# Passively $Q$－switched $\mathrm{Nd}^{3+}$ ：YAG laser with corner cube 

Weiqing Gao（高伟清）${ }^{1}$ ，Gongmin Yao（姚功民）${ }^{1}$ ，Lixin Xu（许立新）${ }^{1}$ ， Yong Cheng（程 勇）${ }^{2}$ ，Hai Ming（明 海）${ }^{1}$ ，and Jianping Xie（谢建平）${ }^{1}$<br>${ }^{1}$ Department of Physics，University of Science and Technology of China，Hefei 230026<br>${ }^{2}$ Optics Teaching and Research Room of Ordnance Noncommissioned Officer School，Wuhan 430064

Received January 9， 2006


#### Abstract

A passively $Q$－switched $\mathrm{Nd}^{3+}:$ YAG laser with corner cube is theoretically and experimentally studied．We analyze the polarization variation in cavity and simulate the peak power，pulse energy and pulse width changed with the rotation angle of corner cube numerically．An experiment is made to verify the theoretical results．With rotating the angle of corner cube about the axis the variation range of peak power is 1.77 MW （from 10.36 to 8.59 MW ），and that of pulse energy is 14.9 mJ （from 159.5 to 174.4 mJ ），the fluctuation of pulse width is 2.95 ns．The experimental results agree with the theoretical analysis to the extent of variation rules．The most dynamic to static energy ratio of $62.5 \%$ is achieved．

OCIS codes：140．3540，140．3580，260．5430．


Because of low saturation intensity，high damaged threshold，and excellent optical，chemical，and thermal characteristics ${ }^{[1]}, \mathrm{Cr}^{4+}$ ：YAG can be used as passive $Q$－ switch in $\mathrm{Nd}^{3+}$ ：YAG laser ${ }^{[2-6]}$ ．Corner cube is insensitive to vibration，impact，and great variety of temperature， and can be used in adjust－free lasers ${ }^{[7,8]}$ ．The combina－ tion of $\mathrm{Cr}^{4+}$ ：YAG with corner cube will obtain adjust－free $Q$－switched laser．Chun ${ }^{[9]}$ and Singh ${ }^{[10]}$ made use of the crossed Porro prism to realize electro－optical $Q$－switched laser．But this kind of resonant cavity is adjust－free only in one dimension．Chen used corner cube as the resonant cavity of passively $Q$－switched laser，and got the pulse energy of 8.8 mJ and the peak power of $400 \mathrm{~kW}^{[11]}$ ．Shi－ mony et al．also realized passively $Q$－switched laser with corner cube ${ }^{[12]}$ ．But none of them thought of the polar－ ization properties of corner cube，and the effects of the polarization on the peak power，pulse energy，and pulse width．We describe the polarization variation with the rotation of corner cube about the axis in cavity ${ }^{[13,14]}$ ，and simulate the peak power，pulse energy，and pulse width changed with the rotation angle of corner cube numeri－ cally．An experiment is executed to verify the theoretical results．

The polarization state of incident light will be changed from linear polarization to elliptical polarization because of the polarization properties of corner cube．When the incident light comes to the bottom of corner cube ver－ tically，Liu et al．deduced the Jones matrix by vector optics as ${ }^{[15]}$

$$
\begin{align*}
& J_{123}=R\left(-120^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \\
& J_{231}=R\left(120^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(180^{\circ}\right) \\
& J_{312}=R\left(0^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \\
& J_{321}=R\left(-60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(-120^{\circ}\right) \\
& J_{132}=R\left(180^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(120^{\circ}\right) \\
& J_{213}=R\left(60^{\circ}\right) \cdot T \cdot R\left(-60^{\circ}\right) \cdot T \cdot R\left(60^{\circ}\right) \cdot T \cdot R\left(0^{\circ}\right), \tag{1}
\end{align*}
$$

where subscripts 123－213 are in correspondence with six reflecting orders of incident light on three flanks of corner cube．$R(\varepsilon)=\left[\begin{array}{cc}\cos \varepsilon & \sin \varepsilon \\ -\sin \varepsilon & \cos \varepsilon\end{array}\right]$ describes the coordinate variation when light is reflected from one flank to an－ other．$T=\left[\begin{array}{cc}r_{\mathrm{p}} & 0 \\ 0 & r_{\mathrm{s}}\end{array}\right]$ is the variation of amplitude and polarization between incident light and reflected light，$r_{p}$ and $r_{\mathrm{s}}$ can be got from Fresnel formula．

