Spectrum analysis of all parameter noises in repetition-rate laser pulse train

Junhua Tang (汤君华) and Yuncai Wang (王云才)

Physics Department, College of Science, Taiyuan University of Technology, Taiyuan 030024

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The theoretical investigation of all parameter noises in repetition-rate laser pulse train was presented. The expression of power spectrum of laser pulse trains with all parameter noises was derived, and the power spectra of pulse trains with different noise parameters were numerically simulated. By comparing the power spectra with and without pulse-width jitter, we noted that pulse-width jitter could not be neglected compared with amplitude noise and timing jitter and contributed a great amount of noise into the power spectrum under the condition that the product of pulse width and angular frequency was larger than 1.

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High repetition-rate pulses with low jitter are very important for high-speed optical communication system. Noise characteristics of these pulse trains are crucial in many applications such as ultrafast optical telecommunication, detection and electric-optical sampling system etc.. The noise of an optical pulse train can be categorized into three basic types: amplitude noise, phase noise, and pulse-width jitter. The amplitude noise is the fluctuation of pulse intensity; the phase noise (timing jitter) is the random variation of pulse repetition time, and the pulse-width jitter is the fluctuation of pulse duration. The amplitude and pulse-width fluctuations can reduce detecting accuracy in optical probe experiments and the timing jitter can degrade time resolution in sampling system. It is necessary to evaluate all kinds of jitter of output pulses. One of the techniques to quantify the noise in these pulse trains is spectral measurement, first introduced by Linde[1]. Several research groups have used this technique to measure and analyze the noise of different lasers[2−4]. And other techniques are also known to analyze the noise of optical pulses, such as time-domain demodulation[5], cross-correlation[6], phase noise measurement[7], and indirect phase comparison method[8]. Besides, the theoretical analysis of laser noise was also reported[9]. However, the measurement and analysis of laser noise were limited in low frequency. Previous studies only considered amplitude and phase noise, not including pulse-width fluctuation. In most diode lasers, however, the pulse-width jitter cannot be neglected. In this paper, we derived the pulse power spectrum with all noise (including amplitude noise, timing jitter, and pulse-width jitter) in theory, analyzed qualitatively the contribution of the pulse-width jitter to the noise of pulse trains.

Considering a constant pulse shape model, including not only amplitude noise and timing jitter, but also pulse-width jitter, the intensity of this period pulse trains can be represented as[10]

$$I(t) = \sum_{n=-N}^{N} (I + I_n) h\left(\frac{t - nT + T_n}{\tau + \tau_n}\right), \tag{1}$$

where $h(t)$ is the normalized dimensionless intensity shape function of the pulse, I the average amplitude, I_n the amplitude fluctuation, T the average repetition period of the pulse, T_n the pulse timing jitter, τ the average full-width at half-maximum (FWHM) pulse-width, τ_n the pulse-width jitter. The power spectrum of pulse intensity $I(t)$ can be described as[11]

$$S_I(\omega) = \lim_{N\to\infty} \frac{\langle |I(\omega)|^2 \rangle}{(2N+1)T}, \tag{2}$$

where $I(\omega)$ is the Fourier transform of the pulse intensity $I(t)$. By the Fourier transform shift and similarity theorems,

$$I(\omega) = \sum_{n=-N}^{N} (I + I_n)(\tau + \tau_n) \times H[(\tau + \tau_n)\omega] \cdot \exp[-i\omega(nT - T_n)], \tag{3}$$

where $H(\omega)$ is the Fourier transform of the pulse intensity shape $h(t)$ and $(\tau + \tau_n)H[(\tau + \tau_n)\omega] = \tau H(\tau\omega)(1 + \frac{V(\omega)}{\tau}\tau_n)$[10], $V(\omega)$ is a function depending on pulse shape. Supposing amplitude noise, timing jitter, and pulse-width jitter are all uncorrected, then

$$\langle |I(\omega)|^2 \rangle = |I|^2 |\tau H(\tau\omega)|^2 (Y_1 + Y_2), \tag{4}$$

