Influence of net gain on the statistical fluctuation in a single-mode laser system

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Using linear approximation method, we calculate the intensity correlation time of a single-mode laser driven by both colored pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts, and analyze the influence of the net gain coefficient a_0 on the statistical fluctuation of the laser system. It is found that for the case that the colored pump noise is long time correlated, the main factor influencing the statistical fluctuation of the laser system is a_0, and the frequency of the modulation signal has negligible effect on the statistical fluctuation of the laser system.

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Generally, people think that noise and its cross-correlation form have much influence on the statistical fluctuation of a laser system. As a result, studies on the influence of the noise and its cross-correlation form on the statistical properties of the laser system have attracted much attention in recent years[1−10]. We have studied the dynamic property and statistical fluctuation of a single-mode laser system driven by both pump noise and quantum noise with cross-correlation between the real and imaginary parts[11−14]. However, in the previous research, we focused on the influence of the noise and its cross-correlation form on the statistical fluctuation of the laser system and hardly considered the influence of the signal on the statistical fluctuation of the laser system. In this paper we introduce a colored pump noise modulated by the signal in a laser system[15]. This noise can describe the laser system better than the common noise, because when the laser system is modulated by signal, the noise of the system is simultaneously modulated by the signal, it affects the dynamic behavior of the laser system remarkably. On the other hand, the noise of driving system has essential and decisive effect on the dynamic behavior of the laser system although the dynamic fluctuation of the system is a result of the interaction between the noise and the nonlinear stochastic system. Hence, it is necessary to study the influence of the parameters of the laser system on the statistical fluctuation of the laser system.

The net gain a_0 is an important parameter describing the statistical properties and working state of the laser system and is a critical value determining whether a laser beam outputs. In this paper, we study the influence of a_0 on the statistical fluctuation of the laser system and provide theoretic foundation and an optimum scheme for the steady output of the laser system.

Langevin equation of amplitude for a loss-noise model of a single-mode laser is given by[16]

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = a_0 r - Ar^3 + \frac{Q}{2r}(1-|\lambda_\mathrm{q}|) + r p_\mathrm{R}(t') + \varepsilon_\mathrm{r}(t'). \quad (1)$$

If we consider the colored pump noise which is modulated by a periodic signal $B\cos\Omega t'$, Langevin equation of intensity for a loss-noise model of the single-mode laser with an input signal can be expressed by

$$\frac{\mathrm{d}I}{\mathrm{d}t'} = 2a_0 I - 2AI^2 + Q(1-|\lambda_\mathrm{q}|) + 2I p_\mathrm{R}(t') B\cos\Omega t' + 2\sqrt{I}\varepsilon_\mathrm{r}(t'), \quad (2)$$

where noises $p_\mathrm{R}(t')$ and $\varepsilon_\mathrm{r}(t')$ are correlated in the following forms:

$$\langle p_\mathrm{R}(t')\rangle = \langle \varepsilon_\mathrm{r}(t')\rangle = 0, \quad \langle p_\mathrm{R}(t') p_\mathrm{R}(s)\rangle = \frac{P}{2\tau}\mathrm{e}^{-\frac{|s-t'|}{\tau}},$$
$$\langle \varepsilon_\mathrm{r}(t')\varepsilon_\mathrm{r}(s)\rangle = Q(1+|\lambda_\mathrm{q}|)\delta(t'-s),$$
$$\langle p_\mathrm{R}(t')\varepsilon_\mathrm{r}(s)\rangle = 0. \quad (3)$$

In Eqs. (1), (2), and (3), r is the amplitude of a laser field, A represents the self-saturation coefficient, I is the laser intensity, B is the amplitude of a modulation signal, Ω is the frequency of the signal, $p_\mathrm{R}(t')$ is the real part of the pump noise, $\varepsilon_\mathrm{r}(t')$ is the quantum noise of phase lock, P and Q are the intensities of the pump noise and quantum noise, respectively, τ is the pump noise self-correlation time, λ_q is the cross-correlation coefficient between the real part and imaginary part of the quantum noise with $-1 \le \lambda_\mathrm{q} \le 1$, $\delta(t'-s)$ is the Kronecker delta function.

Let $I = I_0 + \delta(t')$, where $I_0 = \frac{a_0}{A}$ is the deterministic steady-state intensity, and $\delta(t')$ is the perturbation term. Linearizing Eq. (2) around I_0, then we get

$$\frac{\mathrm{d}\delta(t')}{\mathrm{d}t'} = -\gamma\delta(t') + 2I_0 p_\mathrm{R}(t') B\cos\Omega t' + 2\sqrt{I_0}\varepsilon_\mathrm{r}(t') + Q(1-|\lambda_\mathrm{q}|), \quad (4)$$

where $\gamma = 2a_0$.

