

Evaluation of absolute phase for 3D profile measurement using fringe projection

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Received January 12, 2006

A new method of absolute phase evaluation for three-dimensional (3D) profile measurement using fringe projection is presented, which combines the gray code and the phase shift technique. Two kinds of fringe patterns are projected onto the object surface respectively, one is sinusoidal intensity distribution used for phase demodulation and the other is gray code fringe pattern for unwrapping. These images are acquired by camera and stored into computer. The absolute phase is obtained by analyzing these images. The validity of this method is verified experimentally. The method is superior to other phase unwrapping methods.

OCIS codes: 120.2650, 120.5050, 100.5070.

There are several kinds of optical shape measurement methods for a diffuse object. Specifically, structured light methods have been extensively used recently for measuring the three-dimensional (3D) shape of an object with non-contact, high-speed, high-accuracy and easier implementation^[1-6]. Fringe projection is one of the structured light methods. It is based on the triangulation principle and the full-field imaging techniques. A fringe pattern is projected onto the object and observed by a charge-coupled device (CCD) camera. Since the projection and observation directions are different, the observed fringe pattern is distorted according to the height of the object. The fringe distribution of the observed pattern contains the height information of the object. Therefore, the surface profile of the object can be measured. The fringe projection method includes both projected coded light and sinusoidal fringe techniques. The sinusoidal fringe technique can increase the measurement resolution to the pixel scale of the camera without the need of using high-density fringe patterns. The phase evaluation of the observed pattern plays a very important role in sinusoidal fringe method, because the phase map acquired directly is limited from $-\pi$ to π . It must be unwrapped to retrieve the absolute phase employing suitable phase unwrapping algorithm. Normally, phase unwrapping is carried out along image. When the object has large height steps and/or spatially isolated surfaces, the phase unwrapping may become a critical issue, because the phase jumps at these regions are too large, i.e., they fall outside the range from $-\pi$ to π . Also an error at a given point can propagate along the unwrapping path. In recent years, several methods have been developed to overcome discontinuities^[7-10], especially, temporal phase unwrapping algorithm^[7] and dual-frequency technique^[8] have obtained better effect in absolute phase measurement, but acquiring a set of images with these techniques is time consuming.

The purpose of this paper is to propose a method that is based on combination of the phase shift technique for

sinusoidal fringe pattern and gray code technique for phase unwrapping. It means that the phase shift technique is used to achieve the phase value in the range of 2π , and the gray code fringe pattern is used to evaluate the integer part of the phase. This method offers a simple solution to the problem of high resolution and high speed in 3D profile measurement.

The optical schematic diagram for fringe projection 3D profile measurement system with crossed optical axes geometry is shown in Fig. 1. The optical axis PO of a projector lens crosses the optical axis OC of a CCD camera lens at point O on a reference plane R , from which the object height h is measured. L is the distance between the reference plane R and the exit pupil of projector lens, and d is the distance between the exit pupil of projector lens and the entrance pupil of CCD camera. The fringe pattern is projected onto the object surface by liquid crystal display (LCD) projector. The deformed fringe image is observed by CCD camera. The projective ray PHB strikes the object surface at point H , and crosses the reference plane R at point B . H will be seen to be the point A on the reference plane R when observed through the CCD camera. BA is the deviation of distorted fringe

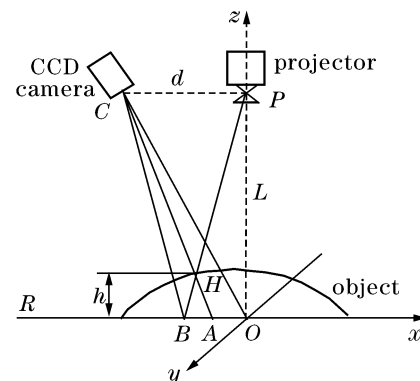


Fig. 1. Optical setup with crossed optical axes.

line opposing to straight fringe line. The object height h is given by

$$h = \frac{\overline{BA} \cdot L}{d + \overline{BA}}, \quad (1)$$

where \overline{BA} represents the offset of point A on the reference plane R due to the object height at point H . For phase measurement, the phase distributions of two different situations can be obtained, denoted as $\varphi_r(x, y)$ and $\varphi_o(x, y)$, respectively corresponding to the absence and presence of the target object on the reference plane. The offset of phase can be denoted as

$$\Delta\varphi(x, y) = \varphi_o(x, y) - \varphi_r(x, y). \quad (2)$$

The shift distance of the fringe can be deduced as

$$\overline{BA} = \frac{\Delta\varphi(x, y)}{2\pi f}, \quad (3)$$

where f is the spatial frequency of fringe on the reference plane R .