where

$$Y_1 = (2N+1)[1 + \frac{\langle I_n^2 \rangle}{I^2} + \frac{V^2(\omega)}{\tau^2} \langle \tau_n^2 \rangle], \tag{5}$$

$$Y_2 = \sum_{n=-N}^{N} \sum_{m=-N}^{N} \{[1 + \frac{\langle I_n \rangle^2}{I^2} + \frac{V^2(\omega)}{\tau^2} \langle \tau_n \rangle^2] \times \langle \exp[i\omega(T_n - T_m)] \rangle \cdot \exp[-i\omega(n-m)T]\}. \tag{6}$$

Supposing the different pulses are statistically independent, substituting Eqs. (4), (5) and (6) into Eq. (2), and divided by $(2N+1)T$, then

$$S_I(\omega) = \frac{I^2|\tau H(\tau\omega)|}{T}\{\sigma_I^2 + V^2(\omega)\sigma_\tau^2 + \sigma_T^2\omega^2 + \frac{1}{4}\sigma_T^4\omega^4 + (1-\sigma_T^2\omega^2 + \frac{1}{4}\sigma_T^4\omega^4)\omega_1 \sum_{k=-\infty}^{+\infty}\delta(\omega-\omega_k)\}, \quad (7)$$

where $\sigma_I^2 = \frac{\langle I_n^2\rangle - \langle I_n\rangle^2}{I^2}$, $\sigma_T^2 = \langle T_n^2\rangle - \langle T_n\rangle^2$, $\sigma_\tau^2 = \langle \tau_n^2\rangle - \langle \tau_n\rangle^2$, σ_I, σ_T, and σ_τ are root-mean-square (RMS) values of amplitude noise I_n, timing jitter T_n, and pulse-width jitter τ_n, respectively. Equation (7) was deduced under the assumption that all noise contents are small.

Supposing pulse shape is Gaussian, then $V^2(\omega) = 1-(\tau\omega)^2 + \frac{1}{4}(\tau\omega)^4$. Compared with $\frac{1}{4}\sigma_\tau^2\tau^2\omega^4$, $\frac{1}{4}\sigma_T^4\omega^4$ can be neglected at high frequency; thus

$$S_I(\omega) = \frac{I^2|\tau H(\tau\omega)|}{T}\{[\sigma_I^2 + \sigma_T^2\omega^2 + (1-\tau^2\omega^2 + \frac{1}{4}\tau^4\omega^4)\frac{\sigma_\tau^2}{\tau^2}] + (1-\sigma_T^2\omega^2)\omega_1 \sum_{k=-\infty}^{+\infty}\delta(\omega-\omega_k)\}, \quad (8)$$

and the noise power can be expressed as

$$\sigma_n^2 = \sigma_I^2 + \sigma_T^2\omega^2 + (1-\tau^2\omega^2 + \frac{1}{4}\tau^4\omega^4)\frac{\sigma_\tau^2}{\tau^2}, \quad (9)$$

where σ_n^2, σ_I^2, σ_T^2, and σ_τ^2 correspond to the RMS of total noise, amplitude noise, timing jitter, and pulse-width jitter, respectively. It is noted that the timing jitter is proportional to ω^2, and the pulse-width jitter is proportional to ω^2 and ω^4. Equation (8) is just the derived mathematic expression.

If the pulse-width jitter is not taken into consideration, i.e., $\sigma_\tau = 0$, then the power spectrum of pulse intensity is represented as

$$S_I(\omega) = \frac{I^2|H(\omega)|}{T}\{[\sigma_I^2 + \sigma_T^2\omega^2] + (1-\sigma_T^2\omega^2)\omega_1 \sum_{k=-\infty}^{+\infty}\delta(\omega-\omega_k)\}. \quad (10)$$

Comparing Eqs. (8) and (10), we note that the pulse-width jitter contributes its power especially in $(\tau\omega)^4$. Thus the pulse-width jitter will contribute a great amount of noise to the pulse power spectrum when $\tau\omega > 1$, where τ is the pulse width and ω is the angular frequency of the harmonic of the pulse repetition rate.

