A novel type of solitons in electron-ion plasmas

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By using one-dimensional self-consistent relativistic fluid model, a novel type of moving relativistic electromagnetic solitons with high intensity in electron-ion plasmas where the ion dynamics is taken into account is investigated numerically. Unlike solitons with single-humped scalar potential found in previous studies, these solitons possess multi-humped scalar and vector potential. The existence of such soliton solutions is investigated in plasmas with different background densities, and the properties of these solitons are presented in detail. We found that the peculiar profile of electron density has alternating regions of humps and dips like a Bragg's grating.

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The technique of ultrashort, ultra-intense laser pulses is developing rapidly in recent years. It makes it possible to produce laser pulses with a few-cycle duration and extremely high intensity. The nonlinear propagation of such pulses in a plasma is a topic of considerable current research interest due to its application in particle accelerators, photon accelerators, and the fast ignition scheme of laser fusion.

Ultraintense laser pulses can propagate like solitons for a very long distance without apparent changes in properties, as have been observed in particle-in-cell (PIC) simulations[1−5]. Electromagnetic solitons with relativistic amplitudes were first investigated by Kozlov *et al.*[6]. Analytical studies on relativistic solitons are usually based on one-dimensional (1D) theories[7], and several novel solutions have been found[8−16]. Shen *et al.* studied subcycle relativistic solitons that could propagate from low density to high density plasmas[12]. The ion motion influence on relativistic soliton has firstly been investigated numerically in Refs. [14] and [15], and the properties of multi-humped solitons are investigated in detail. Poornakala *et al.*[13] studied solitons in cold overdense electron-ion plasmas, and found weakly relativistic single-humped solitons with a significant density cavitation for both immobile and mobile ions. In these previous studies, the number of humps refers to the peak number of vector potential. As for scalar potential, these solitons possess only one peak. That is to say, the scalar potential of solitons found in previous studies has only one peak, which encloses one or more peaks of the vector potential.

In this paper, using 1D relativistic fluid model, we demonstrate a new type of soliton solutions with precisely given field energy and group velocity for given pulse width and plasma density in plasmas where the ion dynamics is taken into account. With the help of the numerical solution of the boundary problem for a set of nonlinear ordinary differential equations derived from 1D relativistic fluid model, we describe the properties of this kind of solitons in detail. Unlike solitons with single-humped scalar potential found in previous studies, this new type of solitons possesses multi-humped scalar and vector potential. The peculiar profile of electron density has alternating regions of humps and dips like a Bragg's grating.

The theory of 1D circularly polarized solitons is usually presented within the relativistic hydrodynamic approximation used to describe both the electron and ion components[13−15]. In this paper, the plasma is assumed to be cold, that is to say, the electron and ion temperatures are zero. To conveniently investigate the properties of solitons, we normalize time through the laser frequency ω_L. Therefore, other variables such as space, velocity, momentum, vector and scalar potentials, and particle density are normalized by c/ω_L, c, $m_\alpha c$, $m_e c^2/e$, and n_c, where m_α is the rest mass of particles with $\alpha = e, i$ denoting electron and ion, and $n_c = m_e\omega_L^2/4\pi e^2$ is the critical density for the laser with a frequency ω_L. In the Coulomb gauge, the Maxwell's equations for the vector and scalar potentials, $\mathbf{A}$ and ϕ, and the hydrodynamic equations for the densities n_α and the canonical momentum $\mathbf{P}_\alpha$ of electrons and ions can be written as

$$\nabla^2\mathbf{A} - \frac{\partial^2}{\partial t^2}\mathbf{A} - \frac{\partial}{\partial t}\nabla\phi = n_e\mathbf{v}_e - n_i\mathbf{v}_i, \quad (1)$$

$$\nabla^2\phi = n_e - n_i, \quad (2)$$

$$\frac{\partial}{\partial t}n_\alpha + \nabla\cdot(n_\alpha\mathbf{v}_\alpha) = 0, \quad (3)$$

$$\frac{\partial}{\partial t}\mathbf{P}_\alpha = -\nabla(\rho_\alpha\phi + \gamma_\alpha) + \mathbf{v}_\alpha\times\nabla\times\mathbf{P}_\alpha, \quad (4)$$

where $\mathbf{P}_\alpha$ is related to kinetic momentum $\mathbf{p}_\alpha$ by $\mathbf{P}_\alpha = \mathbf{p}_\alpha + \rho_\alpha\mathbf{A}$ with the parameter $\rho_\alpha = (q_\alpha/q_e)(m_\alpha/m_e)$, $\gamma_\alpha = (1+|\mathbf{p}_\alpha|^2)^{1/2}$, and $\mathbf{v}_\alpha = \mathbf{p}_\alpha/\gamma_\alpha$ is the fluid velocity.

