Kalman filtering techniques for reducing variance of digital speckle displacement measurement noise

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Target dynamics are assumed to be known in measuring digital speckle displacement. Use is made of a simple measurement equation, where measurement noise represents the effect of disturbances introduced in measurement process. From these assumptions, Kalman filter can be designed to reduce variance of measurement noise. An optical and analysis system was set up, by which object motion with constant displacement and constant velocity is experimented with to verify validity of Kalman filtering techniques for reduction of measurement noise variance.

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Speckle will be observed when a coherent beam is reflected by a diffuse object whose roughness is in a scale of the wavelength. Laser speckle metrology[1] has been developed for many years. Its applications include experimental mechanics, machine control, velocimetry, electronic element package, and so on. Laser speckle metrology has many merits, such as high accuracy, non-contact, full field, and real time. A variety of methods have been proposed to figure out speckle displacement, such as digital image correlation (DIC)[2], digital speckle correlation method (DSCM)[3,4], digital speckle displacement measurement by fuzzy logic[5], digital speckle displacement measurement by thresholding method[6], level-crossing technique[7], laser doppler velocimeters[8], digital image/speckle correlation (DISC)[9], and so on.

These methods, however, do not consider target dynamics in measurement process. They only use speckle images obtained before and after displacement. Target dynamics are assumed to be known in this paper. Target dynamics equation and measurement equation together model a speckle displacement measuring problem. Kalman filter[10] can be designed from the model. From Kalman filtering theory, variance of state filtering estimation error will be less than that of measurement noise. In a linear-Gaussian environment, Kalman filter is optimal. The optimality means that conditional mean square error of its state filtering estimate is minimum one in all estimators. In an arbitrary statistics environment, Kalman filter is the best linear estimator. In addition to optimality, other merits of Kalman filter, such as adaptability, recursive computation etc., make Kalman filter find extensive applications. Park *et al.*[11] proposed a robust discrete time Kalman filter for the dynamic compensation of nonlinearity in a homodyne laser interferometer for high-precision displacement measurement and in real time. Some error factors were considered, and the Kalman filter was designed on the basis of the characteristics of the system modeling errors and sensor noises. This paper makes use of Kalman filter for reduction of variance of measurement noise in a non-interferometric environment.

It is hoped that speckle displacement measurement can be more accurate. However, noises will be inevitably introduced in the imaging and numerical computation processes. There are many approaches to reduce noise. For example, frame averaging is a simple and effective way. The wavelet-packet noise-reduction process could also be employed in improved DSCM[12]. A low-pass morphological filter could be incorporated into the thresholding method to increase the ratio of signal to noise for measuring speckle displacement[6].

Rather than image processing, Kalman filtering technique to suppress measurement noise is employed. To this end, target dynamics are assumed to be known and modeled as an autoregressive process of order 1 (AR(1)) process. Speckle displacement is measured by DISC[9]. The algorithm of DISC is described as follows.

We have the relations

$$\begin{cases} I_1(r) = I_2(r+U_0) \\ I_1(r-U_0) = I_2(r) \end{cases}, \tag{1}$$

where r denotes pixel location, $I_1(r)$ is the speckle image obtained before displacement, $I_2(r)$ is the speckle image obtained after displacement, U_0 denotes speckle displacement. After Taylor expansion of these two equations and neglecting high order components, we get

$$\begin{cases} I_1(r) = I_2(r) + \nabla I_2(r)\cdot U_0 \\ I_2(r) = I_1(r) - \nabla I_1(r)\cdot U_0 \end{cases}, \tag{2}$$

where $\nabla I_1(r)$ and $\nabla I_2(r)$ are spatial gradients of $I_1(r)$ and $I_2(r)$ respectively. Equations (2) can be rearranged as

$$2[I_2(r) - I_1(r)] = [\nabla I_1(r) + \nabla I_2(r)]\cdot U_0. \tag{3}$$

Equation (3) holds for $M \times N$ pixels in a subimage Ω around r and leads to $M \times N$ equations. The least square approximate solution to these equations is then determined by

$$A^{\mathrm{T}} A U_0 = A^{\mathrm{T}} b, \tag{4}$$

where

$$A = \begin{pmatrix} \vdots \\ \nabla I_2(r) + \nabla I_1(r) \\ \vdots \end{pmatrix}_{M.N\times 2\in\Omega},$$

and

$$b = \begin{pmatrix} \vdots \\ 2[I_1(r) - I_2(r)] \\ \vdots \end{pmatrix}_{M.N\times 1\in\Omega}. \quad (5)$$

