Simple Strehl ratio based method for pupil phase mask's optimization in wavefront coding system

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Received May 15, 2006

By applying the wavefront coding technique to an optical system, the depth of focus can be greatly increased. Several complicated methods have already been taken to optimize for the best pupil phase mask in ideal condition. Here a simple Strehl ratio based method with only the standard deviation method used to evaluate the Strehl ratio stability over the depth of focus is applied to optimize for the best coefficients of pupil phase mask in practical optical systems. Results of imaging simulations for optical systems with and without pupil phase mask are presented, and the sharpness of image is calculated for comparison. The optimized pupil phase mask shows good results in extending the depth of focus.

OCIS codes: 070.0070, 090.1000, 100.3020, 110.0110, 120.5060.

Wavefront coding was introduced to increase the depth of focus for optical system in 1995[1]. A cubic phase mask was placed in pupil plane to encode the wavefront of the optical system so that the intermediate image formed could almost be invariant in a large depth of focus, and the intermediate image could be digitally restored to clear one. This technique shows wonderful results in extending the depth of focus[2,3].

Many approaches have already been taken to explore the best type of pupil phase mask in ideal conditions. Both spatial and spatial-frequency domains have already been considered, and many types of pupil phase masks have been proposed. Prasad *et al.*[4] introduced the concept of pupil phase engineering (PPE), Strehl ratio based optimization was applied to search for the best coefficients of pupil phase mask in the ideal condition with only defocus taken into account. The Strehl ratio was expanded in its Taylor series as a function of defocus and the optimization was done by minimizing the coefficients of the Taylor series. However it is much more complicated for practical optical systems, as it may suffer from a lot of aberrations besides defocus. Furthermore, only a part of the coefficients was used, so it was an approximate way. In this paper, a simple Strehl ratio based method is applied to solve these problems and optimize for the best coefficients of a symmetric mixed cubic mask which is of $Z = a(X^3+Y^3)+b(X^2Y+XY^2)$ form in practical optical systems, where (X, Y, Z) is the coordinate of the mask's surface in Cartesian coordinate with Z axis along the optical axis.

An $F/5$ cemented doublet system with a 100-mm focal length and a $\pm 3°$ half field of view is chosen as the object system to extend its depth of focus by wavefront coding so that sharp images for objects from 15 m to infinity can be acquired without changing the position of the image plane. A two-dimensional (2D) view of this system can be found in Fig. 1. Three wavelengths, which are 656.3, 587.6, and 486.1nm, are taken into account with equal weight. The polychromatic sagittal and tangential modulation transfer functions (MTFs) of this system are plot in Fig. 2, where the solid, dashed, dotted, and dash-dot lines correspond to the MTFs with the object at infinity, 50, 30, and 15 m, respectively. The MTFs of different fields of views (0°, 2°, and 3°) are plotted in lines of the same style. Besides defocus, this system suffers from large coma, axial chromatic aberration, and astigmatism.

For the wavefront encoding system, a 3-mm-thick BK7 glass (Schott) is placed 1 mm before the STOP of the original system. The STOP of the new system is set on the first surface of the plate, which will be changed to be $Z = a(X^3 + Y^3) + b(X^2Y + XY^2)$ form for the optimization.

Strehl ratio is the ratio of the on-axis values of PSF with and without aberrations. It can be calculated as

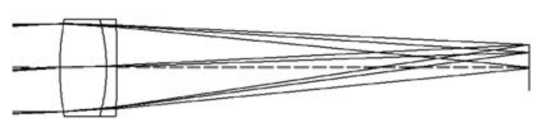

Fig. 1. 2D view of the original cemented doublet system.

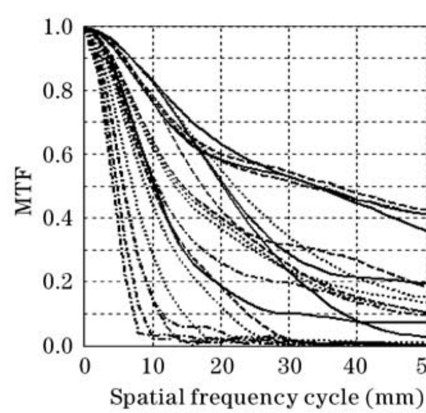

Fig. 2. MTFs of the original cemented doublet system.

