A new approach to measure the ocean temperature using Brillouin lidar

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An approach of lidar measurements of ocean temperature through measuring the spectral linewidth of the backscattered Brillouin lines is presented. An empirical equation for the temperature as a function of Brillouin linewidth and salinity is derived. Theoretical results are in good agreement with the experimental data. The equation also reveals the dependence of the temperature on the salinity and Brillouin linewidth. It is shown that the uncertainty of the salinity has very little impact on the temperature measurement. The uncertainty of this temperature measurement methodology is approximately 0.02 °C.

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Temperature is a very important physical parameter in oceanography. Currently, the sea surface temperatures can be obtained from satellite data, while underwater temperatures have to be measured by fixed buoys in the oceans or expandable bathythermographs. However, these approaches do not allow the rapid, accurate, and real time range-resolved monitoring. Brillouin lidar technique provides a promising solution. Although Brillouin scattering has been proposed as a possible approach for measuring ocean temperature since twenty years $ago^{[1]}$, only recent progress in laser technology could make such an apparatus feasible^[2-4].

Brillouin scattering consists of two frequency shifted Lorentzian lines centered symmetrically with respect to the frequency of the transmitted laser line. Water temperatures can be obtained by measuring the spectral shift (Brillouin shift) of the backscattered Brillouin lines. For seawater, the theoretical temperature uncertainty of this method is approximately $0.2 \, {}^{\circ}C^{[5]}$. However, this resolution in the fresh water is only $0.06 \,^{\circ}C^{[5]}$. Therefore, the uncertainty in the salinity is one of the main factors limiting the accuracy of a Brillouin lidar system. In the laboratory test, the temperature of distilled water has been measured to an accuracy of ± 0.3 °C with continuous wave argon ion laser operating at $514 \text{ nm}^{[6]}$. In the field test, on the other hand, the temperature uncertainty of dirty seawater increased by $\pm 4 \, {}^{\circ}C^{[1]}$. Consequently, this approach using Brillouin shift is not suitable for some applications due to the lower resolution. Experiments have demonstrated that there is a strong dependence of Brillouin linewidth on the temperature^[7]. Nevertheless, the potential for water temperature measurement by use of Brillouin linewidth has not been investigated. This paper is focusing on this subject, and derives an empirical equation for calculating the ocean temperature as a function of Brillouin linewidth and salinity. Moreover, comparison with experimental data is made. The dependence of the temperature on Brillouin linewidth and salinity is also analyzed. Meanwhile, the measurement uncertainty using Brillouin linewidth is compared with the one that using Brillouin shift. It is seen that the method using lidar Brillouin linewidth measurements has higher accuracy and fewer limiting factors.

For a given incident laser wavelength λ , the linewidth of the Brillouin peaks is given by^[8]

$$\Gamma_{\rm B}(S,T) = \frac{1}{4\pi\rho(S,T)} \left(\frac{4\pi n(S,T,\lambda)}{\lambda}\right)^2 \\ \times \left[\frac{4}{3}\eta_{\rm s}(S,T) + \eta_{\rm b}(S,T) + \frac{\kappa}{C_{\rm P}}(\gamma-1)\right] \sin^2\left(\frac{\theta}{2}\right), (1)$$

where n is the refractive index of water, S is the salinity, T is the temperature, ρ is the density of water, η_s is the shear viscosity, η_b is the bulk viscosity, θ is the scattering angle, $\gamma = C_P/C_V$ is the ratio of specific heats, and κ is the thermal conductivity of water. For liquid, the third term on the right side of the equation is usually 2 orders of magnitude smaller than the first or the second term, so we can neglect it.

An empirical equation for the dependence of the refractive index of seawater on temperature, salinity, and wavelength is given by^[9]

$$n(S,T,\lambda) = n_0 + (n_1 + n_2T + n_3T^2)S + n_4T^2 + \frac{n_5 + n_6S + n_7T}{\lambda} + \frac{n_8}{\lambda^2} + \frac{n_9}{\lambda^3},$$
 (2)

where λ is the wavelength in nanometers, T is in degrees Celsius, S is the salinity in parts per thousand, and n_j are constants as given in Ref. [9]. For simplicity, the pressure is assumed to be zero (i.e., atmospheric pressure) for the present calculations, but could be easily included.

