Transverse superresolution with tunable phase-only pupil filters

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A new set of tunable phase-only filters for realizing the transverse superresolution is introduced. The filters, whose significant features are their ability to tune and simplicity, consist of one half-wave plate between two quarter-wave plates, and the half-wave plate is made of two zones that can rotate with respect to each other. Through analyzing the transverse point spread function of such system, it can be concluded that by rotating any zone of the half-wave plate, the transverse resolution can be realized and can be tunable.

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In an attempt to transfer the results obtained with supergain antennas to optical systems for increasing the resolving power, an original way for overcoming the diffraction limit was proposed in 1952 by Francia[1]. After that superresolution plays an important role in modern science and technology, with numerous applications such as astronomy[2], confocal scanning microscopy[3,4], optical storage[5], and free space laser communication[6]. Till now, most of the superresolution filters are all based on annular, multiannular, or leaky annular designs[7,8]. In addition, superresloution by means of polarization pupil mask, radial birefringent filter and polarization-assisted superresolution technique have been reported[9−11].

There lies an obvious disadvantage in the pupil filters that we have mentioned above. It is that once the pupil filter is fabricated, the corresponding superresolution characteristics cannot be changed. However, in some applications, such as particle trapping and manipulation, the superresolution characteristics must be dynamically tunable[12]. So, we must seek the appropriate pupil filters to realize the modification of the superresolution characteristics. In order to overcome the drawback of the conventional pupil filters, tunable axial superresolution by annular binary filters was obtained by Martinez-Corral *et al.*[13]. Recently Canales *et al.*[14] reported that they realized the variable resolution with pupil mask by using the polarizers to modify the transmittance of each zone of the masks. And in our previous work, the superresolution characteristics can be modified by changing the angle between the radial birefringent element and the polarization axis[15].

In this paper, in order to overcome the drawback of the conventional pupil filters, a new set of pure phase filters based on the combination of wave plates is introduced. Through continuously changing the essential parameters of such filters, the transverse superresolution of the optical system can be tunable. At the same time, the superresolution characteristics of such filter, the transverse gain factor G_{T}, the Strehl ratio S are analyzed in detail.

According to the scalar Debye formulation, the normalized complex field amplitude U in the focal region can be written as[16]

$$U\left(v,u\right)=2\int_0^1 P\left(\rho\right)J_0\left(v\rho\right)\exp\left(-\frac{iu\rho^2}{2}\right)\rho\mathrm{d}\rho. \tag{1}$$

The function $P\left(\rho\right)=T\left(\rho\right)\exp\left[i\varphi(\rho)\right]$ defines the complex transmission of the pupil, or pupil function. Here $v=\frac{2\pi}{\lambda}\mathrm{NA}r$ and $u=\frac{2\pi}{\lambda}\mathrm{NA}^2z$ are the radial and axial optical coordinates, respectively, NA is the numerical aperture of the system, r and z denote the radial and axial distances. Within the second order approximation, the transverse and axial intensity distributions can be expressed as[17]

$$I\left(\nu,0\right)=\left|I_0\right|^2-\frac{1}{2}\mathrm{Re}\left(I_0I_1^*\right)\nu^2, \tag{2}$$

$$I\left(0,u\right)=\left|I_0\right|^2-\mathrm{Im}\left(I_0^*I_1\right)u-\frac{1}{4}\left[\mathrm{Re}\left(I_2^*I_0-\left|I_1\right|^2\right)\right]u^2, \tag{3}$$

where $*$ denotes complex conjugate and I_n is the nth moment of the pupil function given by

$$I_n=2\int_0^1 P\left(\rho\right)\rho^{2n+1}\mathrm{d}\rho. \tag{4}$$

In this paper we lay our emphasis on the transverse superresoltuion. The transverse superresolution can be described by Strehl ratio S and transverse gain G_{T}. From Eqs. (2) and (3), the Strehl ratio which is defined as the maximum intensity of the superresolved pattern and the maximum intensity of the Airy pattern, is given by

$$S=\left|I_0\right|^2-\frac{u_{\mathrm{F}}}{2}\mathrm{Im}\left(I_0^*I_1\right). \tag{5}$$

