Parameter analysis of a photonic crystal fiber with raised-core index profile based on effective index method

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Received May 9, 2006

Photonic crystal fibers (PCFs) with a stepped raised-core profile and one layer equally spaced holes in the cladding are analyzed. Using effective index method and considering a raised step refractive index difference between the index of the core and the effective index of the cladding, we improve the characteristic parameters such as numerical aperture and V-parameter, and reduce its bending loss to about one tenth of a conventional PCF. Implementing such a structure in PCFs may be one step forward to achieve low loss PCFs for communication applications.

OCIS codes: 060.2270, 060.2280, 060.2310, 060.2330.

Photonic crystal fibers (PCFs) are single silica materials with holes located in their cladding running along the length of the fiber[1−5] and are usually fabricated by tube-in-tube method[6]. Holes in the cladding give lower refractive index, allowing guidance by total internal reflection. If the core and cladding glasses are similar, the wavelength will depend on cladding index, giving numerical aperture upper bound and allowing the fiber to act as endless single-moded[7]. Conventional PCFs (CPCFs) with high losses, despite of the properties of negative dispersion and large numerical aperture (NA), are not suitable for most of loss-limited applications. The main reason of such a high loss in these fibers is due to lack of physical boundary between their cores and claddings, which causes the propagating light get trapped in the space among the holes in the cladding, creating confinement losses along these fibers[8]. The smaller the holes, more will be the void space, and thus more will be such a loss which appears in the forms of bending and connection losses.

There are different methods to solve this problem. One is fabrication of a PCF with bigger holes in the first row and smaller ones in next rows, as shown in Fig. 1(a)[9]. Although this method to some extent reduces the loss, but still with the presence of void spaces in the silica of the cladding region, the light gets trapped and thus loss may increase, too. Another is fabrication of PCF with high d/Λ, where d is the hole diameter and Λ is the hole spacing, allowing practically an air boundary between the core and the cladding, which can reduce the loss level as the bending loss of the PCF directly depends on Λ^3, as shown in Fig. 1(b)[7,10]. Since increase of d/Λ to 0.8 or more may disturb the single-mode operation of the PCF and might not reduce the loss considerably, the later method will not help so much.

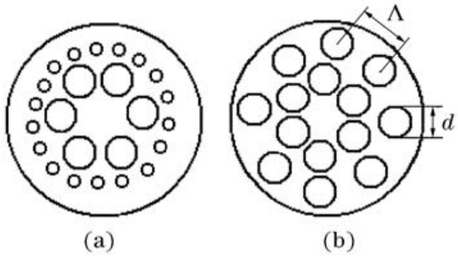

Fig. 1. PCFs with bigger holes in the first row (a) and with large d/Λ value (b).

Hasegawa *et al.*[11,12] have designed and fabricated a similar PCF that had a low loss of 0.41 dB/km. Their analysis was based on full-vector finite element method. Saitoh *et al.*[13] reported a bending-insensitive single-mode PCF of doped core and with two layers of holes of different diameters.

A PCF with a step raised-core (RCPCF) profile[14], and one layer of holes in the cladding is proposed in this paper. The core of RCPCF has an index slightly higher than that of the cladding silica, similar to conventional depressed clad single mode (DCSM) fiber used in fiber communications[15]. The aim of this work is to provide more confinement of the propagating mode within the core region, similar to DCSM fibers, and observe the resulting profile effects on the numerical aperture and the V-parameter. The PCF with an effective step index profile can help light guidance by total internal reflection and beside photonic crystal effect, thereby reduces the trapping of propagating light in the cladding regions, resulting in less confinement losses, as reported in Ref. [8].

The PCF geometry considered is shown in Fig. 2. Due to presence of holes in its cladding region, its equivalent refractive index would be analogous to DCSM fibers. To analyze the proposed profile, mathematically, the

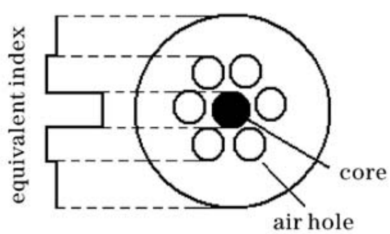

Fig. 2. PCF cross-sectional view.