$$\mathrm{S.R.} = \frac{\mathrm{PSF}(0,0)_{\mathrm{w-a}}}{\mathrm{PSF}(0,0)_{\mathrm{w/o-a}}} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{OTF}(\xi,\eta)_{\mathrm{w-a}}\mathrm{d}\xi\mathrm{d}\eta}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{OTF}(\xi,\eta)_{\mathrm{w/o-a}}\mathrm{d}\xi\mathrm{d}\eta}, \tag{1}$$

where the subscripts w-a and w/o-a represent with and without aberration. With Eq. (1), it can be concluded that Strehl ratio is equal to the normalized volume under the optical transfer function (OTF) of the optical system, so optimization for the best coefficients of pupil phase mask can be done by maximizing the Strehl ratio stability over the depth of focus while making sure that the Strehl ratio is large enough. Here only the standard deviation (STD) method is used to evaluate the stability. Several discrete object distances, field positions are set to stand for the whole space over the depth of focus. With the coefficients a and b of the pupil phase mask which is of $Z = a(X^3 + Y^3) + b(X^2Y + XY^2)$ form, the metric function $\mathrm{MF}(a,b)$ can be defined as

$$\mathrm{MF}(a,b) = \mathrm{STD}(\{\mathrm{S.R.}_{ZF}\}) + \frac{W*(1-\mathrm{S.R.}_{\min})}{\exp(K*(\mathrm{S.R.}_{\min}-\mathrm{S.R.}_{\mathrm{th}}))+1}, \tag{2}$$

where the former term is the STD of the Strehl ratios at all sampled object distances and field positions, the later term is the additional penalty for low Strehl ratio, W and K are constants, $\mathrm{S.R.}_{ZF}$ stands for the Strehl ratio at the sampled object distance Z and sampled field position F, $\mathrm{S.R.}_{\mathrm{th}}$ and $\mathrm{S.R.}_{\min}$ stand for the threshold Strehl ratio and the minimal Strehl ratio.

Three object distances (50, 30, and 15 m) and four field positions (0°, 1°, 2°, and 3°) are set, and the polychromatic Strehl ratio is considered. The threshold Strehl

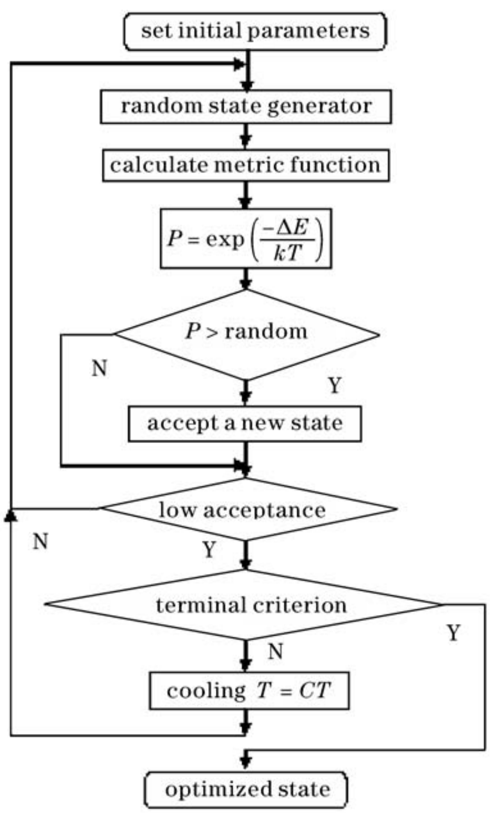

Fig. 3. Flow chart of simulated annealing method[5].

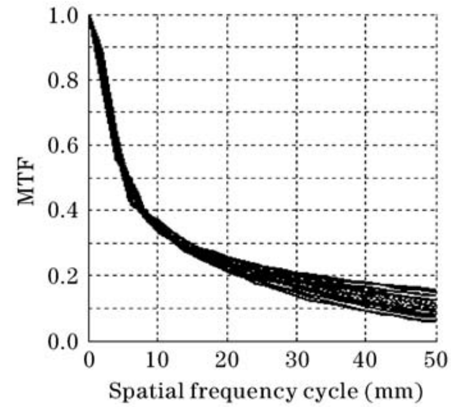

Fig. 4. Unfiltered MTFs of the system applied with the optimized pupil phase mask.

ratio is chosen to be 0.25%, W is 10, and K is 500. A simple simulated annealing method[5] with its flow chart shown in Fig. 3 is applied to search in the coefficients space $a \in [0, 1\times10^{-4}]$, $b \in [-1\times10^{-5}, 0]$ for the minimal value of the metric function. The optimized coefficients are: $a = 9.69543\times10^{-6}$, $b = 0$. A plot of the unfiltered sagittal and tangential polychromatic MTFs of the new system applied with this optimized pupil phase mask is shown in Fig. 4. It is obvious that there is no zero point within 50 cycle/mm for sagittal and tangential MTFs, and they are also much more stable over the depth of focus than the original system.