The transverse gain, which is defined as the ratio between the squared width of the parabolic approximation of the intensity point spread function (PSF) without pupil filter and that with pupil filter, is given by

$$G_{\mathrm{T}}=2\frac{\mathrm{Re}\left(I_0I_1^*\right)-u_{\mathrm{F}}\mathrm{Im}\left(I_0^*I_2\right)}{S}, \tag{6}$$

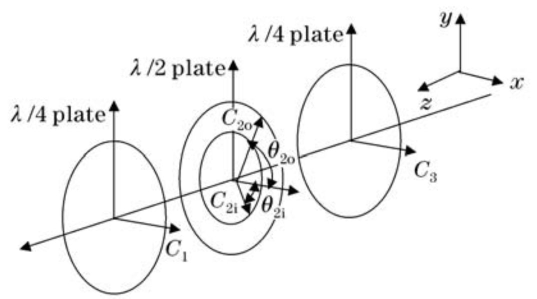

Fig. 1. Schematic of the tunable phase-only pupil filter. The fast axes C_1 of the first $\lambda/4$ plate and C_3 of the second are parallel. A two-zone $\lambda/2$ plate is placed between them, the fast axes $C_{2\mathrm{i}}$, $C_{2\mathrm{o}}$ of the inner and outer zones have the angles $\theta_{2\mathrm{i}}$ and $\theta_{2\mathrm{o}}$ with respect to C_1.

G_{T} is greater than unity for transverse superresolution. In Eqs. (5) and (6), μ_{F} is the displacement of focus in the axial direction.

Combining two identical quarter-wave plates and one half-wave plate together will result in a phase retarder and its retardation depends on the angle between axes of the half-wave plate and the quarter-wave plate[18]. Basing on this principle, a tunable pupil filter, which consists of one half-wave plate between two identical quarter-wave plates, can be designed, as shown in Fig. 1. The two quarter wave plates are parallel and their fast axes, C_1 and C_3, are parallel to the x axis. The half-wave plate is made of two zones that can rotate with respect to each other: an inner circle of radius ρ_1 and an outer annulus of the same size as those of the conventional phase-only mask. And the angles between the x axis and the fast axes of the two zones of the half-wave plate are $\theta_{2\mathrm{i}}$, $\theta_{2\mathrm{o}}$, respectively. With Jones calculus, such device can be written as

$$M = R(-\theta_1)\,\frac{1+i}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$
$$\times R(\theta_1)\,R(-\theta_2)\,i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$\times R(\theta_2)\,R(-\theta_3)\,\frac{1+i}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} R(\theta_3)\,. \quad (7)$$

From Fig. 1, we know $\theta_1 = \theta_3 = 0$, and θ_2 corresponding to $\theta_{2\mathrm{i}}$, $\theta_{2\mathrm{o}}$, so we can get

$$M = \begin{bmatrix} -\cos 2\theta_2 & j\sin 2\theta_2 \\ j\sin 2\theta_2 & -\cos 2\theta_2 \end{bmatrix}. \quad (8)$$

According to the mentioned principle, such system can be regarded as a phase retarder with

$$M' = \begin{bmatrix} \cos\frac{\varphi}{2} + j\sin\frac{\varphi}{2}\cos 2\theta & j\sin\frac{\varphi}{2}\sin 2\theta \\ j\sin\frac{\varphi}{2}\sin 2\theta & \cos\frac{\varphi}{2} - j\sin\frac{\varphi}{2}\cos 2\theta \end{bmatrix}. \quad (9)$$

So $M = M'$, and we can get $\theta = \frac{\pi}{4}$ and $\varphi = 2\pi - 4\theta_2$ ($\theta_2 = \theta_{2\mathrm{i}}$ or $\theta_{2\mathrm{o}}$). This corresponds to an equivalent phase retarder, whose retardation is $\varphi = 2\pi - 4\theta_2$, and the angle between its fast axis and the x axis is $\frac{\pi}{4}$. The amount of the phase is solely determined by the angular position of the half-wave plate. Thus, the tunable phase-only filter is achieved, and by rotating any zone of the half-wave plate ($\theta_{2\mathrm{i}}, \theta_{2\mathrm{o}}$), the phase of such filter can be modified.