In DISC's computation process, components with high order are neglected to obtain its linear equations. DISC requires estimating gradients of pixels, which will also introduce noise due to the discretized and contaminated image. These noises and others such as from temperature instability could be represented as a random variable, i.e., the measurement noise in the measurement equation

$$z_k = x_k + v_k, \quad (6)$$

where v_k is measurement noise, z_k is DISC's measurement, and x_k is the target displacement. In the measurement equation, v_k is assumed to be white. v_k, target position $x_{1:k}$, and target model error are assumed to be mutually independent. These assumptions approach to reality and make Kalman filter designed from the target model, and the measurement equation is the optimal estimator in linear-Gaussian situation and the best linear estimator for arbitrary statistics. Variance of measurement noise v_k will be reduced.

Layout of the measuring system is shown in Fig. 1. Laser speckle images are fetched by a laser digital speckle imaging system. These images are then used by DISC algorithm to compute speckle's displacement. In the end, these speckle displacement measurements are filtered by a Kalman filter.

Two target motion models are taken as examples to show reduction of variance of measurement noise by Kalman filtering. They are constant displacement model and constant velocity model. Because it is assumed that each position coordinate is measured independently, each coordinate is uncoupled from others and can be treated separately. Thus, for each coordinate, the motion of object and the relationship between measurements and states are assumed to be described as follow:

1) constant displacement

$$\begin{cases} x_k = x_{k-1} + w_k \\ z_k = x_k + v_k \end{cases}, \quad (7)$$

where x_k and z_k are target displacement and its displacement measurement at the kth instant. Process noise w_k and measurement noise v_k are assumed to be Gaussian and satisfy $E[v_k w_j] = 0$, $E[w_k w_j] = \sigma_w^2 \delta(k-j)$, $E[v_k v_j] = \sigma_v^2 \delta(k-j)$, $E[w_k] = E[v_k] = 0$; $\delta(k-j)$ is Kronecker delta. The initial state x_0 is also assumed to be independent of Gaussian distribution.

Equation (7) models an object with constant displacement perturbed by noise. When Kalman filter gets to steady state, from discrete algebraic Racatti equation (DARE), we can get

$$P^2 + \sigma_w^2(P - \sigma_v^2) = 0, \quad (8)$$

where P is the variance of x_k's steady-state filtering estimation error. Solution of Eq. (8) is

$$Y = \frac{P}{\sigma_v^2} = \frac{2}{\sqrt{1+4r^2}+1}, \quad (9)$$

where $r = \frac{\sigma_v}{\sigma_w}$. Evidently it must be satisfied that $\sigma_w \neq 0$, $P < \sigma_v^2$. Figure 2 shows the relationship between Y and r.

2) constant velocity[13]

$$\begin{cases} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}_{k-1} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} a_k \\ z_k = x_k + v_k \end{cases}, \quad (10)$$

where x is the position, $\dot{x}$ the velocity, a_k the random acceleration, T the sampling interval, z the position measurement, v the measurement noise, k the kth sampling instant. The motion Eq. (10) models an object with random acceleration perturbing its motion from a straight line.

By steady state analysis[13], the variance of position's steady-state filtering estimation error is always less than that of position measurement noise.

The experimental setup is schematically shown in Fig. 3. A laser with 780-nm wavelength and 10-mW output power was used to illuminate the object. The beam was collimated by a lens installed in the laser head. The average size of speckle was about 30 μm. For objects, we employed the solid surface of paper sheet. They were mounted on an X-Y stage and were subjected to translation with stepping motors controlled by a microcomputer which has a 0.1-μm resolution. The speckle image was produced through a charge couple device (CCD) camera which has 512×512 pixels, which pixel size is 7.5×7.5 (μm), and fill factor is 100%. A V512 image grabber was used in the system.

