aperture optical components' quality of laser system is universal. But it cannot provide effective guidance to eliminate certain frequency segment error. In order to solve this problem, two-dimensional discrete wavelet transform (2D-DWT) is used to separate frequency segment error and detect the corresponding region of certain frequency segment error, which is used as feedback to a machining process. The experimental results show that the corresponding region of certain frequency segment can be found and machining can be guided effectively by using wavelet.

OCIS codes: 220.4880, 100.7410, 120.0120.

The large aperture optical components in laser system are different from conventional optics. We should control wavefront quality in frequency domain. Because conventional characteristic parameters such as peak-valley (P-V) value, root-means-square (RMS) value and Zernike polynomial lack frequency spectrum information[1], the Lawrence Livermore National Laboratory (LLNL) put forward power spectral density (PSD) function parameter and PSD characteristic curve (PCC) in designing construction of National Ignition Facility (NIF)[2−4]. In 1997, when the ISO 10110 optics drawing standards implemented, the PSD also became one new characteristic parameter[5].

But there is limitation in PSD method. We will not know the corresponding region of certain frequency segment error of optical surface under test, namely, PSD cannot provide effective guidance to eliminate certain frequency segment error via machining. Because PSD is based on Fourier transform (FT) that is a global transform.

In order to find the corresponding region of certain frequency segment error and guide machining, we decide to adopt one local transform method — wavelet transform. Wavelet transform belongs to time-frequency analysis. It is a local transform in both time domain and frequency domain that is different from FT. There are two methods probably. 1) We can use the PCC to find unqualified frequency segments and then wavelet is used to find the corresponding region of certain unqualified frequency segment, and finally this information is used as feedback to a machining process[6]. But because it is based on PCC, it only adapts to large aperture optical components and mid frequency analysis, moreover, the PCC is only the characteristic curve of LLNL that would not satisfy other optics' specification. 2) We can use wavelet to separate frequency segment error and detect the corresponding region of certain frequency segment for any size optics and any frequency analysis directly, and then this information is also used as feedback to a machining process. In this paper, we use the latter.

At present, the error of a machined optical surface can be separated into three parts, that is, the surface profile error, the surface waviness and the surface roughness, which correspond to low frequency, mid frequency and high frequency in frequency domain. For the NIF's large aperture optical components, the low frequency implies spatial wavelengths greater than 33 mm or frequency $f < 1/33 = 0.03$ mm^{-1}, the mid frequency implies spatial wavelengths from 33 to 0.120 mm or 0.03 mm^{-1} $\leq f \leq$ 8.33 mm^{-1}, and the high frequency implies spatial wavelength smaller than 120 μm or $f > 8.33$ mm^{-1}. In the following analyses, we will base on NIF's definition. It is obvious even if the frequency distribution range is different, the analysis method will be identical.

In order to separate the frequency segment error and with a view to the two-dimensional property of surface error, we adopt two-dimensional discrete wavelet transform (2D-DWT). Suppose the original signal is S, after we do 2D-DWT (including decomposition and reconstruction), for first-level analysis:

$$\mathrm{S} = \mathrm{A1} + \mathrm{D1}, \tag{1}$$

where A1 is reconstructed approximations of S, D1 is reconstructed details of S (we only choose diagonal details for simplicity, see Wavelet Toolbox3.0.1 Help[7]). It is obvious we obtain two different frequency segments of original signal. For second-level analysis:

$$\mathrm{A1} = \mathrm{A2} + \mathrm{D2}. \tag{2}$$

Also, A2 is reconstructed approximations of A1 and D2 is reconstructed details of A1. From Eqs. (1) and (2), we can obtain

$$\mathrm{S} = \mathrm{A2} + \mathrm{D2} + \mathrm{D1}. \tag{3}$$

From Eq. (3), the original signal S is divided into three parts according to the difference of frequency segment. But we should notice that the dyadic-scale separation of actual frequency segment would not satisfy the predefined distribution range of frequency. So, Eq. (3) is only a nominal result. We would extend this technique to a multilevel analysis.

Based on 2D-DWT analysis result, we can find the corresponding region of certain frequency segment directly. Because of the dyadic-scale characteristic of 2D-DWT, the decomposable scale or frequency is fixed. Two-dimensional continuous wavelet transform can be use to

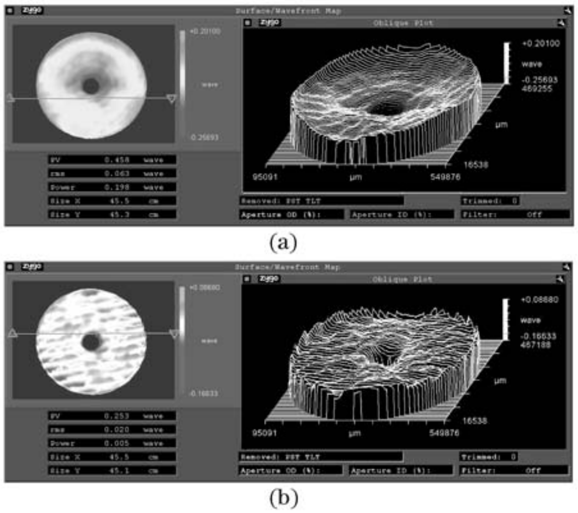

Fig. 1. Surface errors of an identical mirror in (a) direct measurement and (b) measurement after machining.

increase the agility of scale selection after separation based on 2D-DWT.