To illustrate the above results, we numerically simulated power spectrum of pulse jitter according to Eq. (8) setting $\sigma_I = 0.05$, $\sigma_T = 1.1$ ps, $\sigma_\tau = 1.5$ ps, repetition frequency $f = 10$ GHz, average pulse width $\tau = 10$ ps. To illuminate the contribution of pulse-width jitter to noise term, power spectra of pulse trains with (1) only amplitude noise, (2) amplitude noise and timing jitter, (3) all noises (including amplitude noise, timing jitter, and pulse-width jitter) were numerically simulated respectively. Figure 1 shows the spectral power. As expected, repetition frequency occurs at harmonic of 10 GHz. Figure 1(a) shows the power spectrum with only amplitude noise. The noise is a constant in all the spectrum ranges, except that the power falls off in high frequency, which is limited by the bandwidth $|H(\omega)|^2$. Figure 1(b) shows the power spectrum with the amplitude noise and timing jitter, the jitter continuum increases with ω. Under the same condition, Fig. 1(c) shows the power spectrum with all noises. Figure 1(d) presents the third-order power spectrum of pulse trains.

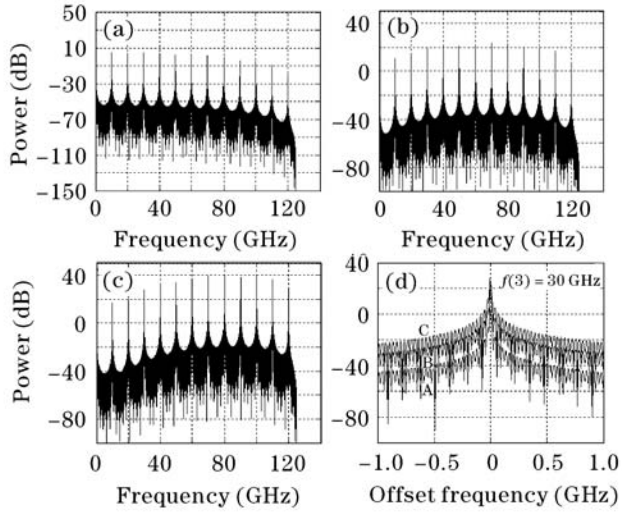

Fig. 1. Numerically simulated power spectra of 10-GHz pulse train ($\tau = 10$ ps) with amplitude noise (a), amplitude noise and timing jitter (b), all noise terms (c). (d) The third-order power spectrum curves from bottom to top are A, B, C, corresponding to spectra with only amplitude noise, amplitude noise and timing jitter, and all noises, respectively.

Comparing Figs. 1(b) and (c), the noise power in Fig. 1(c) is greater than that in Fig. 1(b) at about 20 GHz where $\tau\omega \approx 1.26$, and this becomes markedly at 30 GHz, where $\tau\omega \approx 1.88$. The noise power is consistent well with Eq. (8). Curves in Fig. 1(d) correspond to the third-order ($f_3 = 30$ GHz) power spectrum of the pulse trains with (1) only amplitude noise (A), (2) amplitude noise and timing jitter (B), (3) all noise (amplitude noise, timing jitter, and pulse-width jitter) (C). As shown in Fig. 1(d), the noise powers in curves A, B, and C are about −45, −30, and −23 dB at 29 GHz, respectively. The noise power increases from −30 (not including pulse-width jitter) to −23 dB (considering pulse-width jitter), the difference is approximately 7 dB. It indicates that when $\tau\omega > 1$, the pulse-width jitter contributes a significant amount of noise to the power spectrum. At high frequency (> 80 GHz), pulse noise falls off because of the limited bandwidth $|H(\tau\omega)|^2$.

