An improved algorithm for cloud multiple scattering

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Clouds' radiation characteristics are very important in clouds scene simulation, weather forecasting, pattern recognition, and other fields. Radiation of a cloud mainly comes from its multiple scattering. A new algorithm to calculate multiple scattering, called build-up factor algorithm, is proposed in this paper. In this algorithm, a modified gamma distribution is assumed to describe droplets distribution inside a cloud, then the radiation transport equation is calculated to get the solution of single scattering, and finally, a build-up factor is defined to estimate the multiple scattering contributions. This algorithm considers both single scattered radiance and multiple scattered radiance and needs shorter computing time. It can be used in real time simulations.

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Clouds radiation characteristics are very important in clouds scene simulation, weather forecasting, pattern recognition, and other fields. In most cases, the backgrounds of a flying missile in the space are cloud scenes. In order to detect missiles against cloud backgrounds, to enhance the fidelity of simulation, it is critical to understand clouds' thermal radiation characteristics. First of all, we must know how to calculate thermal radiation from cloud multiple scattering. Thermal radiation from a cloud includes both cloud thermal emissions along the line of sight and the effect of other thermal emissions scattered into the line of sight, such as scattered sunlight, reflected sky, earthshine, and sun glints[1,2]. The popular algorithm to calculate multiple scattering is Monte Carlo method which is a statistical model. It uses statistical method to trace every photon's scattering and absorption in atmosphere instead of radiation transport equation. In order to calculate the final distribution of radiant intensity, enough amounts of photons must be traced, at least more than one million of photons. Therefore Monte Carlo method needs longer computing time and could not be used in real time or online simulation. In order to find a new algorithm of calculating multiple scattering which needs shorter computing time to be applicable to real time simulations, the build-up factor algorithm is described here.

As we know, clouds are composed of water or ice droplets and other particles. In order to develop spatially varying particle size distributions inside the cloud, a modified gamma distribution is assumed to describe the number density for all cloud types according to particle size. Thus, the number density is given by[3]

$$N = \int_0^\infty n(r)\mathrm{d}r = \int_0^\infty ar^\alpha \exp(-br^\nu)\mathrm{d}r = \frac{n_0}{\nu}\lambda^{-\frac{\alpha+1}{\nu}}\Gamma\left(\frac{\alpha+1}{\nu}\right). \quad (1)$$

In Eq. (1), the parameters α, a, b, and ν vary by cloud type[4], and are given in Table 1.

The basic principle of build-up factor algorithm is to approximate multiple scattering in a slab of a given thickness using single scattering enhanced by estimation of the multiple scattering contribution. Let $L(s,\Omega)$ represent the radiance along a path at path length s, radiating in the direction Ω. The radiation transport equation is

$$\frac{\mathrm{d}L}{\mathrm{d}s} = -k_e L + J. \quad (2)$$

The direction Ω can be described by the spherical coordinates θ and ϕ. As it propagates, the radiation suffers extinction governed by the coefficient k_e and picks up radiance due to the scattering of light into the Ω direction, represented by the source term J. The radiation transport equation has the solution[5]

$$L(s,\Omega) = L(s_0,\Omega)\mathrm{e}^{-k_e(s-s_0)} + \int_{s_0}^{s} J_{\mathrm{ext}}(s',\Omega)\mathrm{e}^{-k_e(s'-s_0)}\mathrm{d}s', \quad (3)$$

where

$$J_{\mathrm{ext}}(s',\Omega) = \frac{k_e\omega_0}{4\pi}\int_{4\pi} L_{\mathrm{ext}}(s',\Omega')p(\Omega,\Omega')\mathrm{d}\Omega'. \quad (4)$$

Considering light scattered off of a cloud top, let z measure the distance down from the top of the cloud, which has a vertical thickness of l, and let s be at the cloud top where $z = 0$. Then

$$(s-s') = \frac{z}{\cos(\theta)} = \frac{z}{\mu}, \quad \mu = \cos(\theta). \quad (5)$$

