Variation analysis of turbulence resistance and angular spreading for partially coherent beam in turbulence

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A new factor M is proposed to characterize the similarity of the behavior of a partially coherent beam (PCB) to its coherent counterpart when propagating through atmospheric turbulence. It is shown that there exists a boundary in the range of the source coherent length. The decreasing rates of free space angular spreading and of turbulence distance with the source coherent length are different before and after the coherent length arriving at the boundary value, by which the change trend of M can be concluded.

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It is well known that partially coherent beam (PCB) is less sensitive to the effects of turbulence than fully coherent ones[1−8]. Among those works, Gbur *et al.* gave the generalized expression for spreading of PCBs in atmosphere[4]. Their results have shown that PCB has a larger free space angular spreading θ_{S} in addition to its robust turbulence resistance that is characterized by turbulence distance z_{T}. For PCB, the longer z_{T} and the smaller θ_{S} are desired for applications. However, it seems that both parameters of θ_{S} and z_{T} have the same change trend with the variation of the source coherence, which makes one hard to obtain the desired beam. To trade-off z_{T} and θ_{S}, one firstly should determine change trends of z_{T} and θ_{S} along with the variation of the source coherent length σ_μ.

In this letter we introduce a factor M to describe the similarity of the behavior of PCB to its coherent laser counterpart. It can also be used to evaluate the trends of variations of the two parameters with increasing σ_μ. We find that there exists a boundary in the curve of M. Both the changes of z_{T} and θ_{S} differ in their velocities before and after the value of M arrives at the boundary.

We consider a quasi-monochromatic field propagating from the source plane $z = 0$ into the turbulent half-space $z > 0$. By using the paraxial approximate form of the extended Huygens-Fresnel principle[9,10], the field at a given plane can be expressed as

$$U(\boldsymbol{\rho}, z) = -\frac{ik\exp(ikz)}{2\pi z}\iint U_0(\boldsymbol{\rho}_0)\exp[ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2}{2z}] \times\exp[\psi(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z)]\mathrm{d}^2\rho_0, \quad (1)$$

where $U_0(\boldsymbol{\rho}_0)$ denotes the incident field at the source plane and $U(\boldsymbol{\rho}, z)$ is the filed at a given z plane ($z > 0$). The quantity $\psi(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z)$ denotes a phase function that depends on the atmosphere turbulence and $k = 2\pi/\lambda$ is the wave number. It is assumed that the source is statistically stationary and the medium is statistically homogenous and isotropic. Then the spectral density of the beam can be given by[5]

$$\begin{aligned} I(\boldsymbol{\rho}, z) &= \langle U^*(\boldsymbol{\rho}, z)U(\boldsymbol{\rho}, z)\rangle \\ &= (\frac{k}{2\pi z})^2\iint \mathrm{d}^2\rho_0 \iint \mathrm{d}^2\rho_0' \langle U_0^*(\boldsymbol{\rho}_0)U(\boldsymbol{\rho}_0')\rangle \\ &\times\exp\left[-ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2-(\boldsymbol{\rho}-\boldsymbol{\rho}_0')^2}{2z}\right] \\ &\times\langle\exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z)+\psi(\boldsymbol{\rho}, \boldsymbol{\rho}_0', z)]\rangle, \end{aligned} \quad (2)$$

where the two angle brackets on the right side represent the averages over the filed ensemble and over the ensemble of the turbulent medium, respectively, and the asterisk denotes the complex conjugate. The first average is the cross-spectral density function of the incident field[11] and the second one introduces the influence of the turbulence which can be expressed as

$$\begin{aligned} &\langle\exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z)+\psi(\boldsymbol{\rho}, \boldsymbol{\rho}_0', z)]\rangle \\ &= \exp\{-4\pi^2k^2z\int_0^1\int_0^\infty \kappa\Phi_n(\kappa) \\ &\times[1-J_0(\kappa\xi|\boldsymbol{\rho}_0-\boldsymbol{\rho}_0'|)]\mathrm{d}\kappa\mathrm{d}\xi\}, \end{aligned} \quad (3)$$

where Φ_n is the spatial power spectrum of the refractive-index fluctuations of the turbulent medium and J_0 is the Bessel function of the first kind and zero order.