A formula for calculating the shear viscosity $\eta_{\rm s}$ (Pa·S) of seawater as a function of temperature T, salinity S, and pressure P is given in Ref. [10], which is valid for 0 °C $\leq T \leq 30$ °C, $1\% \leq S \leq 36\%$, and 1 dbar $\leq P \leq 1000$ dbar. Setting P = 0 in the expression yields

$$\eta_{\rm s}(S,T) = 0.1 \left[\sum_{j=0}^{3} Q_{0j} T^j + S \sum_{k=0}^{3} R_k T^k \right], \qquad (3)$$

where Q_{0j} and R_k are the coefficients obtained by fitting to experimental data. These coefficients are given in Ref. [10]. The ratio of bulk viscosity to shear viscosity has a weak dependence on the salinity^[11]. It is given by

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Table 1. Calculated Data for Brillouin Linewidth $\Gamma_{\rm B}$ (GHz) as a Function of Salinity and Temperature at 532 nm

$T (^{\circ}C)$	Salinaty (in Part Per Thousand)							
	0	5	10	15	20	25	30	35
0	1.75072	1.75830	1.76590	1.77350	1.78108	1.78863	1.79617	1.80367
1	1.62633	1.63398	1.64165	1.64931	1.65696	1.66458	1.67217	1.67973
3	1.40925	1.41688	1.42452	1.43215	1.44397	1.44734	1.45489	1.46241
5	1.22894	1.23639	1.24384	1.25128	1.25869	1.26607	1.27342	1.28074
7	1.07964	1.08679	1.09395	1.10109	1.10820	1.11529	1.12234	1.12936
10	0.90319	0.90981	0.91642	0.92301	0.92957	0.93612	0.94263	0.94911
15	0.70386	0.70952	0.71518	0.72081	0.72643	0.73201	0.73758	0.74311
20	0.58420	0.58904	0.59386	0.59867	0.60346	0.60823	0.61298	0.61770
25	0.51229	0.51652	0.52074	0.52495	0.52913	0.53330	0.53745	0.54157
30	0.46487	0.46878	0.47267	0.47655	0.48042	0.48426	0.48809	0.49189

$$\frac{\eta_{\rm b}}{\eta_{\rm s}} = 4.8840 - 0.2488T + 7.33 \times 10^{-3}T^2 -6.1250 \times 10^{-5}T^3.$$
(4)

The expression for the density of seawater as a function of temperature and salinity is based on the international equation of state for seawater diluted with pure water or concentrated by evaporation, which is valid for $-2 \text{ °C} \leq T \leq 40 \text{ °C}$, $0 \leq S \leq 42\%$. The density of seawater is written as^[10]

$$\rho(S,T) = \sum_{i=0}^{5} a_i T^i + S \sum_{j=0}^{5} b_j T^j + S^{3/2} \sum_{k=0}^{5} c_k T^k + d_0 S^2,$$
(5)

where a_i, b_j, c_k , and d_0 are known constants^[10].

Brillouin linewidth $\Gamma_{\rm B}$ and salinity S are taken as the two independent variables. To obtain empirical equation for $T(\Gamma_{\rm B}, S)$, we use Eqs. (1)—(5) to generate a table containing $\Gamma_{\rm B}(S,T)$ at fixed values of S and T for 170 °C backscattering, as shown in Table 1. We fit the data to a power series that contains various powers of Brillouin linewidth, salinity, and their cross terms. According to the calculated data, the water temperature has an inverse dependence on Brillouin linewidth, negative powers of $\Gamma_{\rm B}^{-1}$ to $\Gamma_{\rm B}^{-5}$ are included. The dependence of the temperature on salinity is linear. Therefore, the chosen polynomial which contains 16 terms is fitted to 80 sets of data to determine each coefficient and its standard deviation. We then eliminate all those terms for which the standard deviation of the coefficients is much larger than the coefficient itself. The remaining terms are again fitted to the calculated data. The function $T(\Gamma_{\rm B}, S)$ of the following form is obtained as

$$T(\Gamma_{\rm B}, S) = \sum_{i=0}^{5} t_i \Gamma_{\rm B}^{-i} + S(t_6 \Gamma_{\rm B}^{-1} + t_7 \Gamma_{\rm B}^{-2} + t_8 \Gamma_{\rm B}^{-3}).$$
 (6)

This equation is valid for 0 °C $\leq T \leq 30$ °C and $0 \leq S \leq 35\%$. The coefficients t_i are given in Table 2. The difference between $T(\Gamma_{\rm B}, S)$ from Eq. (6) and the corresponding value of T from Table 1 is shown in Fig. 1.

Table 2. Coefficients in the Empirical Equation for $T(\Gamma_{\rm B},S)$ at 532 nm

$t_0 = -18.9997$	$t_3 = 13.1460$	$t_6 = 3.4390 \times 10^{-2}$
$t_1 = 46.3960$	$t_4 = -2.9485$	$t_7 = -2.8277 \times 10^{-2}$
$t_2 = -29.7296$	$t_5 = 0.4135$	$t_8 = 1.5998 \times 10^{-2}$



Fig. 1. Difference between the temperature T in Table 1 and $T(\Gamma_{\rm B}, S)$ given by Eq. (6).

Comparison of theoretical results with experimental data^[7] for 0 and 35%₀salinities at atmospheric pressure is presented as Fig. 2(a) and (b), respectively. It can be seen that theoretical results obtained from Eq. (6) are in excellent agreement with the experimental data. More measurements to demonstrate the relationship between the temperature and the Brillouin linewidth for various values of salinity are underway.

