Tracking speckle displacement by double Kalman filtering

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A tracking technique using two sequentially-connected Kalman filter for tracking laser speckle displacement is presented. One Kalman filter tracks temporal speckle displacement, while another Kalman filter tracks spatial speckle displacement. The temporal Kalman filter provides a prior for the spatial Kalman filter, and the spatial Kalman filter provides measurements for the temporal Kalman filter. The contribution of a prior to estimations of the spatial Kalman filter is analyzed. An optical analysis system was set up to verify the double-Kalman-filter tracker's ability of tracking laser speckle's constant displacement.

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Laser speckle technique[1] has wide-range applications such as experimental mechanics, machine control, velocimetry, electronic element package, and so on. Some of these applications involve object motion tracking, which estimates motion state evolution from noisy measurements. Kalman filters[2−4] have been applied in object tracking, such as maneuvering target tracking, for a long time. In these object-tracking applications, measurements or observations are available; however, object's motion states such as displacement, velocity and so on, are unknown. The goal is to estimate states from noise-contaminated measurements as accurately as possible. Kalman filter, which represents the problem in time field and state space, can obtain optimal estimation for linear-Gaussian environment. For arbitrary statistics, Kalman filter is the best linear estimator. Nonlinear problem could be approximately solved through extended Kalman filter[5,6] which involves a linearization process. Kalman filter is recursively Bayesian in that it obtains a prior via prediction process and then obtains posterior of states via update process.

This paper presents a speckle tracker consisting of two sequentially-connected Kalman filters. One of them estimates speckle temporal displacement; another Kalman filter estimates speckle spatial displacement. The sequentially-constructed tracker guarantees that the temporal Kalman filter provides initial parameter estimations for the spatial Kalman filter and the spatial Kalman filter provides accurate measurements for the temporal Kalman filter. Speckle displacement could be figured out through such methods as least-square[7], interpolation[8,9], fuzzy correlation[10], and so on. These methods, however, do not consider a prior which could help improving estimation precision. The double-Kalman-filter tracker considers motion dynamics as Markov process and computes speckle displacement through Kalman filtering. The double-Kalman-filter tracker, thus, could track speckle displacement accurately.

The double-Kalman-filter tracker is illustrated in Fig. 1. Speckle displacement in a subimage is estimated by a maximum likelihood method. Speckle displacement in whole image is estimated by the spatial Kalman filter which makes use of subimage's computation as measurement. The spatial Kalman filter's initial state is provided by prediction estimation produced by the temporal Kalman filter at the last time. The temporal Kalman filter makes use of measurements provided by the spatial Kalman filter to estimate speckle displacement at current time. The tracker's sequential structure makes sure that one Kalman filter can utilize another Kalman filter's outputs for a good estimation of speckle displacement. In Fig. 1, I_k and I_{k-1} are two speckle images fetched at discrete time kT and $(k-1)T$, where $k \geq 2$ and T is the time interval. $\hat{x}_k(N|Z_N)$ is the spatial Kalman filter's filtering output. $\hat{x}_k(M_k)$ and $\hat{x}_{k+1}(M_k)$ are the temporal Kalman filter's filtering output and prediction output, respectively. Z^{-1} denotes unit delay.

If I_1 denotes reference speckle image obtained before object motion, I_2 denotes comparison speckle image obtained after object motion, then I_1 and I_2 have the relationship as

$$I_1(r) = I_2(r+U), \tag{1}$$

$$I_1(r-U) = I_2(r), \tag{2}$$

where r denotes the location of a pixel in the speckle image, and U denotes speckle displacement vector. Displacements at pixel r and its neighborhood Ω_r could be assumed a constant U.