Our theoretical model is shown in Fig．1．Resonant cav－ ity is formed by corner cube and fully reflecting flat mir－ ror．The gain material is $\mathrm{Nd}^{3+}: \mathrm{YAG}$ and the $Q$－switch is $\mathrm{Cr}^{4+}$ ：YAG．The polarizer in the resonant cavity can pro－ vide the output beam of linear polarization．The cross section of light beam in cavity is divided into six sym－ metric faculas（three pairs of conjugate faculas）because of the diffraction of three edges of corner cube．Because corner cube has the function of exchanging the situation of light beam on space，the cavity is equivalent to three sub－cavities．The three pairs of conjugate faculas enter into $\mathrm{Nd}^{3+}$ ：YAG and $\mathrm{Cr}^{4+}$ ：YAG at different situations， and pass through them four times each round trip（only twice in flat－mirror cavity）．This increases the gain of cavity．Referring to the Refs．［16］－［18］，we can deduce the peak power and pulse energy of three sub－cavities as


Fig．1．Theoretical model of passive $Q$－switched $\mathrm{Nd}^{3+}$ ：YAG laser with corner cube．

$$
\begin{equation*}
E_{m}(\alpha)=\frac{h \nu A}{2 \gamma \sigma} \ln \left(\frac{1}{R_{m}}\right) \ln \left(\frac{n_{\mathrm{i} m}}{n_{\mathrm{f} m}}\right), m=1,2,3 \tag{3}
\end{equation*}
$$

where $n_{\mathrm{im}}$ is initial inverted population density of each sub-cavity. $n_{\mathrm{t} m}$ and $n_{\mathrm{f} m}$ are the inverted population densities when the photon population density reaches the most and the least, respectively, which are expressed as

$$
\begin{align*}
n_{\mathrm{t} 0 m}= & \frac{\ln \left(\frac{1}{R_{m}}\right)+\delta \ln \left(\frac{1}{T_{0}^{2}}\right)+\partial}{2 \sigma l} \quad m=1,2,3,  \tag{4}\\
n_{\mathrm{i} m}= & \frac{\ln \left(\frac{1}{R_{m}}\right)+\ln \left(\frac{1}{T_{0}^{2}}\right)+\partial}{2 \sigma l} \quad m=1,2,3,  \tag{5}\\
\frac{n_{\mathrm{t} m}}{n_{\mathrm{i} m}}= & \frac{\ln \left(\frac{1}{R_{m}}\right)+\delta \ln \left(\frac{1}{T_{0}^{2}}\right)+\partial}{2 \sigma n_{\mathrm{i} m} l} \\
& +\frac{(1-\delta) \ln \left(\frac{1}{T_{0}^{2}}\right)}{2 \sigma n_{\mathrm{i} m} l}\left(\frac{n_{\mathrm{t} m}}{n_{\mathrm{i} m}}\right)^{\kappa} \quad m=1,2,3,  \tag{6}\\
n_{\mathrm{i} m}- & n_{\mathrm{f} m}=\frac{\ln \left(\frac{1}{R_{m}}\right)+\delta \ln \left(\frac{1}{T_{0}^{2}}\right)+\partial}{2 \sigma l} \ln \left(\frac{n_{\mathrm{i} m}}{n_{\mathrm{f} m}}\right) \\
& +\frac{(1-\delta) \ln \left(\frac{1}{T_{0}^{2}}\right)}{2 \sigma l} \frac{1}{\kappa}\left(1-\frac{n_{\mathrm{f} m}^{\kappa}}{n_{\mathrm{i} m}^{\kappa}}\right) m=1,2,3,(7)
\end{align*}
$$

where $\delta=\sigma_{\mathrm{e}} / \sigma_{\mathrm{g}}, \kappa=\sigma_{\mathrm{g}} / \gamma \sigma$, and $t_{\mathrm{r}}=4 L / c$. The meanings of all parameters are as follows: $h$ represents Plank constant, $A$ area of the beam, $\nu$ frequency of photon, $L$ equivalent length of cavity, $l$ length of operation material, $\sigma_{\mathrm{g}}$ absorption cross section of ground state, $\sigma$ emission cross section, $\sigma_{\mathrm{e}}$ absorption cross section of excited state, $\gamma$ inversion reduction factor, $T_{0}$ initial transmission of $Q$-switch, $\partial$ round trip dissipative optical loss, and $\alpha$ rotation angle of corner cube.

