Precise calculation of the KTP crystal used as both an intracavity electro-optic Q-switch and a second harmonic generator

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A method of precisely calculating the external applied voltage and the optimum type-II phase matching angles for KTP crystal, which is used as both an intracavity electro-optic (EO) Q-switch and a frequency doubler, is presented. The effective EO coefficient along the phase-matching direction is defined to calculate the half-wave voltage and the quarter-wave voltage, and the precise calculation for the phase matching angles in the condition of KTP crystal optimum second harmonic phase matching is theoretically realized.

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Compact, high-power, and high-repetition-rate all-solid-state green lasers are highly desirable for a number of industrial, medical, and military applications. The technology of the acousto-optic (AO) Q switching is one of the main methods of obtaining high-power, high-repetition-rate green laser. However, the total turn-off time and pulse duration (usually > 100 ns) of AO devices are limited by long travelling times of acoustic waves across the optical beam diameter, which hinders their use in high-gain lasers. By contrast, electro-optic (EO) Q switching is a promising technology to achieve high-repetition-rate and high-peak-power green laser because it has the merits of short switching times (about 10^{-9} s) and short pulses (10—20 ns). $LiNbO_3$ and KD^*P crystals are widely used as EO materials for Q switching devices, but they both have some disadvantages. Although $LiNbO_3$ crystal is nonhygroscopic and has a low half-wave voltage, a major drawback is the relatively low optical damage threshold, which limits its use in high-peak-power laser systems. In particular, $LiNbO_3$ crystal is severely suffered from piezoelectric ringing, and a $LiNbO_3$ Pockels cell typically cannot be run at repetition rates above a few kilohertz without attenuating these resonances[1,2]. KD^*P, on the other hand, has a higher optical damage threshold than $LiNbO_3$. However, KD^*P crystal is hygroscopic and requires relatively high half-wave voltage to carry out the EO Q switching in the laser cavity.

Potassium titanyl phosphate ($KTiOPO_4$ or KTP) is a unique material, which has superior properties such as reasonably large nonlinear coefficients, high optical damage threshold, low optical loss, wide acceptance angles, and thermally stable phase-matching, etc.. In general, the KTP crystal is used for several nonlinear optical applications, and in particular, for second harmonic generation (SHG) near 1-μm radiation of Nd^{3+} laser[3]. Moreover, its large linear EO coefficients, low dielectric constant, and no piezoelectric effect except the range of 500 kHz—10 MHz make it attractive for various EO applications[4,5], such as EO modulators[6], Q-switches[7−10], and EO waveguides[11−13].

A single KTP crystal simultaneously used as Q-switch and SHG device has been reported[14−17]. Such operation is especially utilizable for compact diode pumped Q-switched Nd^{3+} lasers, which can realize low loss, high efficiency, and low cost. In this case, we need consider the external half-wave voltage for Q-switching and the phase matching angles at the same time. On one hand, the half-wave voltage depends on the propagation direction of the fundamental wave in KTP crystal, and it would induce the change of principal refractive indices. On the other hand, the phase-matching angles depend on the refractive indices. Hence the two conditions interact and the problem must be solved consistently. Generally, the half-wave voltage was considered under certain phase matching angles, which were calculated under the initial principal refractive indices without applied voltage. There were certain second harmonic (SH) phase shifts when applying the voltage. Although the phase shifts of the SH wavelength by the voltages were within the SHG phase matching acceptance condition, the total SHG conversion efficiencies were decreased evidently. And the incident angles were needed to adjust during the experiments. In this paper, we offer a new method of calculating the voltage and SHG phase-matching angles. The effective EO coefficient along the phase-matching direction is defined. This coupled problem is solved through loop numerical calculation, and we have theoretically realized the precise calculation for phase matching angles in the condition of KTP crystal optimum SH phase-matching, which is different from the values without regard to the external electric field.