Expanding Eqs. (1) and (2) into Taylor series and neglecting the second and higher order components yield

$$I_1(r) = I_2(r) + \nabla I_2(r) \cdot U, \tag{3}$$

$$I_2(r) = I_1(r) - \nabla I_1(r) \cdot U, \tag{4}$$

where $\nabla I_1(r)$ and $\nabla I_2(r)$ are spatial gradients of two speckle images. Equations (3) and (4) can be rearranged as

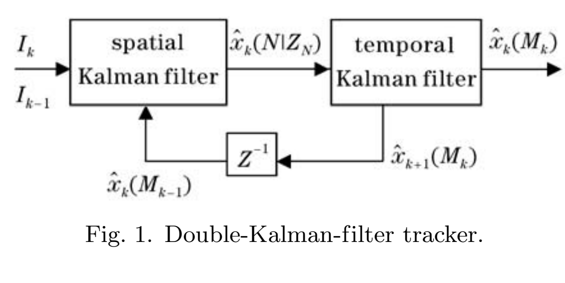

Fig. 1. Double-Kalman-filter tracker.

$$2[I_1(r) - I_2(r)] = [\nabla I_1(r) + \nabla I_2(r)] \cdot U. \tag{5}$$

Thus, Eq. (5) holds for all pixels in the subimage Ω_r of pixel at r:

$$Y = A \cdot U, \tag{6}$$

where $Y = 2[I_1(r_1) - I_2(r_1), \cdots, I_1(r_N) - I_2(r_N)]$, $A = [\nabla I_1(r_1) + \nabla I_2(r_1), \cdots, \nabla I_1(r_N) + \nabla I_2(r_N)]$.

Equation (6) being an over-determined linear system, a random error variable is added into it, yielding

$$Y = A \cdot U + v. \tag{7}$$

Assuming v is normal, so that its probabilistic density is

$$f_V(v) = \frac{1}{2\pi\sqrt{|R|}} \exp\{-\frac{1}{2} v^{\mathrm{T}} R^{-1} v\}, \tag{8}$$

where covariance $R = E\{(v - \bar{v})^{\mathrm{T}}(v - \bar{v})\}$, $\bar{v}$ being mean value of v.

Given data set Y, the likelihood function using Eq. (7) is

$$\begin{aligned} f_{Y/U}(Y/U) &= f_V(Y - AU) \\ &= \frac{1}{2\pi\sqrt{|R|}} \exp\{-\frac{1}{2}(Y - AU)^{\mathrm{T}} R^{-1} (Y - AU)\}. \end{aligned} \tag{9}$$

To maximize the likelihood function $f_{Y/U}(Y/U)$, we can equivalently maximize $\ln(f_{Y/U}(Y/U))$, or minimize

$$J = \frac{1}{2}(Y - AU)^{\mathrm{T}} R^{-1} (Y - AU). \tag{10}$$

Differentiating $\frac{\partial J}{\partial U} = A^{\mathrm{T}} R^{-1} (Y - AU) = 0$, so that the maximum likelihood estimation of U is given by[5]

$$\hat{U}_{\mathrm{ML}} = (A^{\mathrm{T}} R^{-1} A)^{-1} A^{\mathrm{T}} R^{-1} Y, \tag{11}$$

with the estimation error covariance $P = (A^{\mathrm{T}} R^{-1} A)^{-1}$.

At time kT, an image is divided into N subimages, as shown in Fig. 2. Each subimage computes its speckle displacement $z_k(n)$ by using Eq. (11), $n = 1, 2, \cdots, N$ being the subimage index. The spatial Kalman filter makes use of $Z_N = \{z_k(n), n = 1, 2, \cdots, N\}$ as measurements to estimate speckle displacement $x_k(n)$ of subimage n and its output $\hat{x}_k(N|Z_N)$ will be fed into the temporal Kalman filter. Assuming that whole speckle images translated from an equal displacement satisfy $x_k(1) = x_k(2) = \cdots = x_k(N)$ so that

$$\begin{cases} x_k(n+1) = x_k(n) \\ z_k(n) = x_k(n) + u_k(n) + v_k(n) \end{cases}. \tag{12}$$