Here we only consider 1D case, where $\partial_y = \partial_z = 0$. For a circularly polarized laser pulse, the vector potential of the electromagnetic field can be represented by

$$A_y + iA_z = a(\xi)\exp(i[\omega\tau + \theta(\xi)]), \quad (5)$$

with $\xi = x - v_g t$, $\tau = t$[16,17], while all the other quantities, ϕ, n_α, γ_α, and $p_{x\alpha}$, are assumed to only depend on the variable ξ. In this case, the relations $A_x = 0$ and $P_y = P_z = 0$ are satisfied.

Here we choose the boundary condition at the point $\xi = 0$, $a = 0$, $\phi = 0$, $n_\alpha = n_0$, and $p_{x\alpha} = 0$ while n_0 is unperturbed electron or ion density. The longitudinal component of the kinetic momentum, the energy, and the density of each species can be expressed as a function of the potentials as $p_{x\alpha} = (v_g\Phi_\alpha - R_\alpha)/(1-v_g^2)$, $\gamma_\alpha = (\Phi - v_g R_\alpha)/(1-v_g^2)$, and $n_\alpha = v_g(\Phi_\alpha - v_g)n_0/(1-v_g^2)$, where $\Phi_\alpha = 1+\rho_\alpha\phi$, $R_\alpha = [\Phi_\alpha^2 - (1-v_g^2)]^{1/2}$. Then, for an elecron-ion plasma the following closed system of equations for the potential is obtained

$$\phi'' = \frac{v_g n_0}{1-v_g^2}\left(\frac{\Phi_e}{R_e} - \frac{\Phi_i}{R_i}\right), \quad (6)$$

$$a'' + \frac{1}{(1-v_g^2)^2}\left(\omega^2 - \frac{M}{a^4}\right)a = a\left(\frac{n_e}{\gamma_e} + \rho\frac{n_i}{\gamma_i}\right), \quad (7)$$

where $\rho \equiv |\rho_i| = m_e/m_i$, and $M = a^2[(1-v_g^2)\theta' + \omega v_g]$ in Eq. (7) is an integration constant. For an infinite plasma, $M = 0$. Then we can obtain

$$\theta = -\frac{\omega v_g}{1-v_g^2} + \theta_0, \quad (8)$$

where θ_0 is a constant. Substituting Eq. (8) into Eq. (5), we can obtain the phase to be $\exp(i\omega t/(1-v_g^2) - \omega v_g x/(1-v_g^2))$. Thus, the laser frequency is $\omega_L = \omega/(1-v_g^2)$. Since the time variable is normalized by the laser frequency, we obtain $\omega = 1-v_g^2$. And the wavenumber is $k = v_g$. Thus Eq. (7) transforms into

$$a'' + a = \frac{a n_0}{1-v_g^2}\left(\frac{1}{R_e} - \frac{\rho}{R_i}\right). \quad (9)$$

The system described by Eqs. (6) and (9) can be eliminated from Hamiltonian form, and has a first integral

$$W = \frac{a'^2}{2} - \frac{\phi'^2}{2(1-v_g^2)} - \frac{n_0\gamma_e}{1-v_g^2} - \frac{n_0\gamma_i}{\rho(1-v_g^2)}, \quad (10)$$

where W is an integration constant. For an infinite plasma, $W = -(1+1/\rho)n_0$.

In previous studies, various approximations are used to investigate the system described by Eqs. (6) and (9), such as quasineutral approximation, weak nonlinear limit etc.[15]. Under these approximations, the analytical envelope solitons solution with single-humped scalar potential can be obtained. However, for arbitrary amplitudes we need to fully analyze a two-variable problem with respect to a, ϕ described by Eqs. (6) and (9) where the laser amplitude is strongly coupled to the scalar potential. In other words, for a given background density n_0, finding soliton solutions turns out to be an eigenvalue problem in v_g. To investigate this new type of soliton solutions in an electron-ion plasma with a given background density, we solve Eqs. (6) and (9) numerically, due to the complexity of the problem. For a special background density n_0, we solve the system using Runge-Kutta method with initial conditions $\phi' = \phi = 0$, $a' = 0$, $a = a_0$ at $\xi = 0$, where a_0 is a very small value. In this paper, we choose $\rho = 1/1836$.