According to Eq. (6), the quantitative dependence of the temperature on the salinity at atmospheric pressure is given by

$$\frac{\partial T}{\partial S} = t_6 \Gamma_{\rm B}^{-1} + t_7 \Gamma_{\rm B}^{-2} + t_8 \Gamma_{\rm B}^{-3}.$$
 (7)

If the uncertainty in the salinity is 0.5%, the uncertainties in the temperature measurement are from 0.007 to 0.04 °C for Brillouin linewidth values between 1.7 and 0.5 GHz. In contrast, the uncertainty using Brillouin shift is about 0.2 °C^[5]. The dependences of the temperature on the salinity are shown in Fig. 3. Solid curves



Fig. 2. Variation in the temperature T as a function of the Brillouin linewidth $\Gamma_{\rm B}$. (a) Fresh water (S = 0); (b) seawater of salinity (S = 35%).



Fig. 3. Variation in the temperature T as a function of the salinity. $v_{\rm B}$: Brillouin frequency shift; $\Gamma_{\rm B}$: Brillouin linewidth.

denote $T(\Gamma_{\rm B}, S)$ with the Brillouin linewidth $\Gamma_{\rm B}$ as a parameter between 0.5 and 1.7 GHz. Dashed curves represent $T(v_{\rm B}, S)$ with the Brillouin shift $v_{\rm B}$ as a parameter between 7.4 and 7.7 GHz in steps of 0.1 GHz^[5]. It is obvious that the Brillouin linewidth methodology is less sensitive to the variation of the salinity.

The dependence of temperature on Brillouin linewidth is shown in Fig. 4. From Eq. (6) the values of salinity S is from 25%0 to 35%0 in steps of 2%0 (bold solid curve). Another dashed curve denotes the relation between temperature and Brillouin linewidth for fresh water (S = 0) at atmospheric pressure. Quantitatively, from Eq. (6) we find

$$\frac{\partial T}{\partial \Gamma_{\rm B}} = -\sum_{i=1}^{5} i t_i \Gamma_{\rm B}^{-(i+1)} - S(t_6 \Gamma_{\rm B}^{-2} + 2t_7 \Gamma_{\rm B}^{-3} + 3t_8 \Gamma_{\rm B}^{-4}).$$
(8)

Thus the uncertainty in Brillouin linewidth measurements of 1 MHz would lead to uncertainty of 0.015 $^{\circ}\mathrm{C}$



Fig. 4. Temperature as a function of the Brillouin linewidth.

for $\Gamma_{\rm B} = 1.0$ GHz and S = 35%. While, the uncertainty in Brillouin shift measurement of 1 MHz corresponds to temperature uncertainty of approximately 0.06 °C^[5].

As shown in Fig. 4, for the salinity of the actual seawater from 25% to 35% at atmospheric pressure, the curves of the relations between the temperature and Brillouin linewidth are almost superposed. Therefore, in the practical measurements, the fluctuation of the salinity can be ignored. Since there are slight differences between the curves of fresh water and seawater, the average values of the salinity should still be considered.

The uncertainty of measurement ΔT of the temperature depends on the uncertainty of measurements of the Brillouin linewidth $\Delta\Gamma_{\rm B}$ and the salinity ΔS . It can be estimated as

$$\Delta T = \left[\left(\frac{\partial T}{\partial \Gamma_{\rm B}} \right)^2 \left(\Delta \Gamma_{\rm B} \right)^2 + \left(\frac{\partial T}{\partial S} \right)^2 \left(\Delta S \right)^2 \right]^{1/2}.$$
 (9)

Under the conditions of $\Delta S = 0.5\%$, $\Delta \Gamma_{\rm B} = 1$ MHz, and S = 35%, the uncertainties ΔT vary from 0.008 to 0.08 °C for Brillouin linewidth values between 1.7 and 0.5 GHz. For a typical value of seawater $\Gamma_{\rm B} = 1.0$ GHz, the temperature uncertainty is approximately 0.015 °C (≈ 0.02 °C). By contrast, for $\Delta S = 0.5\%$ and $\Delta v_{\rm B} = 1$ MHz, the measurement using Brillouin shift has the uncertainty of 0.2 °C for a corresponding value of $v_{\rm B} = 7.5$ GHz.

In conclusion, a rigorous theoretical relation between the temperature, Brillouin linewidth, and salinity has been obtained. The experimental data are in good agreement with theoretical results. The theoretical function was also employed to quantitatively analyze the dependence of the temperature on the salinity and Brillouin linewidth. It is found that the standard deviation of the salinity in the ocean has little effect on the accuracy of temperature determination. This method has a theoretical uncertainty of 0.02 °C. Under the same conditions, however, the theoretical uncertainty using Brillouin shift is approximately 0.2 °C. Therefore, the measurement of ocean temperature using Brillouin linewidth is a promising approach in the submarine or aircraft laser remote sensing.

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