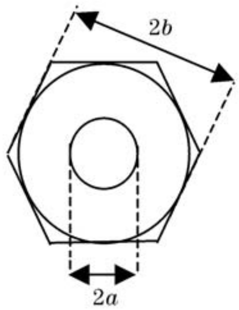

Fig. 3. Hexagonal unit cell in the cladding with air hole of radius a and silica radius of b.

effective index method is used, also for CPCF[16,17]. In this method, the cladding mode field is obtained by solving the scalar wave equation within a cell enclosing an air hole of radius a. For a circular symmetric mode solution, let the hexagonal unit cells in cladding region of the PCF be approximated with a circle of radius b, as shown in Fig. 3.

In a PCF, the modes propagate in the longitudinal directions and are infinitely extended in the transverse direction, often called as space filling modes (SFMs). The effective cladding index of the fundamental SFM is computed by solving the scalar wave equation within the aforesaid unit cell. Therefore, the scalar wave equation in the air hole and silica region is defined as[16,17]

$$\psi = \begin{cases} AI_0(WR) & \text{for air hole} \\ BJ_0(RU) + CY_0(UR) & \text{for silica region} \end{cases}, \tag{1}$$

where $R = r/a$, r being the cylindrical parameter of the fiber. By applying boundary conditions at the edges, the eigenvalue equation is obtained as

$$BJ_1(U) + CY_1(U) = 0, \tag{2}$$

where constants A, B, and C are determined by Bessel functions I_0, I_1, J_0, J_1, Y_0, and Y_1 as

$$B = \frac{A}{J_0(U)}\left[I_0(W) - \frac{WI_1(W)J_0(U) + UJ_1(U)I_0(W)}{U\{J_1(U)Y_0(U) - J_0(U)Y_1(U)\}}\right], \tag{3}$$

$$C = \frac{A[WI_1(W)J_0(U) + UJ_1(U)I_0(W)]}{U[J_1(U)Y_0(U) - J_0(U)Y_1(U)]}. \tag{4}$$

The following parameters are used

$$\begin{cases} U = k_0 a\sqrt{n_\mathrm{s}^2 - n_\mathrm{cl}^2} \\ W = k_0 a\sqrt{n_\mathrm{cl}^2 - n_\mathrm{a}^2} \\ u = k_0 b\sqrt{n_\mathrm{s}^2 - n_\mathrm{cl}^2} \end{cases}, \tag{5}$$

where n_s, n_a, and n_cl are the refractive indices of pure silica, the air, and the cladding respectively, and the outer radius of the circle (in Fig. 3) $b = \Lambda(\sqrt{3}/2\pi)^{0.5}$, is determined by equating filling fraction of hexagonal unit cell with its circular approximation. Λ is the holes spacing and $k_0 = 2\pi/\lambda_0$, where λ_0 is the free space wavelength. The effective index is obtained by replacing cladding index with the index difference.

For our proposed RCPCF structure in Fig. 2, we assume $\Lambda = 2.3$, $d/\Lambda = 0.3$[16], and the index of the core (n'_si) is taken to be 0.005 times higher than the effective silica index of cladding (n_cl) to provide a raised core. A similar solution based on Eq. (1) for fundamental mode of equivalent effective step index profile of RCPCF can be written as

$$\psi = \begin{cases} AJ_0(U_\mathrm{eff}R) & R < 1 \\ BK_0(W_\mathrm{eff}R) & R > 1 \end{cases}, \tag{6}$$

where A and B are the corresponding constants given in Eqs. (3) and (4). The corresponding waveguide parameters for the equivalent step index profile of RCPCF are

$$\begin{cases} U_\mathrm{eff} = k_0\rho\sqrt{n'^2_\mathrm{si} - n^2_\mathrm{eff}} \\ W_\mathrm{eff} = k_0\rho\sqrt{n^2_\mathrm{eff} - n^2_\mathrm{cl}} \\ V_\mathrm{eff} = k_0\rho\sqrt{n'^2_\mathrm{si} - n^2_\mathrm{cl}} \\ \mathrm{NA} = \sqrt{n'^2_\mathrm{si} - n^2_\mathrm{cl}} \end{cases}. \tag{7}$$

With a similar simulation as in CPCFs[16], for the RCPCF, we first determine n_cl and then find out the total effective index (n_eff) of the fiber. Then with the known value of the core index n'_si and by assuming core radius of $\rho = 0.64\Lambda$ as given in Ref. [16], we determine the corresponding NA and effective V-parameter (V_eff) for the RCPCF.