In order to conveniently describe the properties of the new-style solitons, we define the number of nodes of vector potential a as p and the number of humps of scalar potential ϕ as q. First, we consider solitons with a single-humped scalar potential which encloses multi-humped vector potential like Refs. [14] and [15]. In this case, $p = 1, 2, 3, \cdots$ and $q = 1$. Furthermore, p_0 is defined as p when $q = 1$. For plasmas with different background densities $n_0 = 0.5n_c$ and $1.3n_c$, two solitons with $p = q = 1$ are illustrated in subfigures of Figs. 1(a) and (d). Solitons with two and three-humped scalar potentials are illustrated in subfigures of Figs. 1(b), (e) and (c), (f). As can be seen from Fig. 1, those solitons with multi-humped scalar potential change from those with single-humped scalar potential. The peaks of a have a tiny structure at the point of the valley bottom of ϕ. The maxima of amplitude a and scalar potential of solitons with many q increase with the increase of background density, which resembles soltions with single-humped scalar potential. For a same q and different p, the maxima of the scalar and vector potential are almost invariable for solitons in a plasma with a same background density n_0, as illustrated in Fig. 1.

This new style solitons changing from a soliton with $p_0 = 2$ are presented in Fig. 2.

From Figs. 1 and 2, we can find that p and q have a special relation. When p_0 is an odd, $p = q$; while p_0 is an even, $p = 2q$. To investigate the structure of plasmas in the process of forming solitons, the profiles of electron and ion densities are plotted in Fig. 2. Unlike the distribution of electrons for solitons with $q = 1$ concave at the center of the soliton, the distribution is drastically convex such as a δ-function for a soliton with $p = 4$, $q = 2$, as shown in subfigures of Figs. 2(b) and (d). The full

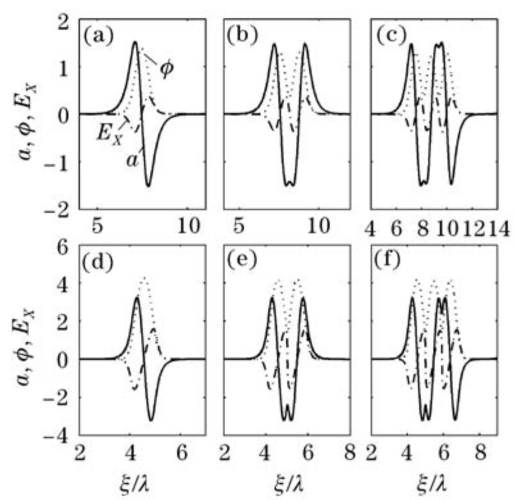

Fig. 1. Profiles of vector potential a, scalar potential ϕ, and longitudinal electric field $E_x = -\phi'$ of six solitons with different p, q are shown in plasmas with different background densities n_0 corresponding to given different group velocities v_g. (a, b, c), and (d, e, f) are for $n_0 = 0.5n_c$ and $1.3n_c$ respectively; (a, d), (b, e), and (c, f) are $p = 1 = 1$, $p = q = 2$, and $p = q = 3$. (a), (b), (c), (d), (e), and (f) correspond to v_g = 0.763945, 0.762604, 0.76266278, 0.42535, 0.4237242, and 0.423731047, respectively.

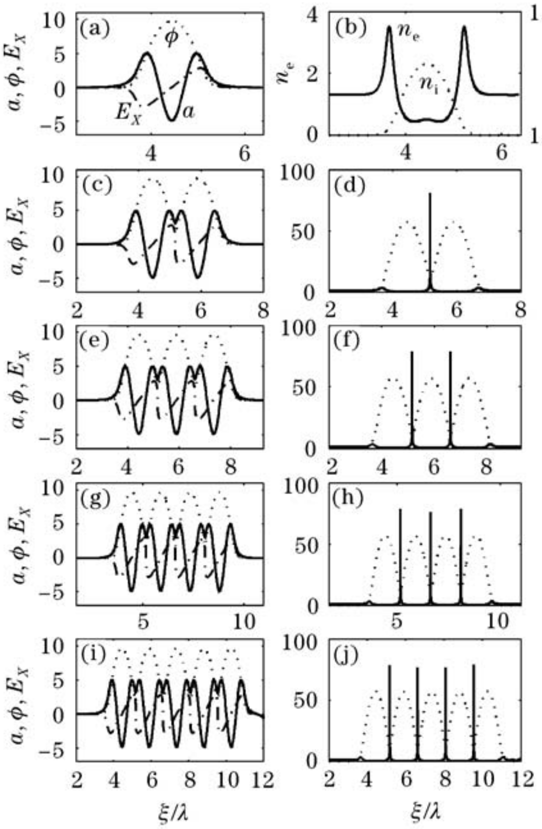

Fig. 2. Five solitons with $p_0 = 2$ and different q are shown in a plasma with background density $n_0 = 1.3n_c$. (a), (c), (e), (g), and (i) are for normalized laser amplitude a, scalar potential ϕ and longitudinal electric field $E_x = -\phi'$; (b), (d), (f), (h), and (j) is for normalized electron density n_e and ion density n_i. (a, b), (c, d), (e, f), (g, h), and (i, j) with $p = 2$, $q = 2$, $p = 2$, $q = 4$, $p = 2$, $q = 6$, $p = 2$, $q = 8$ and $p = 2$, $q = 10$ corresponds to group velocity v_g = 0.498571, 0.498158587, 0.498158892679, 0.498158892451244, and 0.4981588924514133.

