Novel algorithm for simultaneously detecting multiple vapor materials with multiple-wavelength differential absorption lidar

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Differential absorption lidar (DIAL) has been successfully used to detect vapor material, however limited to detect single vapor using two closely spaced wavelengths. The progress in multiple-wavelength lasers motivates the need for detection and estimation algorithms that have the capability for simultaneous detection of multiple materials. In this paper, a simple and accurate algorithm is presented for simultaneously detecting and estimating multiple vapor materials with multiple-wavelength DIAL, which based on the maximum likelihood estimation (MLE) methodology. The performance of the algorithm is evaluated by simulation experiments, the results show that this algorithm can separately identify and quantify vapor material in mixtures and perform quite well.

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Differential absorption lidar (DIAL) has been extensively used to detect chemical vapors and trace atmospheric constituents[1,2]. The recent development in laser tuning technology has made a single laser be rapidly tuned to dozens of wavelengths in a very short period of time[3]. These multiple wavelength lasers offer the ability of simultaneously detecting multiple chemical vapors, which are impossible for conventional two-line DIAL and permit a more thorough spectral probing of the atmosphere.

Warren[4,5] generalized two wavelengths signal-material DIAL to multiple wavelengths in simultaneous multiple materials detection and discrimination. Although He presented an approach for generalizing two wavelengths or more than two wavelengths, this approach has remained tied to the DIAL paradigm of computing the ratios of wavelength return pairs[4]. The fundamental assumption underlying two- or few-wavelength DIAL is that, for sufficiently closely spaced wavelengths, the target reflectance and lidar system response should be approximately constant, and therefore ratioing of return signals should remove these usually unknown parameters. Certainly the DIAL assumption of similar background reflectivity and system response is in general not valid over the extended spectral ranges available from multiple wavelength lasers. Warren has proposed a more complicated method of detection and estimation multi-material (path-integrated) concentration-length product, *CL*, based on likelihood ratio test methodology which directly utilizes the return signal of each wavelength[5]. In order to solve *CL*, the iterative Newton-Raphson method is used to produce numerical solutions in each set of wavelength. This procedure produces a well result if implemented carefully, but takes much longer time than the simple matrix multiplication required in Ref. [4]. Moreover, care is required because that the iterative procedure is very sensitive to initial offsets from differential atmospheric transmission; and the iterative algorithm then produces estimation for *CL* that is very sensitive to noise details[6].

In this paper, a new algorithm based on maximum likelihood estimation (MLE) methodology of multivariate statistical inference theory is proposed, which can solve simultaneously the background absorption, noise covariance matrix, and *CL* of multiple vapors only using the return signal of each wavelength and the absorptivity of detected vapors. Moreover, the algorithm solves *CL* without any iterative method in each set of wavelength, so this algorithm is simple and careless, but it produces a quite well result.

Now, we show how the MLE methodology can be used to develop algorithms for detecting and estimating multiple vapor materials using long-range multiple wavelength DIAL.

Under the vapor-absent hypothesis H_0, the return signal is presented simply as

$$P\left(i,j\right)=G+n_P\left(i,j\right), \tag{1}$$

where i $(i=1,\cdots,N)$ denotes a set of wavelengths, and each set has M wavelengths, indexed by j $(j=1,\cdots,M)$, G denotes an unknown lidar return signal from the topographic target, and $n_P\left(i,j\right)$ describes the zero-mean shot-to-shot fluctuation in the received signal. We take n_P to be normally distributed and independent,

$$\Lambda_P\left(i,j,i',j'\right)\equiv E\left[n_P\left(i,j\right)n_P\left(i',j'\right)\right]=\Lambda\left(j,j'\right)\delta_{ii'}, \tag{2}$$

where E denotes expectation, Λ the wavelength covariance, and $\delta_{ii'}$ the Kronecker delta. The unknown parameter G is, of course, a function of range and lidar system parameters, but those dependencies are irrelevant in the present context.

We set $Q\left(i,j\right)=\frac{1}{2}\ln P\left(i,j\right)$, $H=\frac{1}{2}\ln G$, thus, Eq. (1) becomes

$$Q\left(i,j\right)=H+n_Q\left(i,j\right). \tag{3}$$

The covariance of n_Q is given by

$$\Lambda_Q\left(i,j,i',j'\right) = \frac{1}{4}E\left\{\ln\left[1+\frac{n_P\left(i,j\right)}{G}\right] \times \ln\left[1+\frac{n_P\left(i',j'\right)}{G}\right]\right\}. \tag{4}$$

When the signal-to-noise ratio (SNR) of the system is large enough,

$$\Lambda_Q\left(i,j,i',j'\right) \approx \frac{1}{4G^2}\Lambda\left(j,j'\right)\delta_{ii'}. \tag{5}$$