Figure 4 illustrates the curves of different effective indices such as that of core, cladding, and pure silica with respect to wavelength.

On the basis of data collected from the curves of effective cladding index and total effective index of the

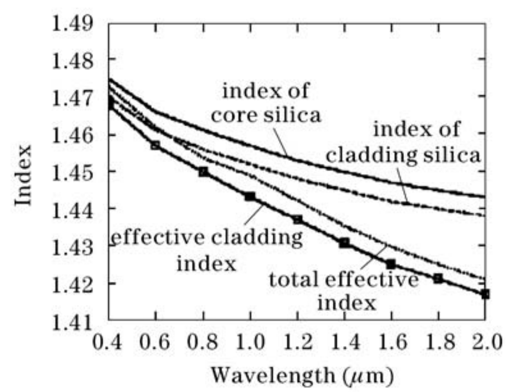

Fig. 4. Refractive indices of the RCPCF versus wavelength.

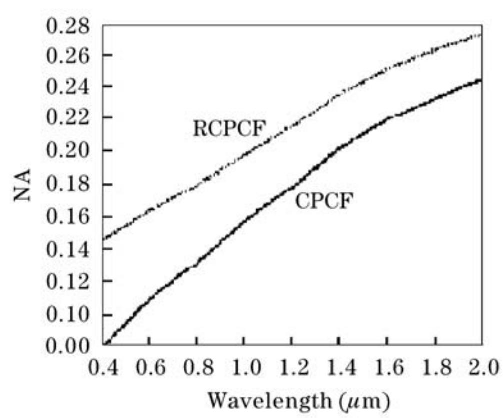

Fig. 5. Comparison of NA of the proposed RCPCF with that of a CPCF.

fiber in Fig. 4, we have plotted the NA variation of RCPCF in terms of wavelengths and compared with the NA of a CPCF having identical parameter values, as shown in Fig. 5. The NA curve of RCPCF shows a higher value than that of the CPCF with an almost same slope at wavelengths higher than 0.8 μm. In the calculated range of wavelengths, covering ultraviolet to far infrared spectra, the refractive index difference between the core and the cladding increases, the NA values will further increase.

To show the profile improvement of the RCPCF in term of its V-parameter, we shall compare it with a CPCF. From Eq. (7), the corresponding expressions for RCPCF and CPCF are rewritten as

$$V_{\mathrm{RCPCF}} = k_0 \rho_{\mathrm{RCPCF}} \sqrt{n_{\mathrm{si}}'^2 - n_{\mathrm{cl}}^2}, \tag{8}$$

$$V_{\mathrm{CPCF}} = k_0 \rho_{\mathrm{CPCF}} \sqrt{n_{\mathrm{si}}^2 - n_{\mathrm{cl}}^2}. \tag{9}$$

Dividing Eq. (8) by Eq. (9), assuming $\rho_{\mathrm{RCPCF}} = \rho_{\mathrm{CPCF}}$ and equal effective cladding indices in both the cases, we obtain

$$\frac{V_{\mathrm{RCPCF}}}{V_{\mathrm{CPCF}}} = \frac{\lambda_{\mathrm{CPCF}}}{\lambda_{\mathrm{RCPCF}}} \left(\frac{n_{\mathrm{si}}'^2 - n_{\mathrm{cl}}^2}{n_{\mathrm{si}}^2 - n_{\mathrm{cl}}^2} \right)^{1/2}, \tag{10}$$

where λ_{RCPCF} and λ_{CPCF} are operating wavelengths of RCPCF and CPCF, respectively. Since the difference between n_{si}' and n_{si} is pratically negligible, Eq. (10) can be approximated as

$$\frac{V_{\mathrm{RCPCF}}}{V_{\mathrm{CPCF}}} \approx \frac{\lambda_{\mathrm{CPCF}}}{\lambda_{\mathrm{RCPCF}}} (\delta)^{1/2}, \tag{11}$$

where $\delta = \Delta_{\mathrm{si}}' / \Delta_{\mathrm{si}}$ is a constant at a given wavelength, and $\Delta_{\mathrm{si}}' = n_{\mathrm{si}}' - n_{\mathrm{cl}}$, $\Delta_{\mathrm{si}} = n_{\mathrm{si}} - n_{\mathrm{cl}}$ are the refractive index differences between core and cladding of RCPCF and CPCF, respectively.