It is well known that the phase of the fundamental waves in the f-(fast) and s-(slow) axis directions agrees with the phase of the SH wave in the f-axis direction in type II phase matching of KTP crystal. Hence, the type II phase matching condition for SHG in a KTP crystal in the absence of the electric field is given by

$$\frac{1}{2}(n_{s,\omega} + n_{f,\omega}) = n_{f,2\omega}, \tag{1}$$

where $n_{s,\omega}$ and $n_{f,\omega}$ are the refractive indices of the s- and f-axis of the fundamental wave; $n_{f,2\omega}$ is refractive index of fast ray for SH wave. The principal refractive indices n_x, n_y, n_z used for calculating the optimum type II phase-matching angles in KTP crystal are given by[18]

$$\begin{cases} n_x = 3.0065 + \frac{0.03901}{\lambda^2 - 0.04251} - 0.01327\lambda^2 \\ n_y = 3.0333 + \frac{0.04154}{\lambda^2 - 0.04547} - 0.01408\lambda^2 \\ n_z = 3.3134 + \frac{0.05694}{\lambda^2 - 0.05658} - 0.01682\lambda^2 \end{cases} . \quad (2)$$

Through refined numerical calculation, the optimum type II phase matching angles were $\theta_i = 90°$, $\phi_i = 24.7°$ for the wavelength of 1064 nm (here, θ_i is the angle between the beam propagation direction and z-axis, ϕ_i is the propagation angle in the x-y plane relative to the x-axis). Namely, the KTP crystal has to be cut at θ_i and ϕ_i for type II phase matching in order to achieve the highest SH conversion efficiency. Actually, the angles also depend somewhat on the quality and the growth method of the crystal.

The KTP crystal is a biaxial crystal with the orthorhombic structure (point group $mm2$). When the electric field is applied along its z-axis, the new principal axes of KTP crystal coincide with the original principal axes of the zero applied field, but the principal refractive indices are as the function of the applied field, which are given as

$$\begin{cases} n'_{\omega,x}(E_z) = n_{\omega,x}(1 + n^2_{\omega,x}\gamma_{13}E_z)^{-1/2} \\ \qquad \approx n_{\omega,x} - \frac{1}{2}n^3_{\omega,x}\gamma_{13}E_z \\ n'_{\omega,y}(E_z) = n_{\omega,y}(1 + n^2_{\omega,y}\gamma_{23}E_z)^{-1/2} \\ \qquad \approx n_{\omega,y} - \frac{1}{2}n^3_{\omega,y}\gamma_{23}E_z \\ n'_{\omega,z}(E_z) = n_{\omega,z}(1 + n^2_{\omega,z}\gamma_{33}E_z)^{-1/2} \\ \qquad \approx n_{\omega,z} - \frac{1}{2}n^3_{\omega,z}\gamma_{33}E_z \end{cases} , \quad (3)$$

where E_z is the electric field parallel to the z-axis, and γ_{13}, γ_{23}, γ_{33} are the EO coefficients of KTP crystal. The values used in the calculation are[18,19]: $\gamma_{13} = 8.8$ pm/V, $\gamma_{23} = 13.8$ pm/V, $\gamma_{33} = 35.0$ pm/V, $n_{\omega,x} = 1.7399$, $n_{\omega,y} = 1.7480$, $n_{\omega,z} = 1.8296$, $n_{2\omega,x} = 1.7790$, $n_{2\omega,y} = 1.7900$, $n_{2\omega,z} = 1.8868$.

In the case of KTP crystal for SHG and Q-switching simultaneously, a fundamental wave propagates along the direction of (θ, ϕ). According to Fresnel's equation, $n'_{\omega,f}$ and $n'_{\omega,s}$, the refractive indices for the f- and s-axis of the fundamental wave in the presence of the electric field, are given by

$$n'_{\omega,s}(E_z) = n'_{\omega,z}, \quad (4)$$

$$n'_{\omega,f}(E_z) = \frac{n'_{\omega,x}n'_{\omega,y}}{\sqrt{n'^2_{\omega,x}\cos^2\varphi + n'^2_{\omega,y}\sin^2\varphi}}. \quad (5)$$