From the model assumption above, the probability density of Q under H_0 is multivariate normal distribution with mean H and block diagonal covariance $\Lambda_Q \otimes I_N$, where $\otimes$ denotes the Kronecker outer product and I_N is the N-dimensional identity matrix,

$$f_0\left(Q\left|H,\Lambda_Q\right.\right) = \prod_{i=1}^{N}\left(2\pi\right)^{-\frac{M}{2}}\left|\Lambda_Q\right|^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\sum_{j,j'=1}^{M}\left[Q\left(i,j\right)-H\right]\Lambda_Q^{-1}\left(j,j'\right)\left[Q\left(i,j'\right)-H\right]\right\}. \tag{6}$$

Denoting by $\hat{H}_0$ and $\hat{\Lambda}_0$ the MLEs of H and Λ_Q under H_0, computing the derivative of $\ln f_0$ using Eq. (6) produces the results

$$\hat{H}_0 = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}Q\left(i,j\right), \tag{7}$$

$$\hat{\Lambda}_0\left(j,j'\right) = \frac{1}{N}\sum_{i=1}^{N}\left[Q\left(i,j\right)-\hat{H}_0\right]\left[Q\left(i,j'\right)-\hat{H}_0\right]. \tag{8}$$

Under the vapor-present hypothesis H_1, the data are assumed to be given by a transmission-modulated version of Eq. (1),

$$P\left(i,j\right) = G\exp\left(-2\sum_{l=1}^{L}\alpha_{jl}CL_l\right) + n_P\left(i,j\right), \tag{9}$$

where L is the number of simultaneous vapor materials, α_{jl} is the absorptivity of the lth material at wavelength index j (taken to be known) and CL_l is the path-integrated concentration of vapor material l between the lidar and topographic target.

After simple transform similar to Eq. (3), we have

$$Q\left(i,j\right) = H - \sum_{l=1}^{L}\alpha_{jl}CL_l + n_Q\left(i,j\right). \tag{10}$$

When the SNR is large enough, the covariance of n_Q is written as

$$\Lambda_Q\left(i,j,i',j'\right) \approx \frac{1}{4G^2\exp\left(-4\sum\limits_{l=1}^{L}\alpha_{jl}CL_l\right)}\Lambda\left(j,j'\right)\delta_{ii'}. \tag{11}$$

In additional to H and Λ_Q, the probability density of Q under H_1 depends on the unknown CL vector, giving

$$f_1\left(Q\left|H,\Lambda_Q,CL\right.\right) = \prod_{i=1}^{N}\left(2\pi\right)^{-\frac{M}{2}}\left|\Lambda_Q\right|^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\sum_{j,j'=1}^{M}\left[Q\left(i,j\right)-H+\sum_{l=1}^{L}\alpha_{jl}CL_l\right] \times \Lambda_Q^{-1}\left(j,j'\right)\left[Q\left(i,j'\right)-H+\sum_{l=1}^{L}\alpha_{jl}CL_l\right]\right\}. \tag{12}$$

The corresponding maximizations of $\ln f_1$ for $\hat{H}_1$, $\hat{\Lambda}_1$, and $\widehat{CL}$ produce the following set of equations,

$$\hat{\Lambda}_1\left(j,j'\right) = \frac{1}{N}\sum_{i=1}^{N}\left[Q\left(i,j\right)-\hat{H}_1+\sum_{l=1}^{L}\alpha_{jl}\widehat{CL}_l\right] \times\left[Q\left(i,j'\right)-\hat{H}_1+\sum_{l=1}^{L}\alpha_{j'l}\widehat{CL}_l\right], \tag{13}$$

$$\hat{H}_1 = \frac{\sum\limits_{j=1}^{M}\hat{\Lambda}_1^{-1}\left(j,j\right)\left[\bar{Q}\left(j\right)+\sum\limits_{l=1}^{L}\alpha_{jl}\widehat{CL}_l\right]}{\sum\limits_{j=1}^{M}\hat{\Lambda}_1^{-1}\left(j,j\right)}, \tag{14}$$

where

$$\bar{Q}\left(j\right) = \frac{1}{N}\sum_{i=1}^{N}Q\left(i,j\right),$$

$$\widehat{CL}_l = \frac{\sum\limits_{j=1}^{M}\alpha_{jl}\hat{\Lambda}_1^{-1}\left(j,j\right)\left[\hat{H}_1-\bar{Q}\left(j\right)-\sum\limits_{l=1,l\neq l}^{L}\alpha_{jl}\widehat{CL}_l\right]}{\sum\limits_{j=1}^{M}\alpha_{jl}\hat{\Lambda}_1^{-1}\left(j,j\right)\alpha_{jl}}. \tag{15}$$

The solution of $\widehat{CL}$ is given above in the N sets of wavelengths, solving a sequential $\widehat{CL}\left(k\right)$ $\left(k=0,1,\cdots\right)$ becomes an iterative procedure with the following steps: 1) begin with $\widehat{CL}_l\left(0\right)=0$ $\left(l=1,\cdots,L\right)$, $\hat{H}_0$ is given by Eq. (7) in the first N sets of wavelengths and set $\hat{H}_1=\hat{H}_0$; 2) evaluate $\hat{\Lambda}_1$ using Eq. (13) in the kth $\left(k=1,2,\cdots\right)$ N sets of wavelengths; 3) re-estimate $\hat{H}_1$ by Eq. (14); 4) solve for $\widehat{CL}_l\left(k\right)$ from Eq. (15); 5) iterate over step 2 through step 4 to solve $\widehat{CL}_l\left(k+1\right)$ in the $\left(k+1\right)$th N sets of wavelengths.