With the same reasoning given in Ref. [18] and for the sake of structural and index profile comparison, we assume $\lambda_{\mathrm{CPCF}} = 2\Lambda$. In case of RCPCF, since more confinement of the mode occurs, thus the field amplitude may be enclosed entirely in the core region, as illustrated in Fig. 6. Therefore, for our profile, we may set $\lambda_{\mathrm{RCPCF}} = (2\Lambda - d)$.

Therefore, Eq. (11) can be written as

$$V_{\mathrm{RCPCF}} = \frac{1}{(1 - d/2\Lambda)} (\delta)^{1/2} V_{\mathrm{CPCF}}. \tag{12}$$

Equation (12) indicates that limiting factor for V_{RCPCF} is the ratio δ which is wavelength dependent and is always greater than unity. Under a practical condition, the multiplying factors of V_{CPCF} in Eq. (12) are always greater than unity, hence $V_{\mathrm{RCPCF}} > V_{\mathrm{CPCF}}$. In a special case, comparing RCPCF and CPCF, for a single-mode operation[19], if $d/\Lambda = 0.3$ and $V_{\mathrm{CPCF}} = \pi$, we obtain from Eq. (12)

$$V_{\mathrm{RCPCF}} = 3.7(\delta)^{1/2}. \tag{13}$$

In the above analysis, we have assumed that the core radii of RCPCF and CPCF are nearly equal that may not be true practically. In a more general form, Eq. (12) can be derived as

$$V_{\mathrm{RCPCF}} = \frac{1}{(1 - d/2\Lambda)} \left(\frac{\rho_{\mathrm{RCPCF}}}{\rho_{\mathrm{CPCF}}} \right) (\delta)^{1/2} V_{\mathrm{CPCF}}. \tag{14}$$

In a separate calculation we have plotted the effective V-parameter of the RCPCF and CPCF fibers in terms of Λ/λ, with parameters values obtained in Fig. 7. The proposed RCPCF has higher V_{eff} than the CPCF at a given wavelength, which verifies Eq. (12). Thus with greater NA, the single-mode operation of RCPCF fiber will be maintained at higher level as comparison with CPCF operation. As the ratio δ increases, the difference between V_{eff} of two types of PCF widens.

The bending loss of the RCPCF can be obtained by[17]

$$\alpha\,(\mathrm{dB/m}) = 4.343 \left(\frac{\pi}{4\rho_{\mathrm{eq}} R_{\mathrm{c}}} \right)^{1/2} \left(\frac{U_{\mathrm{eff}}}{V_{\mathrm{eff}} K_1(W_{\mathrm{eff}})} \right)^2 \times \left(\frac{1}{W_{\mathrm{eff}}} \right)^{3/2} \exp\left\{ -\frac{4R_{\mathrm{c}} W_{\mathrm{eff}}^3 \Delta_{\mathrm{eff}}}{3\rho_{\mathrm{eq}} V_{\mathrm{eff}}^2} \right\}, \tag{15}$$

where the parameters V_{eff}, U_{eff}, W_{eff} are as defined in Eq. (7) for RCPCF and K_1, R_{c}, ρ_{eq}, and Δ_{eff} are second order Bessel function, bending radius, effective core radius, and effective refractive index difference between core and cladding, respectively. Figure 8 illustrates the bending loss of the proposed RCPCF with $d/\Lambda = 0.3$, $R_{\mathrm{c}} = 5$ cm, $\Delta_{\mathrm{eff}} = 0.005$, and $\rho_{\mathrm{eq}} = 0.64\Lambda$ μm. As it is expected, by introducing index difference between the core and the cladding, the bending loss has reduced at the smaller wavelengths. Using parameters given in Eq. (5) with comparative values as that of RCPCF, the bending characteristic curve of a CPCF is plotted in Fig. 9 for

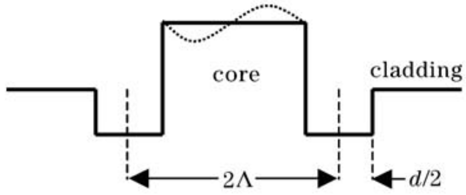

Fig. 6. Cross-section of the RCPCF. Dotted line denotes the field-amplitude of a second order mode.