By combining the formulas of (3) and (5), and using the method of expansion into powerseries, we obtain

$$n'_{\omega,f}(E_z) = n_f\left[1 + n_f^2(\gamma_{23}\cos^2\varphi + \gamma_{13}\sin^2\varphi)E_z\right]^{-1/2}$$
$$\approx n_f - \frac{1}{2}n_f^3(\gamma_{23}\cos^2\varphi + \gamma_{13}\sin^2\varphi)E_z, \quad (6)$$

where

$$n_f = n_{\omega,x}n_{\omega,y}\Big/\sqrt{n^2_{\omega,x}\cos^2\varphi + n^2_{\omega,y}\sin^2\varphi}.$$

Thereby, the phase retardation of the fundamental wave between the f- and s-axis components in the crystal is given by

$$\Gamma = \frac{2\pi}{\lambda}(n'_{\omega,s} - n'_{\omega,f})L = \frac{2\pi}{\lambda}(n_{\omega,z} - n_f)L$$
$$-\frac{\pi}{\lambda}\left[n^3_{\omega,z}\gamma_{33} - n_f^3(\gamma_{23}\cos^2\varphi + \gamma_{13}\sin^2\varphi)\right]V\frac{L}{d}, \quad (7)$$

where L is the KTP crystal length, and d is the KTP crystal thickness across which the voltage $V = E_z d$ is applied. The first term $\left(\frac{2\pi}{\lambda}(n_{\omega,z} - n_f)L\right)$ in Eq. (7), independent of the external electric field, is the static retardation which could be compensated through temperature tuning[6,20]. The half-wave voltage V_π and quarter-wave voltage $V_{\pi/2}$ can be obtained by taking the second term as π and $\pi/2$, respectively. There are

$$V_\pi = \frac{\lambda}{n^3_{\omega,z}\gamma^*}\frac{d}{L}, \quad (8)$$

$$V_{\pi/2} = \frac{\lambda}{2n^3_{\omega,z}\gamma^*}\frac{d}{L}, \quad (9)$$

here, $\gamma^* = \gamma_{33} - (n_f/n_{\omega,z})^3(\gamma_{23}\cos^2\phi + \gamma_{13}\sin^2\phi)$, can be defined as the effective EO coefficient when light propagates along the (θ, ϕ) direction, which is different from the expression in Ref. [20].

At the same time, the SH phase in the presence of the electric field is shifted from the phase-matching condition as given by

$$\delta_s = \frac{\Delta kL}{2} = \frac{\pi}{\lambda}\left\{2n'_{2\omega,f} - (n'_{\omega,f} + n'_{\omega,s})\right\}L, \quad (10)$$

where $n'_{2\omega,f}$ is the refractive index on the f-axis for the SH frequency. And the SHG conversion efficiency can be given by

$$\eta \propto \left[\sin(\frac{\Delta k}{2}L)\Big/(\frac{\Delta k}{2}L)\right]^2. \quad (11)$$

We list the values of $V_{\pi/2}$, the corresponding phase shift of the SH wavelength and conversion efficiency for different values of d/L at the cutting angles of $(90°, 24.7°)$ in Table 1. From the table, we can see that there were certain SH phase shifts under the applied voltages. Although the phase shifts of the SH wavelength by the voltages were within the SHG phase matching acceptance condition, the total SHG conversion efficiencies were decreased evidently. Then we considered loop numerical calculation. Starting with $\phi_i = 24.7°$, the phase matching angle without applied voltage, we can calculate $V_{\pi/2}$ using Eq. (9). With this voltage, we can then calculate the new principal refractive indices under the external electric field with Eq. (3). Hence, the new phase-matching angle also can be calculated with the new principal refractive indices. This procedure should be repeated until the values of $V_{\pi/2}$ and ϕ do not vary. Table 2 lists $V_{\pi/2}$ and ϕ for different values of d/L after double cycle calculation. The phase shifts of the SH wavelength almost are zero and the SHG conversion efficiency can reach the highest.