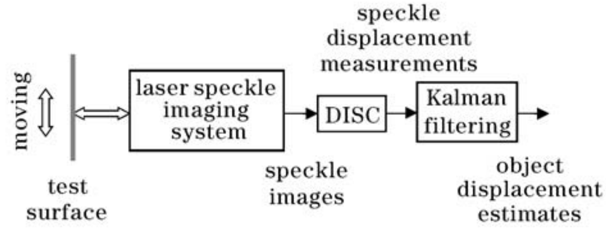

Fig. 1. Layout of the measuring system.

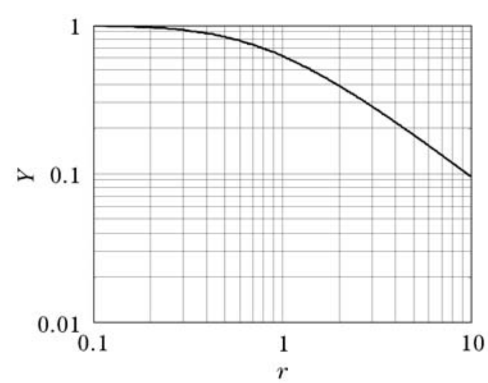

Fig. 2. Relationship between Y and r.

Table 1. Standard Deviation of Measurement Noise σ_n and Estimation Error σ_e in Y Direction (μm(pixels))

Speckle Displacement	0.900 (0.120)	1.800 (0.240)	2.700 (0.360)	3.600 (0.480)	4.500 (0.600)	5.400 (0.720)	6.300 (0.840)	7.200 (0.960)	8.100 (1.080)	Average
σ_n	0.0683	0.063	0.0945	0.1073	0.1305	0.1478	0.1223	0.0953	0.1133	0.105
	(0.0091)	(0.0084)	(0.0126)	(0.0143)	(0.0174)	(0.0197)	(0.0163)	(0.0127)	(0.0151)	(0.014)
σ_e	0.0225	0.0375	0.0278	0.0488	0.0758	0.0908	0.0923	0.0638	0.0503	0.0563
	(0.0030)	(0.0050)	(0.0037)	(0.0065)	(0.0101)	(0.0121)	(0.0123)	(0.0085)	(0.0067)	(0.0075)

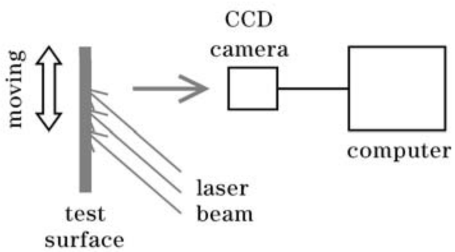

Fig. 3. Schematic diagram of the experimental setup.

The DISC is computed in spatial-domain[9]. Interested area size is 20×20 pixels. The gradient is estimated by Prewitt operators.

Firstly, object motion with constant displacement was conducted in experiment. A steady-state Kalman filter[14] was constructed by two-dimensional (2D) equations expanded by Eq. (7). The experimental results on standard deviation of errors measuring a variety of distance of speckle displacements with and without Kalman filtering are listed in Tables 1 (similar results were obtained in X direction). The sample standard deviation is defined as

$$\text{std} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}, \quad (11)$$

where x_i $(i = 1, 2, \cdots, N)$ are samples, N being the number of samples, and $\bar{x}$ denoting sample mean.

In Table 1, row 1 is a variety of sub pixel speckle displacement measured in experiment. Rows 2 and 3 are standard deviations of error with and without Kalman filtering measuring the speckle displacement in cell with the same column in row 1.

The Kalman filter's state was initialized to be the first measurement[15] and variance of measurement noise was estimated by the sample variance. The variance of process noise was assumed to be 0.0001 pixels^2.

These experimental results indicate that standard deviation of Kalman filtering estimation errors is about half of that of original measurement noise on average, which verified that the variance of measurement noise could be much reduced by using Kalman filtering.