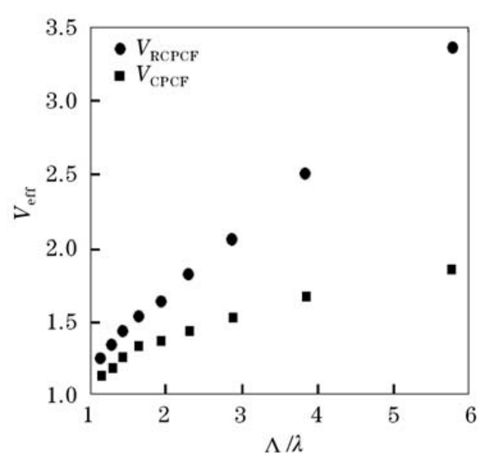

Fig. 7. Effective V-parameters of the proposed RCPCF and CPCF fibers versus Λ/λ.

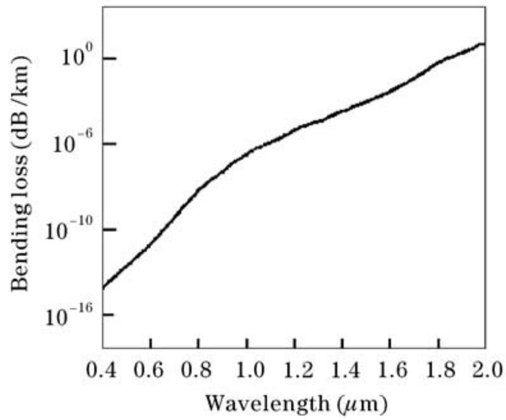

Fig. 8. Bending loss of the RCPCF versus wavelength for $d/\Lambda = 0.3$ at bending radius of 5 cm.

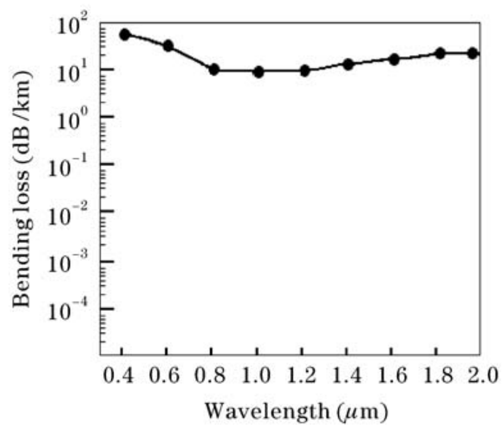

Fig. 9. Bending loss of the CPCF with same parameter values of the RCPCF given in Fig. 8.

silica refractive index $n_\mathrm{s} = 1.45$, and air refractive index $n_\mathrm{a} = 1.0$.

Comparison of Figs. 8 and 9 reveals that at wavelength of 1.6 μm, the amount of loss in the proposed RCPCF (Fig. 8) is about one tenth of the loss obtained in CPCF (Fig. 9).

Saito *et al.*[13] designed and fabricated a PCF with two layers of holes in the cladding with different diameters to control the bending loss, bending loss at 1.55 μm for a bending radius of 10 mm is 0.011 dB/turn. The result in Fig. 8 at 1.55 μm agrees well with the bending loss obtained in Ref. [13], but in former the ratio d/Λ is 0.3 and in the later it is 0.22.

In conclusion, a PCF with a raised-core index is analyzed. Using effective index method, we have shown that the NA and the effective V-parameter of the analyzed profile have improved as compared with the CPCFs. Introducing a stepped raised-index core in the PCF profile would improve the bending loss of the proposed PCF. At a particular wavelength, the calculated bending loss of the raised-core PCF design has shown a reduction to about one tenth of CPCF. The analyzed PCF can be manufactured by the conventional fabrication method. This design may be promising to achieve low loss PCFs for optical wiring applications.

The authors are grateful to the authority of Iran Telecom Research Center for the support of this project. F. E. Seraji's e-mail address is feseraji@itrc.ac.ir.

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