Assuming the phases of the inner circle and the outer annulus are, respectively, φ_1 and φ_2. From Eq. (4) the nth moment of the pupil function can be given by

$$I_n = 2\int_0^{\rho_1} \exp(i\varphi_1)\rho^{2n+1}\mathrm{d}\rho + 2\int_{\rho_1}^{1} \exp(i\varphi_2)\rho^{2n+1}\mathrm{d}\rho, \quad (10)$$

and S, G_{T} can be expressed as

$$S = 1 - 2\rho_1^2 + 2\rho_1^4 + 2\cos(\varphi_1 - \varphi_2)\rho_1^2(1-\rho_1^2) + \frac{1}{2}u_{\mathrm{F}}\sin(\varphi_1 - \varphi_2)\rho_1^2(1-\rho_1^2), \quad (11)$$

$$G_{\mathrm{T}} = [\rho_1^6 + \cos(\varphi_1 - \varphi_2)(\rho_1^2 - 2\rho_1^6 + \rho_1^4) + (1-\rho_1^2)(1-\rho_1^4) - \frac{2}{3}u_{\mathrm{F}}\sin(\varphi_1 - \varphi_2)(\rho_1^6 - \rho_1^2)]/S. \quad (12)$$

Equations (11) and (12) indicate that the superresolution parameters are related with the radius of the inner circle ρ_1 and the phase difference $\varphi_1 - \varphi_2$. Thus if a certain PSF is desired, and the maximum is given, the required phase can be obtained from a fitting process with Eqs. (11) and (12). And for the two-zone pupil mask the required value of the phase can be obtained analytically. It also can be concluded that if the inner radius is fixed, the resolution can be tuned by changing the phase difference $\varphi_1 - \varphi_2$. In order to make the process simple, we assume the phase of the outer zone φ_2 is zero, and φ_1 can be tuned from 0 to 2π.

When the Strehl ratio and the transverse gain are given, the optimized radius and phase of the first zone can be calculated with Eqs. (11) and (12) by global searching algorithms. In our simulation work, we have designed a two-zone phase-only pupil filter, as mentioned above whose phase of the outer zone is zero, and that of the inner zone can be tuned from 0 to 2π. And the radii of them are ρ_1 and 1. Figure 2 shows the Strehl ratio and the transverse gain as functions of radius and phase of the first zone. It can be found that when the radius is in the interval (0.3,0.5), not only the transverse superresolution can be realized but also the Strehl ratio will not be very low. So in our numerical results we assume the radius of the first zone is $\rho_1 = 0.35$, and phase of it is tunable from 0 to 2π. Figure 3(a) shows the transverse gain as a function of the first zone's phases. It can be seen that the gain has a symmetric behavior for $\varphi = \pi$. For the phases in the interval $(0,2\pi)$, the transverse gain is greater than one and the phase-only filter is superresolving in the transverse direction. For such a superresolving phase filter, when the phase of the first zone increases in the interval $(0,\pi)$, the gain increases, but in the interval $(\pi, 2\pi)$ it decreases. This indicates that when the phase of the first zone is π, the transverse superresolution parameter G_{T} has a maximum. The Strehl ratio as a function of the phases is shown in Fig. 3(b). It can be found that, in the interval $(0,\pi)$, the Strehl ratio decreases when the phase