The algorithm discussed here was programmed to evaluate their performance on simulated topographic lidar data from the return signal statistical model of DIAL[7]. Table 1 shows the main system parameters that were chosen. Figure 1 plots the absorption spectra for the agent simulant TEP (triethyl phosphate) and DEMP (diethyl methyl phosphonate) in the spectral range of

Table 1. DIAL Parameters Used in Simulations

Laser Peak Power	0.5 MW
Transmitter Transmittance	0.8
Beam Waist Radius	0.02 m
Beam Divergence	2.5 mrad
Receiver Aperture	0.245 m
Receiver Transmittance	0.6
Range to Target	1000 m
Topographic Reflectivity	0.05
Atmospheric Extinction	2.76×10^{-4} m^{-1}
Index of Refraction Structure Constant	1.4×10^{-16} m$^{-2/3}$

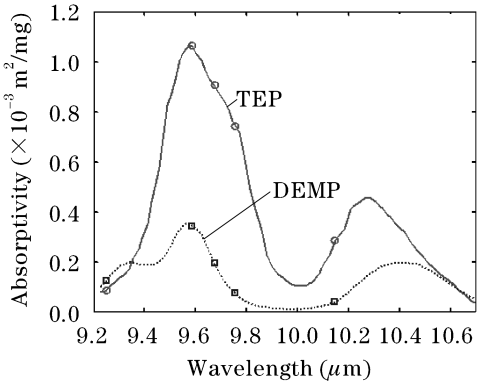

Fig. 1. Absorption spectra of TEP and DEMP.

9.2—10.7 μm[3].

Setting $N = 8$, $M = 5$, $L = 2$ and assuming there are 8 sets of wavelengths in one second (i.e., the burst is fired at 8 Hz), we simulated the procedure of simultaneous detections of TEP and DEMP. In order to evaluate the performance of the algorithm better, we assumed that the concentration-length of both vapors was produced at random and the CL time series were plotted in Fig. 2.

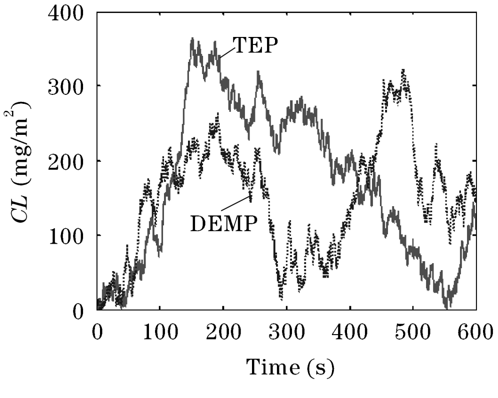

Fig. 2. True CLs of TEP and DEMP.

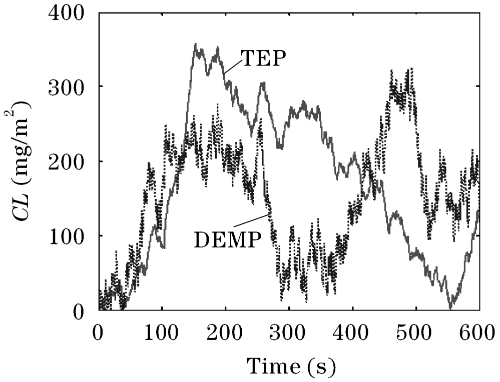

Fig. 3. Calculated CLs of TEP and DEMP.

In our simulated DIAL system, the minimum single-pulse SNR was 4.18. The calculated CL time series of TEP and DEMP were shown in Fig. 3. Comparing Fig. 3 with Fig. 2, the estimated CLs of both vapors tracked the trend of the true CL and were in fairly good agreement with the true series. This simulated results show that our algorithm has the ability to separate and detect the simultaneous presences of TEP and DEMP.

In conclusion, we have developed a MLE algorithm for simultaneously detecting and estimating multiple vapor materials using long-range multiple-wavelength DIAL. The method is based on the assumption that the SNR of return signal is large enough. For case where this assumption is valid, only given the return signal of each wavelength and the absorption spectra of the target vapors (without knowing the atmospheric transmission, target reflectance, lidar system response and so on), the method can separately identify and quantify vapor materials in mixtures and perform quite well. In addition, this algorithm can also be used to range-resolved multiple-wavelength DIAL.

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