Let $L(\Omega) = L(0,\Omega)$ represent the radiation coming out

Table 1. Parameters Used for Droplet Size Distributions by Cloud Type[4]

Cloud Type	α	a	b	ν
Cumulus	3.000	2.604	0.500	1
Stratus	2.000	27.000	0.600	0.5
Stratocumulus	2.000	52.734	0.750	1
Altostratus	5.000	6.268	1.111	3
Nimbostratus	2.000	7.676	0.425	3
Cirrus	6.000	0.011865	1.500	3

of the cloud in the Ω direction. Switching from s to z and ignoring the upwelling background radiation contained in the $L(s_0, \Omega)$ term give

$$L(\Omega) = \int_0^1 J_{\text{ext}}(z, \Omega) \mathrm{e}^{-\frac{k_e z}{\mu}} \frac{\mathrm{d}z}{\mu}. \quad (6)$$

The $L_{\text{ext}}(s, \Omega')$ term contains light that has been multiply scattered, as well as single scattered light. The contribution to the cloud top radiance of single scattered light is easy to calculate. If the light at the top of the cloud comes in from the Ω_0 direction, the illuminating light at the top of the cloud is given by

$$L_0(0, \Omega) = I_0 \delta(\cos\theta' - \cos\theta_0)\delta(\phi' - \phi_0). \quad (7)$$

Applying the attenuation factor $\exp(-k_e z/\mu')$ to get the amount of the illuminating radiation that does not scatter until reaching the z level, and integrating over Ω' give

$$J_{\text{ss}}(z, \Omega) = \frac{k_e \omega_0}{4\pi} \int p(\Omega, \Omega') I_0 \delta(\cos\theta' - \cos\theta_0)\delta(\phi' - \phi_0) \times \mathrm{e}^{-\frac{k_e z}{\mu'}} \mathrm{d}\Omega' = \frac{k_e \omega_0}{4\pi} I_0 p(\alpha) \mathrm{e}^{-\frac{k_e z}{\mu_0}}, \quad (8)$$

where α is the scattering angle between Ω and Ω'. The expression for the single scattered outgoing radiance is

$$L_{\text{ss}}(\Omega) = \frac{\omega_0}{4\pi} I_0 p(\alpha) \frac{\mu_0}{\mu + \mu_0} \left(1 - \mathrm{e}^{-k_e I \frac{\mu + \mu_0}{\mu \mu_0}}\right). \quad (9)$$

In order to estimate the multiple scattering contributions, we define a build-up factor $B(z)$ that estimates the ratio of the multiply scattered radiation from the z level to the single scattered radiation. To define B, consider the fraction f_e of light single scattered at z that escapes out of either the cloud top or bottom without scattering again,

$$f_e(z) = \frac{\int_{4\pi} J_{\text{ss}}(z, \Omega') \mathrm{e}^{-\frac{k_e z}{\mu'}} \mathrm{d}\Omega'}{\int_{4\pi} J_{\text{ss}}(z, \Omega') \mathrm{d}\Omega'}. \quad (10)$$

For the radiation first scattered at z, the fraction f_e escapes without further interactions, while the fraction $f_r = (1 - f_e)$, either is absorbed or scattered. Since ω_0 gives the ratio of scattering to extinction, the fraction $\omega_0 f_r$ of the light survives the second scattering without being absorbed, and the fraction $\omega_0 f_r f_e$ escapes the cloud. In the same way, the fraction of the light that escapes after scattering n times is $(\omega_0 f_r)^{n-1} f_e$. Summing up the contributions and dividing by the fraction escaping after only one scattering gives the build-up factor

$$B(z) = \frac{1}{f_e} \sum_{n=1}^{\infty} f_e (\omega_0 f_r)^{n-1} = \frac{1}{(1 - f_r \omega_0)} = \frac{1}{1 - (1 - f_e)\omega_0}. \quad (11)$$

