Effects of cross-correlated noises on the intensity fluctuation of the single-mode laser system

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Received December 7, 2005

A single-mode laser model with cross-correlated additive and multiplicative noise terms is considered, and the effects of correlation between noises on the relaxation time and the intensity correlation function are studied. Using the projection operator method and taking into account the effects of the memory kernels of the intensity correlation function, the analytic expressions for the relaxation time and the correlation function are derived. Based on numerical computations, it is found that the self-correlation time and the cross-correlation time have the same effects on the single-mode laser system.

OCIS codes: 140.3570, 140.0140, 140.3430.

It is known that the single-mode dye laser model with additive white noise and multiplicative white noise is often used as a prototype to investigate the laser fluctuations[1−5]. Using this method, the previous studies have revealed that the consideration of additive and multiplicative noises simultaneously is of importance to deeply understand statistical properties of the single-mode laser system[6]. In most of these existing theoretical studies, the multiplicative noise (pump noise) and the additive noise (quantum noise) are both modeled as Gaussian white noise and are treated as uncorrelated. Recently, the effects of correlation between additive and multiplicative noises on the statistical fluctuation of the single-mode laser model have attracted close attention[7−10]. Xie *et al.*[11] considered the correlation between the two noises and studied the effects of correlation intensity λ on the relaxation time T_c and the intensity correlation function C. But that study only considered that the self-correlation time τ_0 and the cross-correlation time τ of the two noises are both zero. In this paper, we consider that τ_0 and τ are not zero and investigate the effects of τ_0 and τ on the single-mode laser system.

The complex field-amplitude E of the cubic model of a single-mode laser system can be described by Langevin equation (LE)[3],

$$\frac{\mathrm{d}E}{\mathrm{d}t} = a_0 E - A\left|E\right|^2 E + \tilde{p}(t)E + \tilde{q}(t), \tag{1}$$

where a_0 and A are real and respectively stand for the net gain and the self-saturation coefficients, $\tilde{p}(t)$ is the pump noise and $\tilde{q}(t)$ is the quantum noise. Performing the polar coordinate transform $E = x\mathrm{e}^{i\varphi}$, Eq. (1) can be transformed into two coupling LEs for the field-amplitude x and phase φ. By decoupling them, the LE of x can be obtained as[12]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a_0 x - Ax^3 + \frac{D}{2x} + xp(t) + q(t). \tag{2}$$

We only consider the intensity fluctuation of the laser system. Assuming I is the laser intensity ($I = x^2$), Eq. (2) is readily written for I as

$$\frac{\mathrm{d}I}{\mathrm{d}t} = (2a_0 - AI)I + D + 2I^{1/2}q(t) + 2Ip(t). \tag{3}$$

The multiplicative noise $p(t)$ and the additive noise $q(t)$ are considered to be Gaussian-type noise,

$$\langle q(t)q(t')\rangle = 2D\delta(t-t'), \tag{4}$$

$$\langle p(t)p(t')\rangle = \frac{Q}{\tau_0}\exp(-\left|t-t'\right|/\tau_0) \to 2Q\delta(t-t'), \tag{5}$$

and

$$\langle p(t)q(t')\rangle = \langle q(t)p(t')\rangle = \frac{\lambda\sqrt{QD}}{\tau}\exp\left[-\left|t-t'\right|/\tau\right] \to 2\lambda\sqrt{QD}\delta(t-t'), \tag{6}$$

where Q and D are the multiplicative and additive noise intensities respectively, τ_0 and τ are the self-correlation time and cross-correlation time respectively, λ is the intensity of correlation between $p(t)$ and $q(t)$. Applying the Novikov theorem[13] and the Fox's approach[14], the approximate Fokker-Planck equation corresponding to Eq. (3) reads

$$\frac{\partial P(I,t)}{\partial t} = L_{\mathrm{FP}}P(I,t), \tag{7}$$

with

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial I}F(I) + \frac{\partial^2}{\partial I^2}B(I). \tag{8}$$