As shown in Fig. 4, difference exists between theoretical analysis results and experiment results (also see in Table 1). This difference is caused by two factors: One is that the assumed variance of process noise is not the actual one. The other is that distributions of process noise and measurement noise are only approximately Gaussian due to, such as, not enough sampling, and therefore the Kalman filtering estimation is sub-optimal.

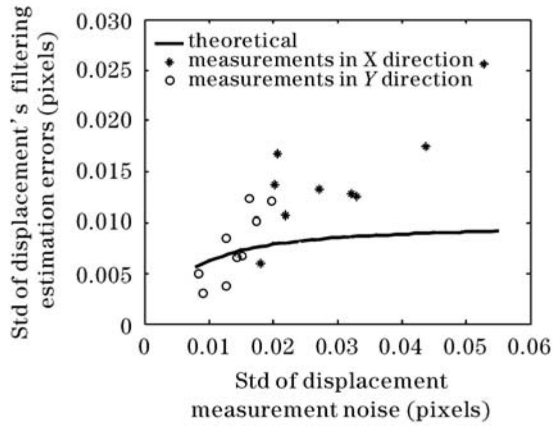

Fig. 4. Relationship between std of measurement noise and estimation errors, std of process noise is constant and equals to 0.01 pixels.

Additionally, object motion with constant velocity was also conducted in the experiment. Object moved in the direction of 45°. We used a steady-state Kalman filter[14] constructed by 2D equations expanded by Eq. (10) to estimate the position of the object.

The covariance matrixes of process and measurement noise were assumed to be: $Q = \text{Diag}(0, 1, 0, 1) \times 10^{-4}$ pixel/T^2, $R = \text{Diag}(9, 1.96) \times 10^{-4}$ pixel^2 (element values of R are taken according to averaged variance of measurement noise in X direction and Y direction obtained from the former experiment results as shown in Table 1); T is sampling interval. The filter is initialized on the basis of the first displacement measurements[15], and velocity is initialized to be 0.99 pixels/T.

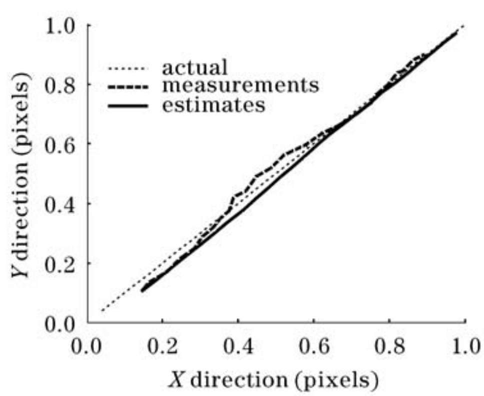

Fig. 5. Trajectories of actual object motion, unbiased measurements and estimates. std of displacement measurement noise = (0.0609 0.0452) pixels in X and Y directions. std of displacement's filtering estimation errors = (0.0415 0.0288) pixels in X and Y directions.

Figure 5 shows trajectory of object motion, as well as trajectories of measurements and estimations of object motion positions. Standard deviations of original displacement measurement noise were 0.0609 pixels in X direction and 0.0452 pixels in Y direction. Standard deviations of displacement's filtering estimation errors were reduced to 0.0415 pixels and 0.0288 pixels respectively. Thus, the estimated position tracks object trajectory more accurately than original measurements.

In conclusion, Kalman filtering technique is proposed to be integrated into the mechanism of measuring digital speckle displacement for the purpose of reducing the variance of measurement noise. Modeling is crucial in application of Kalman filter. Target dynamics are assumed to be known. A simple relationship between measurement and displacement state is used to form a measurement equation. In the measurement equation, measurement noise represents the effect of disturbances introduced in measuring process. These assumptions are available due to their approaching to reality. Experiment results verified that variance of filtering estimation error could be much reduced (about half of) compared with that of original measurement noise, which is basically consistent with Kalman filtering theory for object tracking. An improved-precision estimation of object trajectory with constant velocity perturbed by random acceleration could be achieved by the proposed scheme.

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