According to the steady-state mean intensity correlation function (SSMICF) $C(t)$ defined by

$$C(t)=\lim_{t'\to\infty}\frac{\overline{\langle I(t')I(t'+t)\rangle}-\overline{\langle I(t')\rangle}^2}{\overline{\langle I(t')\rangle}^2}=\lim_{t'\to\infty}\left(\frac{\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')I(t'+t)\rangle\mathrm{d}t'-\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')\rangle^2\mathrm{d}t'}{\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')\rangle^2\mathrm{d}t'}\right),$$

the SSMICF can be solved by Eq. (4)

$$C(t)=\left(\frac{2Q(1+|\lambda_\mathrm{q}|)}{\gamma I_0}+\frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)}\right)\mathrm{e}^{-\gamma|t|}+\frac{B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\cos\Omega t+\frac{2\Omega B^2P}{\tau^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\sin\Omega|t|, \quad (5)$$

where $k_1=\gamma-\tau^{-1}$, $k_2=\gamma+\tau^{-1}$ and $\tau\neq\frac{1}{2a_0}$.

Let $t=0$, so the steady-state mean normalized intensity fluctuation (SSMMNIF) $C(0)$ is obtained as

$$C(0)=\frac{2Q(1+|\lambda_\mathrm{q}|)}{\gamma I_0}+\frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)}+\frac{B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}. \quad (6)$$

According to the intensity correlation time (ICT) T defined by

$$T=\int_0^\infty\phi(t)\,\mathrm{d}t, \quad (7)$$

where $\phi(t)=\frac{C(t)}{C(0)}$, and putting $C(t)$ and $C(0)$ into Eq. (6), we have

$$T=\frac{2Q(1+|\lambda_\mathrm{q}|)}{\gamma^2I_0C(0)}+\frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)C(0)}+\frac{B^2P(3\Omega^2+\gamma^2-\tau^{-2})}{(k_1^2+\Omega^2)(k_2^2+\Omega^2)(1+\tau^2\Omega^2)C(0)}. \quad (8)$$

Because the use of unified colored noise approximation in solving Eq. (1), we further discuss the influence of a_0 on the statistical fluctuation of the laser system in the cases $\tau\gg1$ and $\tau\ll1$.

By choosing a_0 as a parameter in the case of $\tau\ll1$, we plot the curve of T versus Ω as shown in Fig. 1. It can be seen that: 1) when a_0 takes different values and Ω is increasing, the curves of T versus Ω experience a changing process from the monotonic rise first, then to a maximum, and finally to a monotonic descending, as shown in Figs. 1(a)—(d). This illustrates that when the laser system works under different a_0 values, the specific frequency of the input signal leads T to a maximum, which corresponds to the minimum statistical fluctuation. 2) When a_0 approaches zero, the value of T increases to a maximum rapidly near $\Omega=0$, then monotonically descends with increasing Ω, as shown in Fig. 1(a). Therefore, the input signal with a low frequency leads T to increase and the statistical fluctuation of the laser system to decrease near the threshold. 3) Within the range $0.01<a_0<14$, as a_0 increases, the whole curve of T versus Ω falls, its maximum decreases, and the position moves towards the increased Ω, as shown in Fig. 1(b). This demonstrates that when the laser system works far away from the range of the threshold, T is shorter but the fluctuation is greater. 4) Within the range $14<a_0<23.3$, as a_0 increases, the maximum in the curve of T versus Ω increases and the position of the maximum moves towards the increased Ω, as shown in Fig. 1(c). For example, in Fig. 1(c), two curves of T versus Ω are corresponding to $a_0=17$ and $a_0=20$, respectively, and two intersections a ($\Omega=10.5$) and b ($\Omega=38.8$) between the two curve can be observed. When a_0 increases from 17 to 20 within the range $0<\Omega<10.5$, T becomes smaller but the statistical fluctuation augments. Within the range $10.5<\Omega<38.8$, T increases but the statistical fluctuation decreases. In the range $\Omega>38.8$, T becomes smaller again, and the statistical fluctuation continuously augments. So, the operating state of the laser system and the frequency of the modulation signal jointly affect the statistical fluctuation of the laser system. 5) When a_0 increases to 23.3, the curve of T versus Ω presents an acute peak at $\Omega=20$, which corresponds to a longer T and smaller statistical

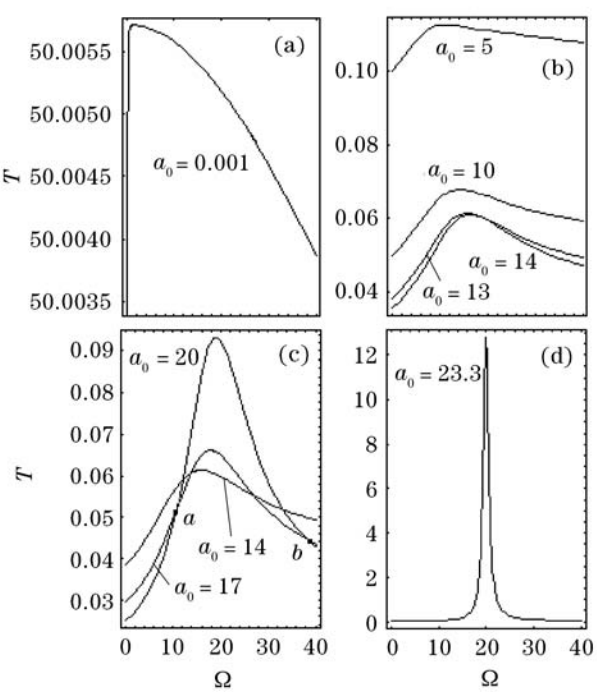

Fig. 1. Curves of T versus Ω for different values of a_0 with parameters of $A=1$, $B=20$, $Q=0.001$, $P=0.001$, $\lambda_\mathrm{q}=0.5$, $\tau=0.01$ ($\tau\ll1$).