We measured an aperture $\phi = 450$ mm, $f/8$ plano-mirror by a ZYGO phase shifted interferometer. The effective charge coupled device (CCD) imaging sensor is a 220×218 array. The sampling period is $\Delta = 2.068$ mm/pixel, wavelength is 632.8 nm. The measured results are shown in Fig. 1(a). Figure 1(b) shows the surface error of the second time measurement after machining again of identical mirror.

Above all, we should find the corresponding scale factor of frequency. There is a relation between scale and frequency[7]:

$$F_a = \frac{F_{\mathrm{c}}}{a \cdot \Delta}, \tag{4}$$

where a is scale, Δ is the sampling period of CCD imaging sensor, F_{c} is the center frequency of a certain analysis wavelet, F_a is the frequency corresponding to the scale a. Referring to wavelet toolbox of Matlab, we select bior3.7 wavelet as the analysis wavelet for 2D-DWT. The center frequency of bior3.7 wavelet is calculated to be $F_{\mathrm{c}} = 0.93358$ mm^{-1}.

The sampling frequency $F = 1/\Delta = 0.4836$ mm^{-1}, it belongs to mid frequency range. From Eq. (4), when $a = 2$, $F_2 = 0.2257$ mm^{-1}; similarly, when $a = 4$, $F_4 = 0.1129$ mm^{-1}; when $a = 8$, $F_8 = 0.0565$ mm^{-1}; when $a = 16$, $F_{16} = 0.0282$ mm^{-1}. It is obvious that $F_{16} \approx 0.03$ mm^{-1} which is just the initial frequency of mid frequency range. So, when we do 2D-DWT for four-level analysis, we can separate the low frequency segment and mid frequency segment of original surface error data approximately:

$$\mathrm{S} = \mathrm{A4} + (\mathrm{D4} + \mathrm{D3} + \mathrm{D2} + \mathrm{D1}), \tag{5}$$

where S is original signal, D1 is the reconstructed details of first-level. D2 is the reconstructed details of second-level. D3, D4 may be deduced by analogy. D1, D2, D3 and D4 make up of mid spatial frequency segment of S approximately. A4 is the reconstructed approximations of S or the low spatial frequency segment.

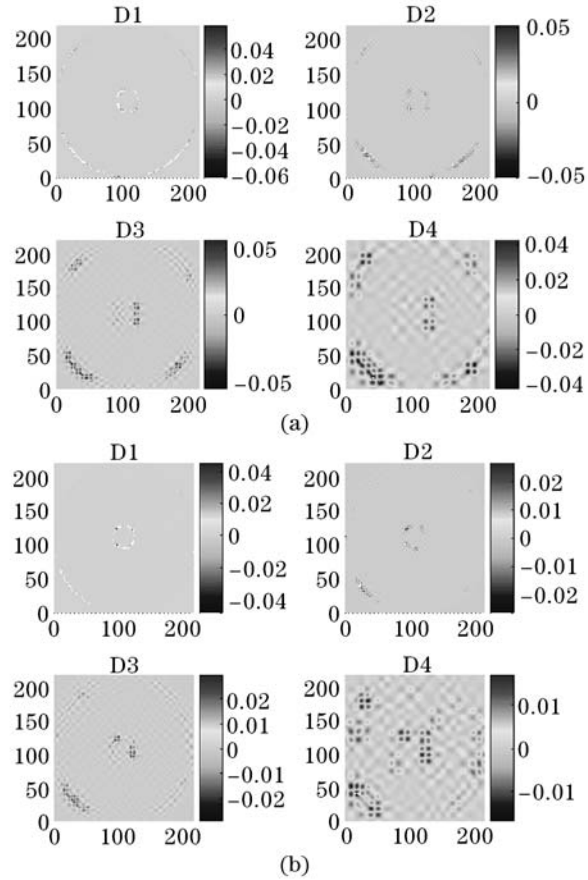

Fig. 2. 2D-DWT analysis results corresponding to the mirror (a) before and (b) after machining.

Figure 2(a) shows the mid frequency analysis results using wavelet toolbox of Matlab[7] (we only choose mid spatial frequency segment to analyze for example, it is also the most important part of frequency range in general analysis). The plot gives a clear picture of where happened corresponding to certain frequency segment. Then we can use this information as feedback to a machining process.

Figure 2(b) shows analysis results of Fig. 1(b) by using wavelet toolbox. Because the maximal absolute values and the distribution range in the plot become small relative to Fig. 2(a), the mid-spatial frequency segments, especially the center frequency of analysis frequency segment close to low frequency, obtain improvement clearly.

From the results of this method, we find that it is feasible to find the corresponding region of certain frequency segment and guide machining. But we do not adequately account for the frequency response of the instrument making the measurement. The complex optical system of the interferometers makes calculation problematic[2]. Thus the conventional method is to determine the optical transfer function (OTF) experimentally. But it is difficult to obtain a high-quality test pattern such as step-height standard.

We have studied the relation of Collins formula and wavelet transform. The study results show that Collins formula can be interpreted as wavelet transform, which simplifies the OTF's calculation and integrates with our current method properly. Next, we will study an actual interferometer and give an experimental result.

The authors would like to thank the Institute of Optics and Electronics of Chinese Academy of Sciences in Chengdu for the assistance in offering measurement data of large mirrors. Z. Yang's e-mail address is sun8_1@163.com.

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