For subimage n, $v_k(n)$ is its computation error which is treated as measurement noise. Assuming $E[v_k(i)v_k(j)] = r\delta(i - j)$ and $E[v_k(n)] = 0$, $\delta(x)$ is Kronecker delta function, $u_k(n)$ is some deterministic term. In scalar case, the spatial Kalman filter is

$$\begin{aligned} \hat{x}_k(n+1) &= \hat{x}_k(n) \\ &+ \frac{p_k(n+1)}{r}[z_k(n+1) - u_k(n) - \hat{x}_k(n)], \end{aligned} \tag{13}$$

with $p_k(n) = \frac{p_k(0)}{1 + k(p_k(0)/r)}$ and the initial value $\hat{x}_k(0)$. Here, $p_k(0)$ is variance of initial state estimation, r being measurement variance. An equivalent equation of Eq. (13) can be obtained by induction as

$$\begin{aligned} \hat{x}_k(n) &= \frac{1}{1 + np_k(0)/r} \\ &\times [\hat{x}_k(0) + (p_k(0)/r) \sum_{i=1}^{n} [z_k(i) - u_k(n)]]. \end{aligned} \tag{14}$$

From this equation, we can see that $\hat{x}_k(n)$ consists of two parts one of which comes from a prior $\hat{x}_k(0)$ and another comes from measurements $z_k(i)$, $i = 1, \cdots, n$. The ratio $p_k(0)/r$ decides how much two parts contribute to the estimation. Figure 3 indicates that if $p_k(0)/r$ is small then $\hat{x}_k(n) \approx \frac{1}{1+np_k(0)/r}\hat{x}_k(0)$, otherwise $\hat{x}_k(n) \approx \frac{p_k(0)/r}{1+np_k(0)/r} \sum_{i=1}^{n} [z_k(i) - u_k(n)]$.

Another temporal Kalman filter is used to track speckle displacement with time going on. Because all subimages translated from the same displacement at time kT, satisfy $x_k(1) = x_k(2) = \cdots = x_k(N) = x_k$, the process equation and measurement equation can be given as

$$\begin{cases} x_{k+1} = f(x_k) + w_k \\ m_k = x_k + \varepsilon_k \end{cases}, \tag{15}$$

where m_k is the spatial Kalman filter's output $\hat{x}_k(N|Z_N)$, ε_k is $\hat{x}_k(N|Z_N)$'s estimation error, and w_k is process noise. The function $f(\cdot)$ is speckle temporal

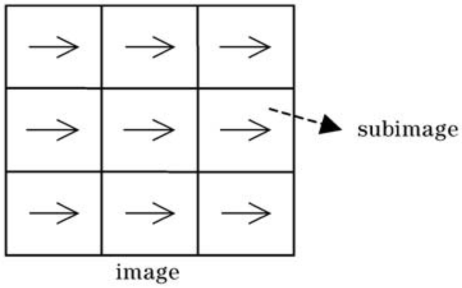

Fig. 2. A speckle image is divided into subimages which all translate equal displacement.

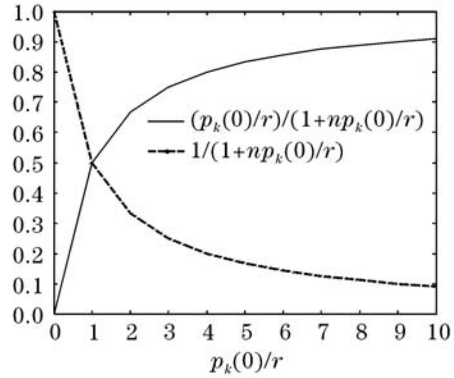

Fig. 3. Coefficients in Eq. (14) changing with the parameter $p_k(0)/r$.

dynamic model. Assuming $E[\varepsilon_i w_j] = 0$, $E[w_i w_j] = \sigma_w^2 \delta(i-j)$, $E[\varepsilon_i \varepsilon_j] = \sigma_\varepsilon^2 \delta(i-j)$, $E[\varepsilon_k] = 0$, and $E[w_k] = 0$. If the function $f(\cdot)$ is not linear, an extended Kalman filter[5,6] could be applied.

By unit delay, the temporal Kalman filter's prediction estimation of speckle displacement $\hat{x}_{k+1}(M_k)$ (where $M_k = \{m_i, i = 2, \cdots, k\}$) is fed back to the spatial Kalman filter as a prior $\hat{x}_{k+1}(0) = \hat{x}_{k+1}(M_k)$. The temporal Kalman filter's filtering estimate of speckle displacement $\hat{x}_k(M_k)$ is as the tracker's outputs.