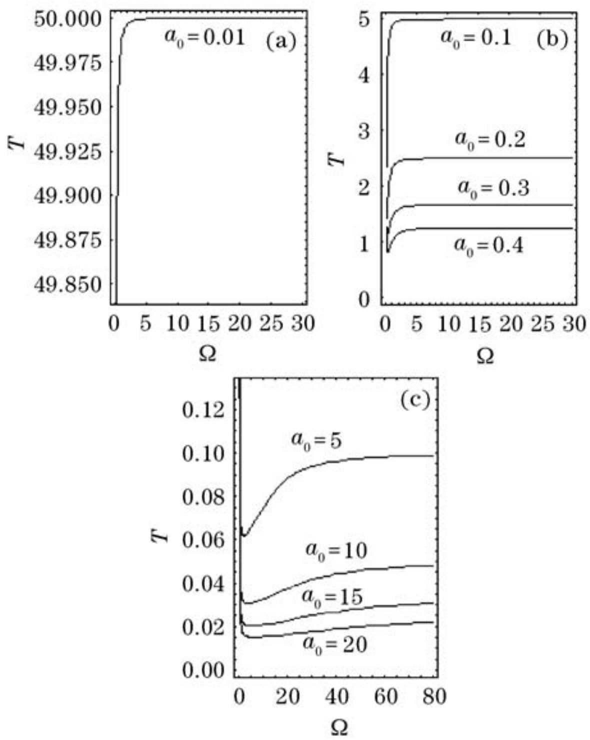

Fig. 2. Curves of T versus Ω for different values of a_0 with parameters of $A = 1$, $B = 20$, $Q = 0.001$, $P = 0.001$, $\lambda_q = 0.5$, $\tau = 100$ ($\tau \gg 1$).

fluctuation, as shown in Fig. 1(d).

On the contrary, we choose a_0 as a parameter in the case of $\tau \gg 1$ and plot the curves of T versus Ω as shown in Fig. 2. The curves show that: 1) within the range $0.01 \le a_0 < 0.3$ and with increasing Ω, T rapidly ascends and finally reaches a saturation value near $\Omega = 0$. As a_0 increases, the whole curve of T versus Ω falls. 2) Within the range $0.3 \le a_0 < 100$ and with increasing Ω, T rapidly descends to a minimum near $\Omega = 0$, then monotonically ascends and finally reaches a saturation value. As a_0 increases, the whole curve of T versus Ω falls. The above analysis reveals that when the colored correlation of pump noise is long time correlated and the frequency of the modulation signal is low, the modulation signal affects the statistical fluctuation of the laser system only. However, when the frequency of the modulation signal increases to a certain extent, the change has slight influence on the statistical fluctuation, and if the laser system works near the threshold in this instance, the change has stronger restraint effect on the statistical fluctuation of the laser system.

In summary, for the case that the colored pump noise is short time correlated, 1) when the laser system works under different values of a_0, the frequency of the modulation signal remarkably affects the statistical fluctuation of the laser system, i.e., the specific frequency of the input signal results in the maximum of T and the minimum of the statistical fluctuation of the laser system; 2) the influence of a_0 on the statistical fluctuation of the system is comparatively complicated, thus when the laser system works near the threshold, it has a stronger restraint effect on the statistical fluctuation. Generally speaking, when the laser system works far away from threshold, T of the laser system decreases, but the statistical fluctuation of the laser system increases. Moreover, if the laser system works far away from threshold in the range $14 < a_0 < 23.3$, the specific range of the frequency corresponding to T prolongs, and the statistical fluctuation of the laser system decreases.

For the case that the colored pump noise is long time correlated, 1) when the laser system works under different values of a_0, the modulation signal slightly affects the statistical fluctuation of the laser system due to a lower frequency, and when the frequency is increased to a threshold, its change also has little influence on the statistical fluctuation of the laser system; 2) the main factor affecting the statistical fluctuation of the laser system is a_0. In this instance, when the laser system works near the threshold, it has strong restraint effect on the statistical fluctuation of the laser system, and when the laser system works far away from the threshold, the statistical fluctuation augments.

Furthermore, in the cases of $\tau \gg 1$ and $\tau \ll 1$, when the laser system works near the threshold, it has strong restraint effect on the statistical fluctuation of the laser system.

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