In Eqs. (2) - (7), there are

$$
\begin{align*}
& \ln \left(\frac{1}{R_{1}}\right)=\frac{1}{2} \ln \left(\frac{1}{r_{123}(\alpha)}\right)+\ln \left(\frac{1}{r_{321}(\alpha)}\right)  \tag{8}\\
& \ln \left(\frac{1}{R_{2}}\right)=\frac{1}{2} \ln \left(\frac{1}{r_{231}(\alpha)}\right)+\ln \left(\frac{1}{r_{132}(\alpha)}\right)  \tag{9}\\
& \ln \left(\frac{1}{R_{3}}\right)=\frac{1}{2} \ln \left(\frac{1}{r_{312}(\alpha)}\right)+\ln \left(\frac{1}{r_{213}(\alpha)}\right) \tag{10}
\end{align*}
$$

where $r_{123}(\alpha)$ to $r_{213}(\alpha)$ are the ratios of transmissivity of the polarizer corresponding with six orders mentioned above. They are related with polarization and the rotation angle of corner cube. Think of the order 123 and assume that the incident light is linearly polarized, namely $\overrightarrow{E_{\mathrm{i} 1}}=E_{01}\left[\begin{array}{c}\sin \alpha \\ \cos \alpha\end{array}\right]$. Then the output light is

$$
\overrightarrow{E_{\mathrm{o} 1}}=\left[\begin{array}{c}
E_{\mathrm{p} 1}  \tag{11}\\
E_{\mathrm{s} 1}
\end{array}\right]=J_{123} E_{\mathrm{i} 1}=J_{123} E_{01}\left[\begin{array}{c}
\sin \alpha \\
\cos \alpha
\end{array}\right]
$$



Fig. 2. Ratios of transmissivity of the polarizer, $r_{123}(\alpha)$ to $r_{213}(\alpha)$, changed with rotation angle of corner cube $\alpha$.

Then we get $r_{123}(\alpha)$ as

$$
\begin{equation*}
r_{123}(\alpha)=\frac{\left|E_{\mathrm{p} 1} \sin \alpha+E_{\mathrm{s} 1} \cos \alpha\right|^{2}}{\left|E_{\mathrm{p} 1}\right|^{2}+\left|E_{\mathrm{s} 1}\right|^{2}} \tag{12}
\end{equation*}
$$

By the same method, we can get all the expressions of $r_{123}(\alpha)$ to $r_{213}(\alpha)$. Numerical simulations of the variation of $r_{123}(\alpha)$ to $r_{213}(\alpha)$ with rotation angle of corner cube are shown in Fig. 2. When corner cube rotates from $0^{\circ}$ to $90^{\circ}, r_{123}(\alpha)$ to $r_{213}(\alpha)$ are changed with rotation angle of corner cube differently, which is caused by the variation of polarization in cavity. The variation range is about $40 \%$. From Eqs. (2) and (3), we get peak power, pulse energy, and pulse width as

$$
\begin{align*}
P(\alpha) & =P_{1}(\alpha)+P_{2}(\alpha)+P_{3}(\alpha)  \tag{13}\\
E(\alpha) & =E_{1}(\alpha)+E_{2}(\alpha)+E_{3}(\alpha)  \tag{14}\\
\tau(\alpha) & =\frac{E(\alpha)}{P(\alpha)} \tag{15}
\end{align*}
$$

Numerical simulations of the variation of peak power, pulse energy, and pulse width with the rotation of corner cube are shown in Figs. 3-5. We can find that peak


Fig. 3. Numerical simulations of the variation of peak power with $\alpha$.


Fig. 4. Numerical simulations of the variation of pulse energy with $\alpha$.


Fig. 5. Numerical simulations of the variation of pulse width with $\alpha$.
power and pulse energy are changed with the rotation angle of corner cube, while pulse width remains the same on the whole. The fluctuation of peak power is about 1.5 MW and that of pulse energy is about 20 mJ .

The experimental configuration is the same as theoretical model shown in Fig. 1. An $8-\mathrm{mm} \mathrm{Nd}^{3+}$ :YAG is pulse-pumped by xenon lamp whose capacitance is 100 $\mu \mathrm{F}$. The length of cavity is 380 mm and the initial transmission of $\mathrm{Cr}^{4+}$ :YAG is $25 \%$. Because of the polarization properties of corner cube, p component light, whose oscillating plane is parallel to incident plane, passes through the polarizer and continues to resonate in cavity, while s component light, whose oscillating plane is vertical to incident plane, is extracted. The intensities of p and s component lights are changed when corner cube is rotated, which results in the variations of output peak power and pulse energy.