For verification of the double-Kalman-filter tracker, an optical analysis system was set up, as shown in Fig. 4. The light beam emitted from a laser diode with 633-nm wavelength and 10-mW output power was diffusely reflected by a solid minute-roughness surface. The object was mounted on an X-Y stage and was subjected to translation with stepping motors controlled by a microcomputer by which a very small displacement of the object could be conducted in any direction. The minimum speckle displacement is 0.1 μm. The reflected lights were captured by a charge coupled device (CCD) camera forming digitalized speckle image. The CCD camera has 512×512 pixels; pixel size is 7.5×7.5 (μm) and fill factor is 100%. While the surface was accurately conducted to do two-dimensional (2D) translation by the controller, a sequence of speckle images were captured at regular time by an image grabber and stored for further analysis using a microcomputer.

Object was conducted to do constant displacement in experiment. The target could be given by

$$\begin{cases} x_{k+1} = x_k + w_k \\ m_k = x_k + \varepsilon_k \end{cases} . \tag{16}$$

From discrete algebraic Racatti equation (DARE) of Eq. (16), we can obtain

$$P_{\hat{x}}^2 + \sigma_w^2 (P_{\hat{x}}^2 - \sigma_\varepsilon^2) = 0, \tag{17}$$

where $P_{\hat{x}}$ is the variance of the steady-state filtering estimation error of x_k. Assuming $E[w_i w_j] = \sigma_w^2 \delta(i-j)$, $E[\varepsilon_i \varepsilon_j] = \sigma_\varepsilon^2 \delta(i-j)$, the solution of Eq. (17) is

$$\frac{P_{\hat{x}}}{\sigma_\varepsilon^2} = \frac{2}{\sqrt{1+4h^2}+1}, \tag{18}$$

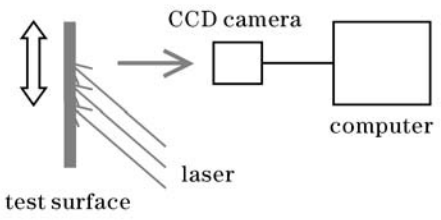

Fig. 4. Schematic diagram of the experimental setup.

where $h = \frac{\sigma_\varepsilon}{\sigma_w}$.

It is concluded from Eq. (18) that $P_{\hat{x}}$ must be less than σ_ε^2 if $\sigma_w \neq 0$. Figure 5 shows two Kalman filters' outputs $\hat{x}_k(N|Z_N)$ and $\hat{x}_k(M_k)$ while tracking temporal speckle constant displacement 1.5 μm. Due to its reduced variance, temporal Kalman filtering outputs fluctuate less than spatial Kalman filtering outputs.

Mean square root (RMS) of estimation error is used to indicate tracking accuracy,

$$\mathrm{RMS} = \sqrt{\frac{1}{N}\sum_i^N (\hat{x}_i - x_i)^2}. \tag{19}$$

The interest area is 25×25 pixels. The image is divided into subimages whose sizes are all 10×10 pixels. Subimages are partially overlapped and the total number of subimages is 10. The ratio $p_k(0)/r$ is about 2.

Table 1 lists two trackers' experimental results in terms of RMS, Table 2 is counterpart of Table 1 with unit being pixel. Tracker 1 is the proposed double-Kalman-filter tracker; tracker 2 employs Eq. (11) for whole speckle image with R being identity. Two numbers in every parenthesis are speckle displacements in X direction and Y direction of the Cartesian coordinate system as illustrated in Fig. 6. For every speckle displacement, all experiment results are from data after conducting five run and thirty times per run tracking. The initial state of the temporal Kalman filter is set to (0,0) for every tracking.