The drift coefficient $F(I)$ and the diffusion coefficient $B(I)$ are given by

$$F(I) = 2(a_0 + \frac{Q}{1+2a_0\tau_0} - AI)I + \frac{3\lambda\sqrt{DQ}}{1+2a_0\tau}I^{1/2} + 2D, \tag{9}$$

$$B(I) = 2\frac{Q}{1+2a_0\tau_0}I^2 + 4\frac{\lambda\sqrt{DQ}}{1+2a_0\tau}I^{3/2} + 2DI. \tag{10}$$

It should be pointed out that the above approximate Fokker-Planck equation is valid only for the case of $1+2a_0\tau_0>0$ and $1+2a_0\tau>0$. The steady-state probability density function $P_{\rm st}(I)$ can be obtained directly from Eq. (7) as

$$P_{\rm st}(I)=N\Big(I+\frac{2\lambda(1+2a_0\tau_0)}{(1+2a_0\tau)}\sqrt{\frac{DI}{Q}}+\frac{D(1+2a_0\tau_0)}{Q}\Big)^{\alpha_1(\lambda)}\exp(\beta(I)), \quad (11)$$

where

$$\alpha_1(\lambda)=\frac{a_0(1+2a_0\tau_0)}{2Q}+\frac{DA(1+2a_0\tau_0)^2}{2Q^2}-\frac{2AD\lambda^2(1+2a_0\tau_0)}{Q^2(1+2a_0\tau)^2}-\frac{1}{2}, \quad (12)$$

$$\beta(I)=\frac{2A\lambda(1+2a_0\tau_0)^2\sqrt{DI}}{(1+2a_0\tau)Q^{3/2}}-\frac{A(1+2a_0\tau_0)}{2Q}I+\alpha_2(\lambda), \quad (13)$$

$$\alpha_2(\lambda)=\frac{\alpha_3(\lambda)}{\alpha_4(\lambda)}\times\arctan\frac{2\sqrt{I}+2\lambda(1+2a_0\tau_0)\sqrt{D}/\big((1+2a_0\tau_0)\sqrt{Q}\big)}{\alpha_4(\lambda)}, \quad (14)$$

$$\alpha_3(\lambda)=\frac{8AD^{3/2}\lambda^3(1+2a_0\tau_0)^4}{(1+2a_0\tau)^3Q^{5/2}}+\frac{2\lambda\sqrt{DQ}(1+2a_0\tau_0)}{(1+2a_0\tau)Q}-\frac{2a_0\lambda\sqrt{D}(1+2a_0\tau_0)^2}{(1+2a_0\tau)Q^{3/2}}-\alpha_5(\lambda), \quad (15)$$

$$\alpha_4(\lambda)=2\left[\frac{D(1+2a_0\tau_0)}{Q}-\frac{\lambda^2D(1+2a_0\tau_0)^2}{Q(1+2a_0\tau)^2}\right]^{1/2}, \quad (16)$$

$$\alpha_5(\lambda)=\frac{2\lambda AD^{3/2}(1+2a_0\tau_0)^3}{(1+2a_0\tau)Q^{5/2}}+2\left[\frac{2A\lambda D^{3/2}(1+2a_0\tau_0)^3}{(1+2a_0\tau)Q^{5/2}}+\frac{2\lambda\sqrt{D}(1+2a_0\tau_0)}{(1+2a_0\tau)Q^{3/2}}\right], \quad (17)$$

and N in Eq. (11) is the normalization constant.