For the imaging simulation, the original image is first decomposed into 3 channels, i.e., the RGB channels. Each channel's Fourier spectrum is multiplied by its corresponding color's OTF, and an inverse Fourier transform is applied to form the intermediate image. To get the final image, multiply the intermediate Fourier spectrum with the filter spectrum, and take an inverse Fourier transform. The flow of the imaging simulation process for the wavefront coding system is shown in Fig. 5. The following assumptions are kept in the whole imaging simulation process. 1) No magnification, vignetting, and noise are taken into account. 2) A 128×128 spoke image is used as the original object image. 3) A CCD with a pixel size of 10 μm is assumed. 4) Only one filter, which is constructed from the mean OTF of object at 50, 30, and 20 m with their each field positions, is used over the depth of focus for a single channel, and the module is limited to be lower than 10.

The simulated images of the original cemented doublet system are shown in Fig. 6, and the restored images of the system applied with the optimized pupil phase mask are shown in Fig. 7. Only the simulated images with 4 object positions (infinity, 50 m, 30 m, 15 m), and 2 field

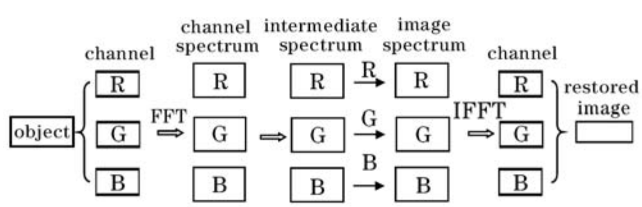

Fig. 5. Imaging simulation process for the wavefront coding system. FFT: fast Fourier transform; IFFT: inverse fast Fourier transform.

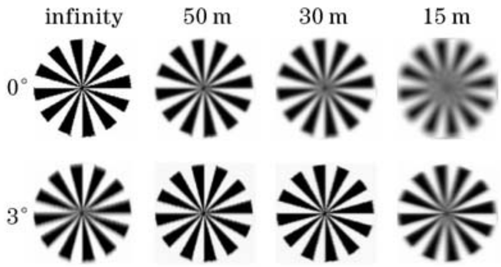

Fig. 6. Simulated images of the original cemented doublet system.

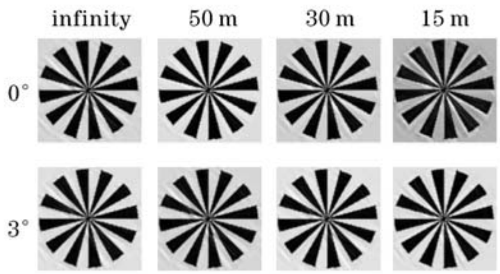

Fig. 7. Restored images of the system applied with optimized pupil phase mask.

positions (0° and 3°) are presented. One can conclude that the original cemented doublet system cannot work well with object at 30 m, or even at 50 m. Lots of information of the object space will be lost in the imaging process. Though there are few brightness fluctuations in the restored images, the wavefront coding system works well within the object at 15 m and sharp image is acquired. The imaging quality is much more stable, and much more details of the object can be distinguished, i.e., much more information of the object is acquired.

The modules of the filter with normalized spatial frequencies for RGB channels are shown in Fig. 8. The average gain of this filter is 5.54. One may notice that the sagittal and tangential modules are lower than those in diagonal areas. That is the characteristic of pupil phase mask which is of rectangularly separable form.

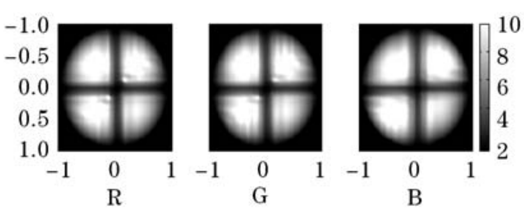

Fig. 8. Modules of RGB channels' filters for the wavefront coding system.

Table 1. Sharpness of the Final Images Acquired by Systems with and without Phase Mask

Distance		Infinity	50 m	30 m	15 m
0°	without	0.59074	0.21729	0.12967	0.04344
	with	0.35772	0.40736	0.3609	0.27977
3°	without	0.23814	0.37416	0.36338	0.14713
	with	0.36617	0.34333	0.38361	0.39523

A simple method with the sharpness of the image defined according to energy of image gradient function[6] can be taken to evaluate the imaging quality. That is

$$f(I) = \sum_X \sum_Y \{[I(x+1, y) - I(x, y)]^2 + [I(x, y+1) - I(x, y)]^2\}, \quad (3)$$

where $I(x, y)$ is the gray value at (x, y) of the image I. The ratios of final images' sharpness to sharpness of the original object image are listed in Table 1. It is obvious that the wavefront coding system has its advantage in sharpness with respect to the original cemented doublet system, and it is much more stable over the depth of focus.

In this paper, a simple Strehl ratio based method for pupil phase mask's optimization is proposed. It is convenient to apply this method to practical optical systems with only the standard deviation method taken into account, and still good imaging quality is acquired and the depth of focus is extended.

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