In phase measurement technique, a sinusoidal fringe pattern is used and the intensity distribution on the object surface is given by

$$I(x, y) = a(x, y) + b(x, y) \cos[\omega x + \varphi(x, y)], \quad (4)$$

where ω is the mean spatial frequency of the fringes along the x -direction, $a(x, y)$ and $b(x, y)$ are the average brightness and the fringe contrast at lateral location (x, y) . These two terms also represent the additive and the multiplicative noises in the image that are responsible for the degradation in measurement accuracy. The phase term $\varphi(x, y)$ represents the local shift of fringes resulting from surface relief variations. From this phase term, the lateral shift \overline{BA} can be found. To determine $\varphi(x, y)$, several images are captured with a known phase step introduced between consecutive images. In the four-step algorithm with a phase step of $\pi/2$, four images captured can be represented by

$$I_k(x, y) = a(x, y) + b(x, y) \cos\left[\omega x + \varphi(x, y) + \frac{k-1}{2}\pi\right], \quad (5)$$

where $k = 1, 2, 3, 4$.

Then, the phase term in Eq. (3) can be derived as

$$\varphi(x, y) = \tan^{-1} \left[\frac{I_2(x, y) - I_4(x, y)}{I_3(x, y) - I_1(x, y)} \right]. \quad (6)$$

Since $a(x, y)$ and $b(x, y)$ are automatically canceled out by the subtraction and division in the calculation, their influences are minimized. Therefore this technique is insensitive to both additive and multiplicative noises. In addition, because of the independent determination of the phase shift at each (x, y) location, this technique enables pure, high-accuracy pixel-to-pixel measurements.

The phase obtained from Eq. (6) is indeterminate. It has to be added $2\pi n$, because the arctangent is defined

over a range from $-\pi$ to π . The absolute phase is given by

$$\varphi_u(x, y) = \varphi(x, y) + 2\pi n(x, y), \quad (7)$$

where $n(x, y)$ is an integer, it denotes the order of the projecting fringes. Phase unwrapping is a procedure that determines integer $n(x, y)$.

In order to determinate $n(x, y)$, according to Ref. [3] and our library condition, we utilize an additional gray code fringe pattern. The fringe pattern is generated by computer and comprises several arrays of rectangular fringe, with each fringe identified uniquely by its gray scale and arranged in a specific sequence as shown in Fig. 2. The width of each fringe is equal to the period of sinusoidal fringes. The black fringe is used as beginning fringe and followed by different gray scale fringes and white fringe as the last. These fringes are encoded with two-bit code “ ij ”, with “ i ” and “ j ” respectively denote the period of the fringe and gray scale of the fringe (see Fig. 2).

Phase unwrapping is made along perpendicular fringes. Firstly the deformed gray fringe code image is segmented as N (the number of gray scale in the fringe pattern) regions based on adaptive thresholds by segmentation algorithm, namely $j = 0, 1, \dots, N-1$. Secondly, the number of period in deformed fringe image is determined by taking counting the fringe of every region respectively. Supposing M is the maximum number of fringes that corresponds to “ k ” region, and then the number of period in deformed fringe image is M . The “ k ” regions are used as beginning fringe of every period, namely the range between adjacent “ k ” regions is in the same period, the corresponding periods are respectively $i = 0, 1, \dots, M-1$. Thus we can obtain the expression of $n(x, y)$ as

$$n(x, y) = Ni(x, y) + [j(x, y) - k]_{\text{comp}}, \quad (8)$$

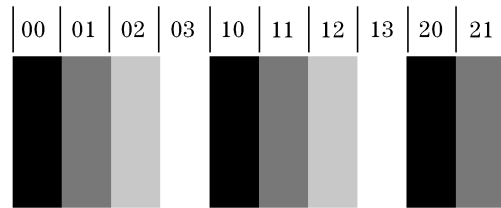


Fig. 2. Codes of gray fringe pattern.