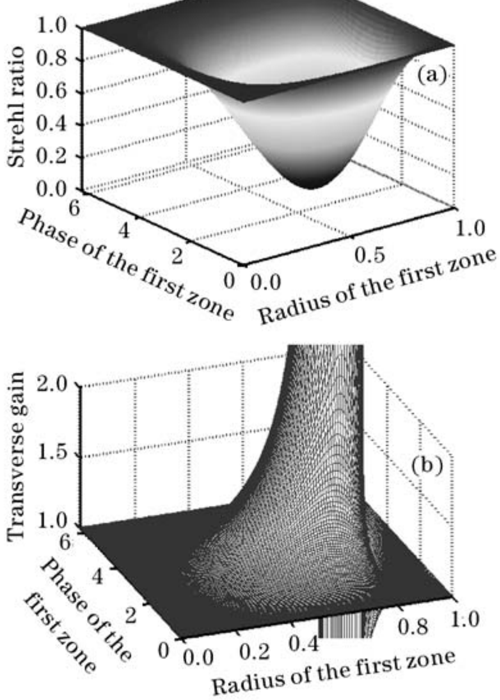

Fig. 2. Strehl ratio (a) and transverse gain (b) as functions of the radius and phase of the inner zone.

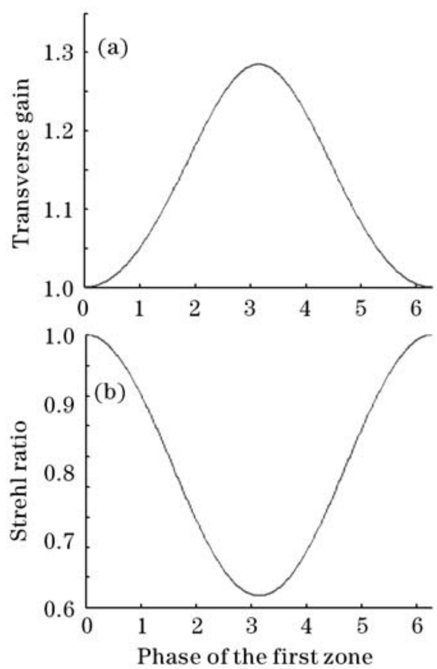

Fig. 3. Transverse gain (a) and Strehl ratio (b) as functions of the phase of the inner zone for a filter of radius $\rho_1 = 0.35$, phase of the outer zone is zero.

increases, but in the interval $(\pi, 2\pi)$ it is opposite. Figures 2 and 3 indicate that it is not possible to attain simultaneously high gain and high Strehl ratio, as expected for any kind of superresolution pupil filters. Moreover, when the phase of the first zone is π, though the transverse gain is the maximum, the Strehl ratio is the minimum. So for some superresolution system with high Strehl ratio needed, π phase is not the ideal selection.

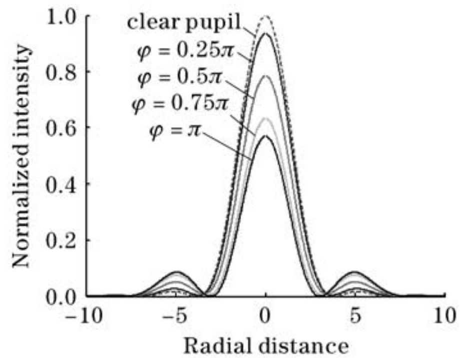

Fig. 4. Transverse PSFs corresponding to different phases of the first zone: 0.25π, 0.5π, 0.75π, and π. The transverse PSF of the clear pupil system is also plotted.

Figure 4 depicts the variation of the transverse intensity PSF for the system with clear pupil, and those the tunable phase-only pupil filters specified by $\varphi = 0.25\pi$, 0.5π, 0.75π, and π are shown respectively. According to the Rayleigh criterion, the proposed system shows better transverse superresolution compared with that without pupil filters. It is clear seen that the center lobe of the pattern with pupil filter becomes smaller than that of the Airy spot. In other words, superresolution is realized with such pupil filters. It also can be found that the the first ring of the superresolution pattern becomes a little wider and brighter (higher intensity) than that of the well known Airy spot. It can be concluded that the transverse PSF behavior can be controlled at will by rotating the inner zone of the phase-only pupil filter as predicted.

By the numerical analysis, the focal patterns with such filters that are produced by rotating the inner zone are given. From the patterns it can be clear seen that the transverse superresolution is tunable. So it can be concluded that with the designed phase-only filters the transverse resolution can be controlled at will by rotating the inner zone of the filter.

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