It is indicated from experimental results listed in Tables 1 and 2 that double-Kalman-filter tracker could accurately track constant distance speckle translation. There are some comments on these results: 1) from Kalman theory, the standard deviation of filtering estimation error is always less than that of measurement noise; 2) the

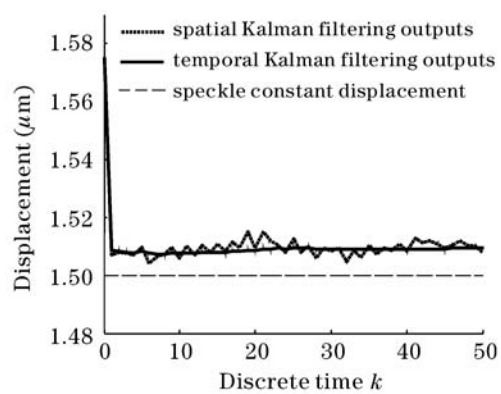

Fig. 5. Kalman filtering outputs (the initial displacement estimation is (1.575,1.575) μm).

Table 1. RMS of Two Trackers Tracking Speckle Constant Displacement I (Unit: μm)

Actual Displacement	(0.6,0.6)	(1.5,1.5)	(3,3)	(4.5,4.5)	(6,6)	(7.5,7.5)	Average
RMS of Tracker 1	(0.0270, 0.0248)	(0.0217, 0.0210)	(0.0398, 0.0412)	(0.0570, 0.0607)	(0.0772, 0.0833)	(0.0960, 0.1020)	(0.0531, 0.0555)
RMS of Tracker 2	(0.0630, 0.0510)	(0.1762, 0.0743)	(0.1545, 0.0375)	(0.0953, 0.0555)	(0.1830, 0.1358)	(0.1267, 0.1417)	(0.1331, 0.0826)

Table 2. RMS of Two Trackers Tracking Speckle Constant Displacement II (Unit: pixels)

Actual Displacement	(0.08,0.08)	(0.2,0.2)	(0.4,0.4)	(0.6,0.6)	(0.8,0.8)	(1.0,1.0)	Average
RMS of Tracker 1	(0.0036, 0.0033)	(0.0029, 0.0028)	(0.0053, 0.0055)	(0.0076, 0.0081)	(0.0103, 0.0111)	(0.0128, 0.0136)	(0.0071, 0.0074)
RMS of Tracker 2	(0.0084, 0.0068)	(0.0235, 0.0099)	(0.0206, 0.0050)	(0.0127, 0.0074)	(0.0244, 0.0181)	(0.0169, 0.0189)	(0.0178, 0.0110)

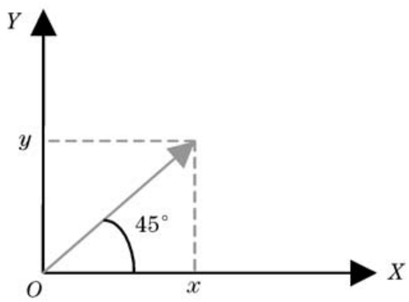

Fig. 6. Speckle displacement: (x, y).

variance of states at initial time should be set to an appropriate number for the spatial Kalman filter so that the prediction estimation of state of the temporal Kalman filter at the last time contributes to the spatial Kalman filter's state estimation appropriately. This setting could depend on the confidence on accuracy of the applied system model; 3) the term $u_k(n)$ in Eq. (12) is used to depress the bias. $u_k(n)$, a factor relevant to tracking performance in terms of RMS, could be obtained through some statistical way.

In conclusion, a method to track laser speckle displacement is presented. A maximum likelihood method is used to compute subpixel speckle displacement. The displacement computation is then filtered by the spatial Kalman filter, which is set up through dividing image into subimages and employing a simple, approximation-to-reality model. The spatial Kalman filter and the temporal Kalman filter are sequentially connected together. The initial state estimation $\hat{x}(0)$ of the spatial Kalman filter is provided by the temporal Kalman filter. The contribution of $\hat{x}(0)$ to estimation of the spatial Kalman filter depends on the ration of $\hat{p}_0$ to r, $\hat{p}_0$ being variance of initial state estimation, r being measurement variance. Tracking laser speckle's constant displacement was conducted using an optical analysis system. The experimental results verified the double-Kalman-filter tracker.

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