The stationary normalized intensity correlation function of the state variable is defined by[15]

$$C(\Delta t)=\lim_{t\to\infty}\frac{\langle\delta I(t+\Delta t)\delta I(t)\rangle}{\langle(\delta I)^2\rangle}=\frac{\langle\delta I(t+\Delta t)\delta I(t)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}}. \quad (18)$$

Based on this equation, the associated relaxation time is

$$T_{\rm c}=\int_0^\infty C(\Delta t){\rm d}\Delta t. \quad (19)$$

In terms of the adjoint operator $L_{\rm FP}^+$ of the operator given by Eq. (8), $\delta I(t+\Delta t)$ can be expressed as $\delta I(t+\Delta t)=\exp(L_{\rm FP}^+\Delta t)\delta I(t)$. Thus one can rewrite Eq. (18) and get the associated Laplace transform

$$\tilde{C}(\omega)=\int_0^\infty\exp(-\omega\Delta t)C(\Delta t){\rm d}\Delta t=\frac{1}{\langle(\delta I)^2\rangle_{\rm st}}\left\langle\delta I\frac{1}{\omega-L_{\rm FP}^+}\delta I\right\rangle_{\rm st}. \quad (20)$$

Using the projection operator method used by Fujisaka and Grossmann[16] to deal with the Laplace resolvent $\omega-L_{\rm FP}^+$ in Eq. (20), we have the following continued fraction expression[15,16],

$$\tilde{C}(\omega)=\frac{1}{\omega+\mu_0+\frac{\eta_1}{\omega+\mu_1+\frac{\eta_2}{\omega+\mu_2+\cdots}}}, \quad (21)$$

in which

$$\mu_i=-\frac{\langle\delta I_iL_{\rm FP}^+\delta I_i\rangle_{\rm st}}{\langle(\delta I_i)^2\rangle_{\rm st}}, \quad (22)$$

$$\eta_i=-\frac{\langle(\delta I_i)^2\rangle_{\rm st}}{\langle(\delta I_{i-1})^2\rangle_{\rm st}}, \quad (23)$$

$$\delta I_{i+1}=J_{i+1}L_{\rm FP}^+\delta I_i, \quad (24)$$

with starting $\delta I_0=\delta I$ and $J_0=1$. The operator J_i is determined by

$$S_{i-1}=J_{i-1}-J_i=\frac{\delta I_{i-1}}{\langle(\delta I_{i-1})^2\rangle_{\rm st}}\langle\delta I_{i-1}|, \quad (25)$$

where the operator $\langle\delta I_i|$ acting on $\varphi(I)$ means the scalar product

$$\langle\delta I_i|\varphi(I)=\langle(\delta I_i\varphi(I))\rangle_{\rm st}=\int P_{\rm st}(I)\delta I_i\varphi(I){\rm d}I, \quad (26)$$

where the projection operator S_i projects $\varphi(I)$ onto the subspace associated with the variable δI_i, the projector J_i projects onto the space orthogonal to the space containing δI_i. The basic idea behind the method used to lead a continuous fraction expansion is to identify δI_i as a slow variable and in J_i space it slaves the remaining fast variables[17]. Fujisaka *et al.*[16] have pointed out that earlier experience with the Duffing oscillator or with the laser fluctuation has shown that the effects of higher orders of memory are not significant. Setting $\eta_2=0$, the first-order approximation of $T_{\rm c}$ is

$$T_{\rm c}=\left[\mu_0+\frac{\eta_1}{\mu_1}\right]^{-1}, \quad (27)$$

and the first-order approximation of $\tilde{C}(\omega)$ is

$$\tilde{C}(\omega)=\frac{\omega+\mu_1}{(\omega+\mu_0)(\omega+\mu_1)+\eta_1}, \quad (28)$$

where

$$\mu_0=\frac{\langle B(I)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}}, \quad (29)$$

$$\eta_1=\frac{\langle G(I)F'(I)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}}+\mu_0^2, \quad (30)$$

$$\mu_1 = -\frac{\left\langle G(I)\left[F'(I)\right]^2\right\rangle_{\rm st}}{\eta_1 \left\langle (\delta I)^2\right\rangle_{\rm st}} + \frac{\mu_0^3}{\eta_1} - 2\mu_0. \quad (31)$$

Performing the Laplace converse transformation of Eq. (28), we get

$$C(\Delta t) = (1+\Delta)\exp(-\alpha_- \Delta t) + \Delta \exp(-\alpha_+ \Delta t), \quad (32)$$

where

$$\Delta = \frac{\mu_1 - \alpha_-}{\alpha_+ - \alpha_-}, \quad (33)$$

$$\alpha_\pm = \frac{1}{2}\left\{\mu_0 + \mu_1 \pm \left[(\mu_1 - \mu_0)^2 - 4\eta_1\right]^{1/2}\right\}. \quad (34)$$

Let $\tau_0 = 0$ and $\tau = 0$, the above results fall back to Eqs. (18)—(20) presented in Ref. [11].