width of half-maximum (FWHM) of n_e for a soliton with $q = 2$ in a plasma of $n_0 = 1.3n_c$ is about 0.0035λ. The corresponding FWHM in time dimension is λ/v_g, for λ = 1 μm it is about 23.3 attosecond. It maybe provide a new mechanism for producing an attosecond electrons pulse.

With the increase of q, the distribution of electrons has alternating regions of sharp humps and flat pits like a Bragg's grating. The electron density becomes a comb-function when p is very large, as shown in Fig. 2. The FWHM of every peak of n_e is approximately equal for a soliton with a same q. With the increase of q, the FWHM of n_e increases from about 0.0035λ, 0.0036λ, 0.0037λ, and 0.0038λ. The intervals between peaks of n_e are equal, about 1.4567λ. However, with respect to the distribution of ions, except its hump number increasing with the increase of q, it looks like that for solitons with single-humped scalar potential. The maxima of electron and ion densities corresponding to solitons with multi-humped scalar potential are almost constant for different q.

In the following, we will discuss what plasmas such solitons can exist in. We look for the group velocity which

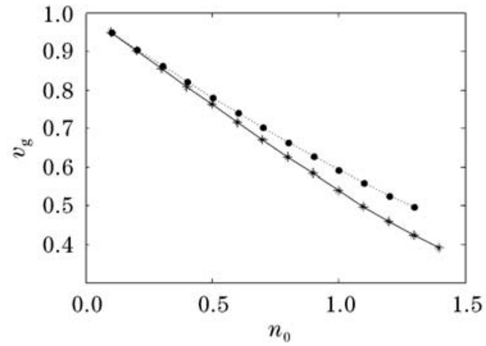

Fig. 3. Group velocity v_g versus the plasma background density n_0 for $p = 2$, $q = 2$ and $p = 4$, $q = 2$. The stars and points refer to the eigenvalues for $p = 2$, $q = 2$ and $p = 4$, $q = 2$, respectively.

can make the laser pulse propagate like such solitons in plasmas with different background densities. Here the group velocities are found corresponding to every other $0.1n_c$ from $0.1n_c$ to the density where solitons breaks, as shown in Fig. 3.

The group velocity v_g versus the plasma background density n_0 for $p = 2$, $q = 2$ and $p = 4$, $q = 2$ are plotted in Fig. 3. By calculating in detail, we found that new-style solitons for $p_0 = 1$ and 2 can exist in plasmas with background densities lower than approximately $n_0 = 1.4n_c$ and $1.3n_c$. Beyond this range, the solitons breaks. Furthermore, for a large p_0, the laser pulse requires a larger group velocity to propagate like these new solitons. For a given p_0, solitons with different q posses group velocities whose values have a tiny difference, as illustrated in captions of Figs. 1 and 2. Although it is perhaps difficult to form solitons with muli-humped scalar potential because of the requirement for a very exactly defined group velocity, we are interested in the structure of electron density like Bragg's grating. Even if the solitons did not form, the structure of electron density would have a form described in Fig. 2. When the laser pulse propagates at a group velocity close to that, a soliton can form.

In conclusion, using 1D relativistic fluid model, we have shown ultrashort relativistic solitons with multi-humped scalar and vector potential in electron-ion plasmas where the ion dynamics is taken into account. By numerically solving the boundary problem described by a set of nonlinear ordinary differential equations, the properties of this new-style solitons are demonstrated in detail. We found that solitons with $p_0 = 1$ and 2 can only exist in plasmas with background densities $0 < n_0 \leq 1.4n_c$ and $0 < n_0 \leq 1.3n_c$, respectively. For a soliton with $p = 4$, $q = 2$, the profile of electron density has a FWHM of dozens of attoseconds in time dimension, which can provide a new mechanism producing an attosecond electron pulse. Another interesting phenomenon is the structure of electron density which has alternating regions of sharp humps and flat pits like a Bragg's grating. The results are expected to be useful in understanding the nonlinear propagation of localized laser pulses in plasmas such as those in inertial confinement fusion and astrophysical environments.

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