Table 1. Voltage ($V_{\pi/2}$), Phase Shift (δ_s) of SHG and Conversion Efficiency (η) with Different Values of d/L at the Phase-Matching Angles of (90°, 24.7°)

d/L		1/1	1/2	1/3	1/4	1/5
d = 1 mm	$V_{\pi/2}$ (V)	3657	1829	1219	914	731
	δ_s	44.3	46.7	49.1	51.5	53.9
	η (%)	81.6	79.7	77.8	75.8	73.9
d = 2 mm	$V_{\pi/2}$ (V)	3657	1829	1219	914	731
	δ_s	46.7	51.5	56.3	61.1	65.9
	η (%)	79.7	75.8	71.7	67.4	63.0

Table 2. Loop Numerical Calculation for the Voltages ($V_{\pi/2}$), Phase-Matching Angles (ϕ), Phase Shifts (δ_s) for SHG and Conversion Efficiency (η) with Various Values of d/L

d/L		1/1	1/2	1/3	1/4	1/5
d = 1 mm	$V_{\pi/2}$ (V)	3644	1825	1217	913	731
	ϕ (deg.)	26.1	25.4	25.2	25.1	25
	δ_s	0.1	0.1	0.1	0.2	0.4
	η (%)	100	100	100	100	100
d = 2 mm	$V_{\pi/2}$ (V)	3650	1827	1218	914	731
	ϕ (deg.)	25.4	25.1	25	24.9	24.9
	δ_s	0.1	0.2	0.3	0.8	0
	η (%)	100	100	100	100	100

Table 3. Values of Crystal Size (d/L), Phase-Matching Angles (ϕ), and $V_{\pi/2}$ for Simultaneous SHG and Q-Switching of KTP in Literatures

Ref.	[14]	[15]	[17]	[18]	[20]
d/L	$\frac{1\,\text{mm}}{5\,\text{mm}}$	$\frac{1\,\text{mm}}{5\,\text{mm}}$	$\frac{2\,\text{mm}}{10\,\text{mm}}$	$\frac{3\,\text{mm}}{7\,\text{mm}}$	1/1
ϕ (deg.)	23.4	23.4	23.6	25	25.8
$V_{\pi/2}$ (V)	738	750—800	647	1370	3130

Comparing Tables 1 and 2, we can find the phase-matching angles and quarter-wave voltages in the case of simultaneous Q-switching and SHG of KTP crystal are slightly different from the values through direct calculation. In the case of $d/L = 1/5$ (d = 2 mm), the phase-matching angle ϕ with an electric field changes by 0.2° from the phase-matching angle without the applied electric field. It is very small. This can be understandable because the change of the principal refractive indices induced by the applied electric field could be the magnitude of 10^{-5}, by which the corresponding changes of phase-matching angles caused are also small. However, the small shifts in $V_{\pi/2}$ and ϕ should be considered when cutting KTP for both Q-switching and frequency doubling in order to achieve the highest conversion efficiency.

Additionally, Table 3 summarizes the relevant experimental and calculated values in order to compare with our results. As mentioned in Refs. [15—18], the incident angles of the KTP crystal were all adjusted during experiments. Comparing Tables 2 and 3, we can see that the calculated values for simultaneous Q-switching and frequency doubling in this paper coincide with the values in Refs. [14—18, 20] to a certain extent. The small difference can be understandable by reason of the electrode contact loss and the non-uniform field in the crystal.

In conclusion, we have presented a new method of calculating the voltage and phase-matching angles for KTP crystal which is used simultaneously for Q-switching and SHG. Through loop numerical computing, the precise calculation for optimum SH phase matching of KTP crystal has been theoretically realized and there are no phase shifts of the SH wavelength. The phase-matching angles for different-sized KTP crystals under the external electric field will be different from the calculated values without regard to the applied field. The change of phase-matching angle should be considered for crystal cutting to achieve the highest SH conversion efficiency.

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