The amount of multiple scattered lights can be estimated using the build-up factor. Scattering from large droplets has a pattern that is highly forward peaked. To determine how the scattering is peaked, we use the forward directivity integral of the Mie phase function, defined by

$$f = \frac{1}{2} \int_{-1}^{1} p(\alpha) \cos(\alpha) \mathrm{d}\cos(\alpha). \quad (12)$$

Then the phase function is approximated by assuming that a fraction of the light given by f is scattered in the forward direction and that the rest of the light, $(1-f)$, is scattered isotropically. In the interaction represented by Eq. (2), the extinction term $k_e L$ is the sum of the part $k_a L$ that is absorbed and the part $k_s L = \omega_0 k_e L$ that is scattered. We assume that the fraction f of scattered light is scattered precisely in the forward direction. This is handled mathematically by adding the amount $k_s f L$ back on the right hand side of Eq. (2)

$$\frac{\mathrm{d}L}{\mathrm{d}s} = -k_e L + J + k_s f L = -k_e(1 - \omega_0 f)L + J. \quad (13)$$

Thus the extinction coefficient k_e in the radiation transport equation will be replaced with the quantity $k_e(1 - \omega_0 f)$.

With these approximations, we calculate the cloud top radiance as a sum of single scattered and multiple scattered radiance

$$L(\Omega) = L_{\text{ss}}(\Omega) + L_{\text{ms}}(\Omega), \quad (14)$$

where single scattered radiance is

$$L_{\text{ss}}(\Omega) = \frac{k_e \omega_0}{4\pi} \int_0^1 \int_{4\pi} p(\Omega, \Omega') L_0(0, \Omega') \times \mathrm{e}^{-\frac{k_e(1-\omega_0 f)z}{\mu'}} \mathrm{d}\Omega' \mathrm{e}^{-\frac{k_e(1-\omega_0 f)z}{\mu}} \frac{\mathrm{d}z}{\mu}, \quad (15)$$

and multiple scattered radiance is

$$L_{\text{ms}}(\Omega) = \frac{k_e \omega_0}{4\pi} \int_0^1 \int_{4\pi} [B(z) - 1](1 - f) L_0(0, \Omega') \times \mathrm{e}^{-\frac{k_e(1-\omega_0 f)z}{\mu'}} \mathrm{d}\Omega' \mathrm{e}^{-\frac{k_e(1-\omega_0 f)z}{\mu}} \frac{\mathrm{d}z}{\mu}. \quad (16)$$

In the above equations, the term -1 occurs in the factor $B - 1$ to remove single scattering from the multiple scattering factor and $(1 - f)$ is used to give the fraction of the scattered radiation that has not been assigned to the forward direction.

This paper describes the build-up factor algorithm used to calculate multiple scattering. Firstly assume a modified gamma distribution to describe the number density of particles inside a cloud. Secondly, calculate the radiation transport equation to get the solution of single scattering. Finally, define a build-up factor to estimate the multiple scattering contributions. So this method needs shorter computing time and can be used in real time or online simulations. It can be used in calculation of cloud thermal radiation, cloud scene simulation, and applications where the infrared radiation from a cloud clutter background is needed. It will be helpful to system designers, data takers, and analysts.

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References

1. Y. Deng, Q. Lu, and Q. Luo, Chin. Opt. Lett. **4,** 45 (2006).
2. R. Haimes, M. Giles, J. Schroeder, A. Berk, and D. Robertson, Proc. SPIE **1098,** 71 (1989).
3. F. X. Sheng, J. T. Mao, J. G. Li, A. S. Zhang, J. G. Sang, and N. X. Pan, *Atmospheric Physics* (in Chinese) (Beijing University Press, Beijing, 2003) p.303.
4. D. G. Kerr, *"A validation study of cloud scene simulation model temporal performance"* (Air Force Institute of Technology, Ohio, 1999) p.35.
5. R. J. Thornburg, J. G. DeVore, and J. Thompson, "Review of the CLDSIM cloud radiance simulator" PL-TR-93-2232 (1994) p.25.