The expression of the mean square width of the beam can be given as[4]

$$\overline{\rho^2(z)} = \sigma_I^2 + \sigma_J^2\cdot z^2 + F_2\cdot z^3. \quad (4)$$

It can be seen from Eq. (4) that the beam spreading in turbulence is composed of three terms. The quantity σ_I in the first term is the normalized root-mean-square (RMS) width of the intensity in the source plane, which characterizes the effective source size[11], and the quantity σ_J in the second term represents the normalized RMS width of the power J radiated by the source per unit solid angle, which is a measure of the angular spread of the beam in free space. Both of them represent the diffractive spreading of the PCB in free space, while the third term shows the turbulence contribution. The quantity F_2 describes the influence of turbulence.

According to the Eq. (4), one can study the beam spreading behavior in atmosphere turbulence. Gbur *et al.*[4] have introduced a quantity $z_{\rm T}$ to quantify the effect of the turbulence, which is defined as the distance at which the spreading due to the turbulent medium accounts for 10% of the magnitude of $\overline{\rho^2(z)}$

$$\frac{\overline{\rho^2(z_{\rm T})}_{\rm turb} - \overline{\rho^2(z_{\rm T})}_{\rm free}}{\overline{\rho^2(z_{\rm T})}_{\rm turb}} = \frac{1}{10}, \tag{5}$$

where $\overline{\rho^2(z_{\rm T})}_{\rm turb}$ is given by the Eq. (4) and $\overline{\rho^2(z_{\rm T})}_{\rm free}$ is given by the first two terms of the Eq. (4). In order to determine the value of $z_{\rm T}$, simplifications have been made by neglecting either the first term or the second term, respectively, for the different kinds of turbulences with two different scales. Their results only provide an approximation to the real case. Here we give the complete solution of Eq. (5) in $z_{\rm T}$ by employing the mathematical manipulations for cubic equation and the Cardano formula, which is expressed in the form

$$z_{\rm T} = \sqrt[3]{-(\frac{b^3}{27a^3} + \frac{d}{2a}) + \sqrt{\frac{b^3 d}{27a^4} + \frac{d^2}{4a^2}}} + \sqrt[3]{-(\frac{b^3}{27a^3} + \frac{d}{2a}) - \sqrt{\frac{b^3 d}{27a^4} + \frac{d^2}{4a^2}}} - \frac{b}{3a}, \tag{6}$$

with

$$a = 9F_2, \quad b = -\sigma_J^2, \quad d = -\sigma_I^2,$$

where a, b, and d are just the mathematical symbols that appear in the calculation. For the beams generated by Gaussian Schell-model sources[11] and the Tatarskii spectrum of the turbulence model[9], one has

$$\sigma_I^2 = \frac{\omega_0^2}{2},\ \sigma_J^2 = \frac{2}{k^2}(\frac{1}{\omega_0^2} + \frac{1}{\sigma_\mu^2}),\ F_2 = 1.095 C_n^2 l_0^{-1/3}, \tag{7}$$

where ω_0 is the minimum spot size and σ_μ is the source coherence length that characterizes the effective spectral coherence width of the source[11]. The quantities C_n^2 and l_0 are the structure parameter of the index of refraction and the inner scale of turbulence, whose typical values are 10^{-14} m$^{-2/3}$ and 0.01 m, respectively[10]. We use $\theta_{\rm S}$ to describe the angular spreading in free space replacing the quantity σ_J in the remainder of this paper because the former is easy to be understood.