Furthermore, consider that pulse-width jitter and the pulse noise are related to pulse width. While $\tau = 5$ ps, the corresponding power spectra without and with pulse-width jitter are shown in Figs. 2(a) and (b), respectively. We observed that the power spectrum in Fig. 2(a) is the same as that in Fig. 1(b), that is to say, pulse width has nothing to do with pulse power spectrum without

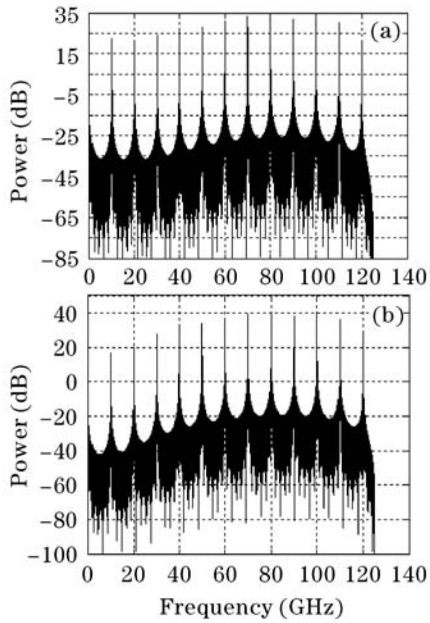

Fig. 2. Numerically simulated pulse power spectra of 10-GHz pulse train ($\tau = 5$ ps) with only amplitude and timing jitter (a) and all noise terms (b).

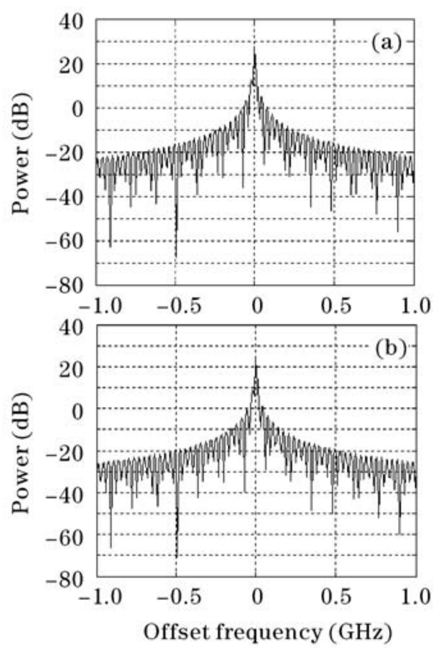

Fig. 3. Third order ($f_3 = 30$ GHz) power spectrum of pulse of different pulse width with all noises. (a) $\tau = 10$ ps; (b) $\tau = 5$ ps.

pulse-width jitter.

Also, for the same angular frequency ω (or same repetition frequency f), the noise power in Fig. 2(b) ($\tau = 5$ ps) is smaller than that in Fig. 1(c) ($\tau = 10$ ps). Larger τ obtains larger $\tau\omega$ which results in larger contribution of pulse-width jitter to noise power. This can be indicated by comparing the noise power in Figs. 3(a) and (b), which corresponds to the third-order power spectrum of pulse trains. The noise power reaches −23 dB in Fig. 3(a) and −26 dB in Fig. 3(b) at 29 GHz, respectively. The difference is about 3 dB.

If one could acquire the total noise power σ_n^2 that corresponds to three different values of n, where n corresponds to the harmonic number of power spectrum; then σ_I^2, σ_T^2, and σ_τ^2 can be determined by solution of the set of three equations such as Eq. (9).

In conclusion, based on the definition of power spectrum we have derived the mathematic expression of harmonic number dependence of pulse power spectrum. We have demonstrated the pulse power spectrum with noise (including amplitude fluctuation, timing jitter, and pulse-width jitter) as a function of ω. By comparing the pulse power spectra with (1) only amplitude noise, (2) amplitude noise and timing jitter, (3) all noises (amplitude noise, timing jitter, and pulse-width jitter), we qualitatively analyzed the contribution of every noise term to the total noise power. Especially, by comparing the noise power with and without pulse-width jitter, we indicated that the pulse-width jitter, compared with amplitude fluctuation and timing jitter, could not be neglected and contributed a great amount of noise into the power spectrum when $\tau\omega > 1$. The requirements of all noise parameters determination are also proposed in this paper.

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