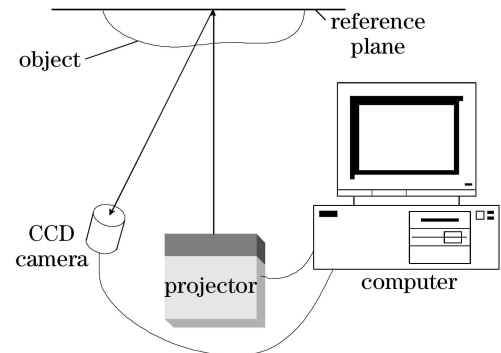


Fig. 3. Schematic of the measurement system.

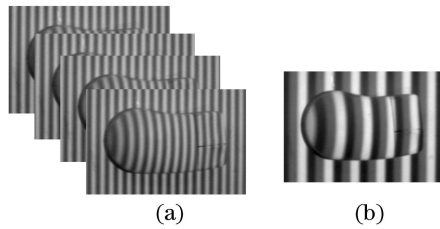


Fig. 4. Deformed fringe images of phase shift (a) and gray code (b).

where $[j(x, y) - k]_{\text{comp}}$ is the complementary operation with modulus N .

The proposed approach was demonstrated experimentally by using the apparatus shown in Fig. 3, which used the cross-axes geometry. Firstly, 4 patterns of sinusoidal fringe with the phase shift of one fourth of the period 2π and a gray code fringe pattern were produced, and stored in computer. Then these patterns were projected in two different situations by a projector (TLP-560, Toshiba), corresponding to the absence and presence of the target object on a reference plane. Finally these patterns were acquired by CCD camera (MTV1881EX, Mintron) and stored in computer. One of the tested objects was a mouse. The fringes were deformed in proportion to the object height, as shown in Fig. 4. The number of gray scale was 3 in Fig. 4(b), namely $N = 3$ in Eq. (8).

With the above algorithm, programmed in MATLAB 6.5, the absolute phase distributions of the object surface and the reference plane could be evaluated respectively from Eqs. (8), (6), and (7). The map of phase offset was reconstructed as shown in Fig. 5.

Using this method, a set of objects whose heights were known (a 16.30-mm-high bar and a 11.50-mm-high cylinder) were also measured. Figure 6 shows the 3D representation of the experimental results. The average measurement error of the bar was 0.31 mm, and that of the cylinder was 0.21 mm. The relative error of the whole measurement was in the bounds of 1.9%. The measurement results show the reliability of using the proposed phase unwrapping method.

In order to be able to increase the height measuring range without any loss in resolution, the period of gray code can be expanded, namely increasing the number of gray scale. Since region segmentation is based on gray information with 256 levels, if the difference between the

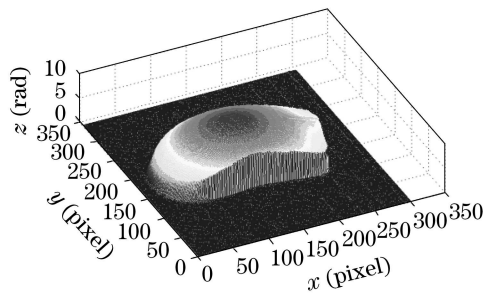


Fig. 5. Result of phase evaluation.

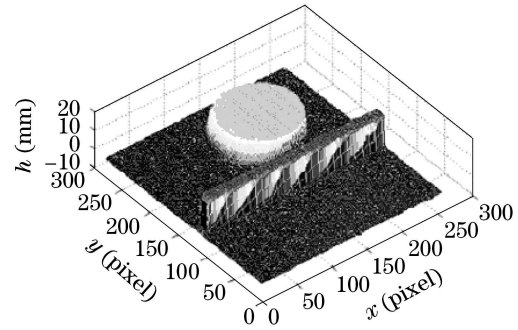


Fig. 6. Result of the shape measurement of standard objects.

gray values of adjacent fringes were not big enough, the edge of region would be blurred easily, thereby brought error for phase unwrapping. The number of gray scale can be adjusted depending on the shape of object under tested.

In conclusion, a novel method of absolute phase evaluation was presented for measuring 3D profile using fringe projection. This technique combined the gray code fringe with the phase shifting technique. It was suitable to measuring more gradient objects. Compare with traditional unwrapping procedures, the procedure using this method is relatively simple, independent of path, and free of propagating errors. In addition, the speed of measurement with the proposed method was faster in comparison with temporal phase unwrapping algorithm, because it needs only one additional image. The experimental results indicated that the method was valid.

This work was supported by the National Natural Science Foundation of China under Grant No. 50535030 and 50275120. M. Huang's e-mail address is huangmtao@163.com.

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