Using Eqs. (29)—(31), we can calculate μ_0, η_1, and μ_1.

The effects of τ_0 and τ on $T_{\rm c}$ (Eq. (27)) and the intensity correlation function (Eq. (32)) of the single-mode laser system can be analyzed by the numerical calculation. We only consider the positive correlation ($\lambda > 0$) in this paper. The results are plotted in Figs. 1—3 respectively.

Figure 1 shows that the curves of $T_{\rm c}$ as functions of the net gain a_0 with different values of τ_0 (Fig. 1(a)) and τ (Fig. 1(b)). We can find that $T_{\rm c}$ exhibits a single-peak with the increase of a_0. This means that with the increase of a_0, the decay rate of the intensity fluctuation in the stationary state turns over, from slowing down to speeding up. This conclusion is the same as the result in Ref. [11]. The difference only lies in the position of peak. Owning to the effects of τ_0 and τ, the position of peak is not near the threshold[11] ($a_0 = 0$) but far above the threshold. It is clear that τ_0 and τ have the same effects on $T_{\rm c}$, I.e., the larger τ_0 or τ is, the larger $T_{\rm c}$ is.

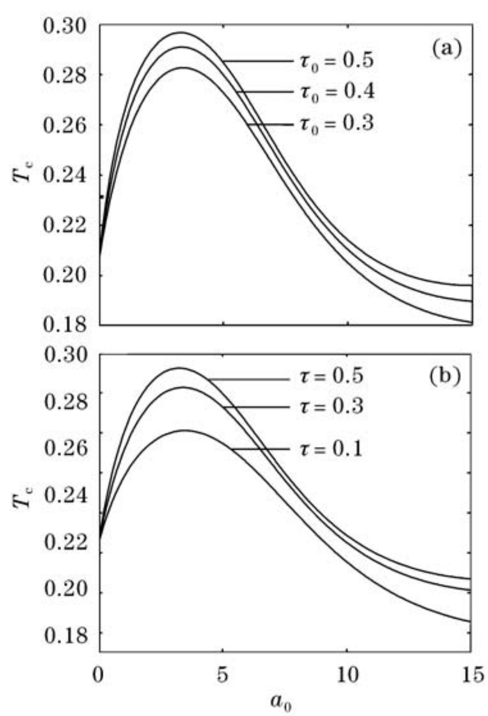

Fig. 1. Relaxation time $T_{\rm c}$ as a function of the net gain a_0. (a) $Q = 1.78$, $D = 2$, $\lambda = 0.8$, and $\tau = 0.3$, τ_0 takes 0.3, 0.4, 0.5, respectively; (b) $Q = 1.78$, $D = 2$, $\lambda = 0.8$, and $\tau_0 = 0.3$, τ takes 0.1, 0.3, 0.5, respectively.