For a given effective source size σ_I, both $z_{\rm T}$ and $\theta_{\rm S}$ decrease with increasing σ_μ. To characterize the both parameters, we introduce a new factor M to describe the similarity between the behaviors of the PCB propagating through turbulence and that of the fully spatially coherent beam, which is defined as

$$M = \frac{\theta_{\rm S}/z_{\rm T}}{\theta_{\rm SC}/z_{\rm TC}}, \tag{8}$$

where the numerator $\theta_{\rm S}/z_{\rm T}$ represents the ratio of $\theta_{\rm S}$ to $z_{\rm T}$ for the PCB while the denominator $\theta_{\rm SC}/z_{\rm TC}$ denotes such ratio for the fully spatially coherent beam. The quantities $\theta_{\rm SC}$ and $z_{\rm TC}$ denote the free space angular spreading and turbulence resistance for a fully spatially coherent beam, respectively. The factor M can also be written as

$$M = \frac{\theta_{\rm S}}{\theta_{\rm SC}} \cdot \frac{z_{\rm TC}}{z_{\rm T}} = \frac{M_1}{M_2}, \tag{9}$$

in which $M_1 = \theta_{\rm S}/\theta_{\rm SC}$ and $M_2 = z_{\rm T}/z_{\rm TC}$ respectively represent the ratios of $\theta_{\rm S}$ and $z_{\rm T}$ for the PCB to its counterpart for a coherent laser beam. We plot the curves of variations of M_1, M_2, and M with various σ_μ as shown in Fig. 1. The source and conditional parameters used in calculation are $\lambda = 628$ nm, $\omega_0 = 0.01$ m, $C_n^2 = 10^{-14}$ m$^{-2/3}$, and $l_0 = 0.01$ m, and the value of σ_μ is ranged from 0.001 to 0.05 m.

Evidently, it can be shown that there are two limiting cases. One is the case of σ_μ closing to infinity, which represents a spatially fully coherent source, while the other is the case of σ_μ approaching to zero that means a spatially incoherent source[11]. One can see from Fig. 1 that firstly the value of M_1 is much greater than unity, that is to say, $\theta_{\rm S}$ for PCB is much larger than its coherent counterpart $\theta_{\rm SC}$. The value of M_1 decreases rapidly with increasing value of σ_μ, but falls off slowly when the value of σ_μ is over about 0.015 m, which means that the behavior of the GSM beam is closing to its coherent limiting case. Finally we get $\theta_{\rm S} \approx \theta_{\rm SC}$ and the value of M_1 equals to unity when the value of σ_μ equal to or is much greater than that of σ_I.

The change of M_2 indicated by Fig. 2 is similar to that of M_1. Since the PCB has longer $z_{\rm T}$ than that of the coherent laser beam, the value of M_2 is also greater than unity. The value of M_2 decreases rapidly with the value of σ_μ increasing, but drops slowly when the value of σ_μ is over about 0.015 m. This phenomenon arises from the fact that with the increase of coherent length the PCB with higher coherence is less stable in turbulence than one with lower coherence[6], which leads to a shorter $z_{\rm T}$. The value of M_2 is closing to unity when the coherent length is arriving at its coherent limiting case.

Figure 3 shows the change of M that includes the information of both M_1 and M_2. It is shown that the value of M increases rapidly with increasing σ_μ, starting to decrease very slowly and finally approaches to unity.

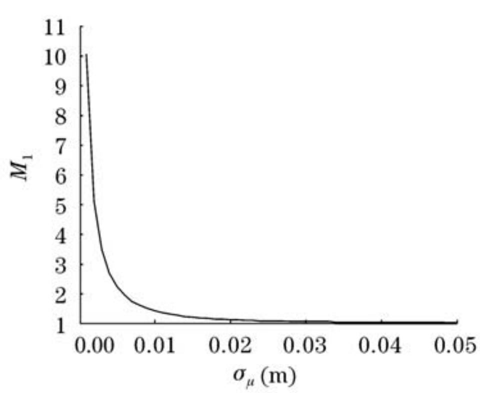

Fig. 1. Variation of M_1 with source coherence length σ_μ.