Rotating corner cube every $10^{\circ}$, we measure peak power, pulse energy, and pulse width as shown in Figs. 6 - 8 . The most and the least peak powers are 10.36 and


Fig. 6. Experimental results of the variation of peak power with $\alpha$.


Fig. 7. Experimental results of the variation of pulse energy with $\alpha$.


Fig. 8. Experimental results of the variation of pulse width with $\alpha$.


Fig. 9. Recorded passively $Q$-switched pulse waveform.
8.59 MW, respectively. The variation range of peak power is about 1.77 MW. The highest and lowest pulse energies are 174.4 and 159.5 mJ , respectively. The adjustable range of energy is 14.9 mJ . The static energy is 279 mJ and ratio of the most dynamic to static energy is $62.5 \%$. The pulse width fluctuates near 20 ns and the fluctuation is 2.95 ns. Figure 9 is the recorded waveform, and the pulse width is 20.71 ns .

Comparing theoretical results in Figs. 3-5 with experimental results in Figs. 6-8, we can find that the variation ranges of theoretical and experimental peak powers are 1.5 and 1.77 MW , the variation ranges of theoretical and experimental pulse energies are 20 and 14.9 mJ . Besides, the variation rules of peak power and pulse energy are the same on the whole. There are still some differences between theoretical and experimental results. They are because that the saturation property
of $\mathrm{Cr}^{4+}$ :YAG is not isotropic, which depends on how the crystal is cut. In addition, experimental error is also the reason.
In conclusion, we analyze the polarization property of corner cube theoretically, simulate the variation of peak power, pulse energy, and pulse width with the rotation angle of corner cube numerically. An experiment is carried on to verify the theoretical results. By rotating the corner cube, the highest pulse energy of 174.4 mJ is achieved, the lowest energy is 159.5 mJ , and the range of adjustment is 14.9 mJ . The experimental results are in correspondence with theoretical analysis to the extent of variation rule. The differences between theoretical and experimental results are because of the saturation property of $\mathrm{Cr}^{4+}$ :YAG and experimental error.
W. Gao's e-mail address is weikin@mail.ustc.edu.cn.

## References

1. J. Dong, P. Deng, and J. Xu, Opt. Mater. 14, 109 (2000).
2. Y. Shimony, Y. Kalisky, and B. H. T. Chai, Opt. Mater. 4, 547 (1995).
3. R. S. Afzal, A. W. Yu, J. J. Zayhowski, and T. Y. Fan, Opt. Lett. 22, 1314 (1997).
4. J. Dong, P. Deng, Y. Liu, Y. Zhang, J. Xu, W. Chen,
and X. Xie, Appl. Opt. 40, 4303 (2001).
5. X. Tian, P. Yan, Q. Liu, M. Gong, and Y. Liao, Chin. Opt. Lett. 2, 536 (2004).
6. J. Li, C. Wang, Y. Wang, Q. Wei, D. Fu, X. Lin, A. Yao, D. Cui, and Z. Xu, Chin. Opt. Lett. 2, 590 (2004).
7. M. S. Scholl, J. Opt. Soc. Am. A 12, 1589 (1995).
8. M. Shen, S. Wang, L. Hu, and D. Zhao, Appl. Opt. 43, 4091 (2004).
9. M. K. Chun and E. A. Teppo, Appl. Opt. 15, 1942 (1976).
10. I. Singh, A. Kumar, and O. P. Nijhawan, Appl. Opt. 34, 3349 (1995).
11. J. Chen, Appl. Opt. 21, 4329 (1982).
12. Y. Shimony, Z. Burshtein, and Y. Kalisky, IEEE J. Quantum Electron. 31, 1738 (1995).
13. M. A. Player, J. Mod. Opt. 35, 1813 (1988).
14. S. E. Segre and V. Zanza, J. Opt. Soc. Am. A 20, 1804 (2003).
15. J. Liu and R. M. A. Azzam, Appl. Opt. 36, 1553 (1997).
16. A. Szabo and R. A. Stein, J. Appl. Phys. 36, 1562 (1965).
17. J. J. Degnan, IEEE J. Quantum Electron. 25, 214 (1989).
18. X. Zhang, S. Zhao, Q. Wang, Q. Zhang, L. Sun, and S. Zhang, IEEE J. Quantum Electron. 33, 2286 (1997).