Figure 2 shows that the curves of C as functions of a_0

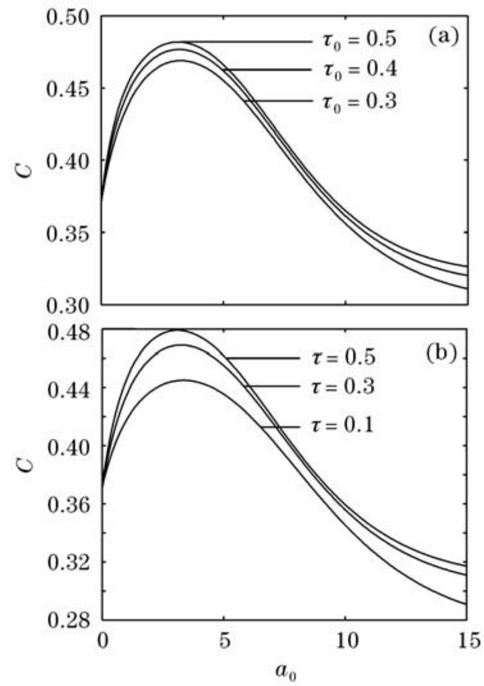

Fig. 2. Intensity correlation function C as a function of the net gain a_0. (a) $Q = 1.78$, $A = 1$, $D = 2$, $\lambda = 0.8$, and $\tau = 0.3$, τ_0 takes 0.3, 0.4, 0.5, respectively; (b) $Q = 1.78$, $A = 1$, $D = 2$, $\lambda = 0.8$, and $\tau_0 = 0.3$, τ takes 0.1, 0.3, 0.5, respectively.

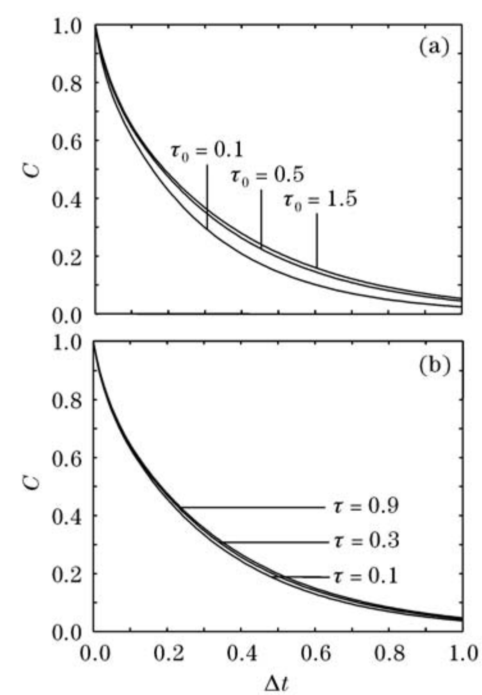

Fig. 3. Intensity correlation function C as a function of time interval Δt. (a) $Q = 1.78$, $A = 1$, $D = 2$, $\lambda = 0.5$, and $\tau = 0.5$, τ_0 takes 0.1, 0.5, 1.5, respectively; (b) $Q = 1.78$, $A = 1$, $D = 2$, $\lambda = 0.5$, and $\tau_0 = 0.5$, τ takes 0.1, 0.3, 0.9, respectively.

with different values of τ_0 (Fig. 2(a)) and τ (Fig. 2(b)). It is known that C is a measure of correlation between intensities at time t and $t + \Delta t$. As shown, the curves exhibit a peak with the increase of a_0. Far above the threshold, the correlation becomes weaker and weaker with the increase of a_0. In Fig. 2, it is obvious that τ_0 (τ) plays a positive role in C, i.e., the larger τ_0 (τ) is, the larger C is. From Figs. 1 and 2, we can find that τ_0 and τ have the same effects on the single-mode laser system, with the increase of correlation time, the decay rate of C becomes slower and slower.

As might be excepted, this result that C decays with time interval Δt (see Fig. 3), is the same as that in Ref. [11]. The effects of τ_0 and τ on C are exhibited in Fig. 3 too. That is, the larger τ_0 or τ is, the larger C is.

In conclusion, τ_0 and τ have the same effects on T_c and C. T_c and C increase with the increase of τ_0 or τ in the case of positive correlation ($\lambda > 0$). With the increase of a_0, T_c and C have a peak, the result of which is the same as that in Ref. [11]. Owning to the effects of τ_0 and τ, the position of peak is far above the threshold.

This work was supported by the National Natural Science Foundation of Yunnan Province under Grant No. 2005A002M. B. Wang's e-mail address is hnitwb@163.com.

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