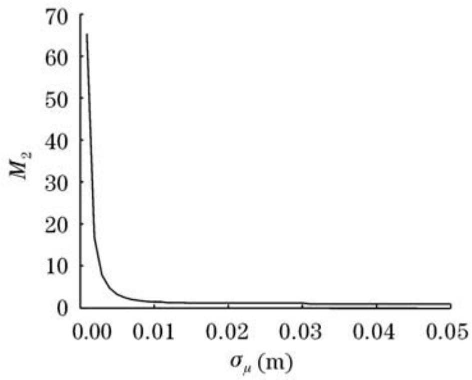

Fig. 2. Variation of M_2 with source coherence length σ_μ.

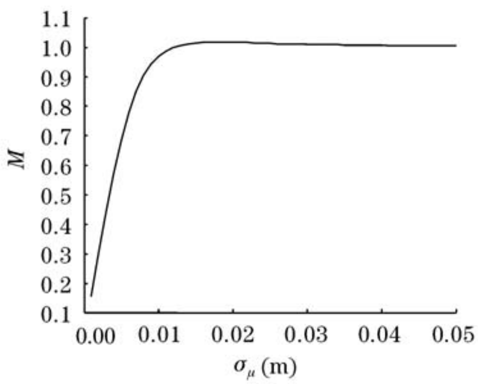

Fig. 3. Variation of M with source coherence length σ_μ.

After that the value of M keeps at unity regardless of the further increment of σ_μ. Let us analyze why the change of M differs from that of M_1 and M_2. As one can see that both θ_S and z_T for the PCB drop with increasing σ_μ. However, the descending rate of θ_S is faster than that of z_T, which leads to increase numerator on the right side of Eq. (9) and further causes the increment of M. However, when the value of σ_μ is arriving at 0.012 m, the ratio of θ_S to z_T is closing to its coherent counterpart and the value of M approximately equals to unity. That means the behavior of such PCB is similar to a spatially coherent laser. Nevertheless, at this time both parameters still continuously descend, in which the only difference is just that θ_S keeps its descending rate while the descending rate of z_T becomes slow that can be seen from Figs. 1 and 2. Therefore, the value of M keeps increasing and exceeds unity that is the ideal value of M. When the value of σ_μ arrives at about 0.018 m, the factor M has its maximum value of about 1.015. After that, the descending rate of θ_S becomes slow and is gradually synchronized with that of z_T. Then the value of θ_S/z_T begins to decrease and further the value of M starts to descend though its value is still greater than unity. When σ_μ is enough large for the PCB to be considered as a spatially fully coherent laser, the values of θ_S and z_T are approximately equal to those of θ_{SC} and z_{TC}, then the value of M equals to unity.

In summary, the factor M we defined can be used to describe the trends of the variations of θ_S and z_T for PCB case with the various source coherent lengths. One can see from the above analysis that there exists a boundary that is $\sigma_\mu = 0.018$ m for given effective source size $\sigma_I = 0.007$ m and particulate turbulence spectrum Φ_n. With increasing σ_μ, the decreasing rate of θ_S is generally faster than that of z_T when the value of σ_μ is less than 0.018 m, i.e., $S_{\theta_S} > S_{z_T}$, in which S_{θ_S} and S_{z_T} represent the decreasing rates of θ_S and z_T with increasing σ_μ, respectively. When the value of σ_μ is greater than 0.018 m, S_{θ_S} becomes slower and gradually approaching to S_{z_T}, and finally one gets $S_{\theta_S} \approx S_{z_T}$. Then the value of M equals to unity and the beam can be treated as a spatially fully coherent laser beam. The results we obtained are just under such circumstances as given effective source size and particulate turbulence spectrum as well as some fixed corresponding parameters in the calculation on influence of turbulence. Changing source and turbulence parameters may give different boundaries of σ_μ. They characterize the same interesting phenomena on the PCB source in despite of possible different values of boundary. The result of this paper can be helpful for practical applications when a trade-off between the robust z_T and the smaller θ_S for PCB is required.

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