Hiding an image in cascaded Fresnel digital holograms

Shaogeng Deng (邓绍更), Liren Liu (刘立人), Haitao Lang (郎海涛), Weiqing Pan (潘卫清), and Dong Zhao (赵 栋)

Laboratory for Optical Information, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

Received October 27, 2005

A system of two separated computer-generated holograms termed cascaded Fresnel digital holography (CFDH) is proposed and its application to hiding information is demonstrated by a computer simulation experiment. The technique is that the reconstructed image is the result of the wave Fresnel diffraction of two sub-holograms located at different distances from the imaging plane along the illuminating beam. The two sub-holograms are generated by an iterative algorithm based on the projection onto convex sets. In the application to the hiding of optical information, the information to be hidden is encoded into the sub-hologram which is multiplied by the host image in the input plane, the other sub-hologram in the filter plane is used for the deciphering key, the hidden image can be reconstructed in the imaging plane of the CFDH setup.

OCIS codes: 090.1760, 100.2000, 070.4560, 050.1940, 260.1960.

Techniques of optical information security have been extensively studied in recent years since Refregier and Javidi proposed the double-random phase encoding technique[1]. Digital watermarking of two-dimensional (2D) image or three-dimensional (3D) image by double-random phase encoding was also successfully verified[2]. A technique of watermarking based on a modified joint-transform correlator was proposed in Ref. [3], in which the hidden information is revealed as a result of a spatial correlation between two concealograms, its good performance has been shown.

In this letter we propose an alternative approach with an inherently simple system architecture, which may offer a useful approach for a practical optical encryption or security system (i.e. hiding a picture, watermarking or product authenticity verification). It can be called cascaded Fresnel digital hologram (CFDH), which means the reconstructed image is yielded by the Fresnel diffraction of two sub-holograms located at different distances from the imaging plane. Compared with that in Refs. [3,4], it is an architecture without lenses, which minimizes the hardware requirement and is easier to implement. The optical setup of the system, shown in Fig. 1, is a cascaded Fresnel hologram optical setup with three planes: the input plane P_1 in which the input mask $g_1(x_1, y_1)$, which is the product of a phase-only function $h_1(x_1, y_1)$ and an intensity function $A_1(x_1, y_1)$, is displayed; the filter plane P_2 in which the filter mask $H_2(x_2, y_2)$, another random phase-only function statistically independent of $h_1(x_1, y_1)$, is displayed; and the imaging plane P_3 in which the camera should record the output predefined image. The distances between the adjacent planes are z_1 and z_2, which satisfy the Fresnel approximation according to the size of the aperture. In the application of the CFDH to the hiding of a picture, the complex function $g_1(x_1, y_1)$ is the result of the phase-only function $h_1(x_1, y_1) = \exp[j\phi(x_1, y_1)]$ multiplied by the host image $A_1(x_1, y_1)$, $g_1(x_1, y_1)$ is Fresnel transformed to the filter plane P_2 and multiplied by the phase-only function H_2, and after the second Fresnel diffraction, an image can be obtained in the imaging plane P_3 which will be modified so that it is close as much as possible to the expected image by an iterative procedure.

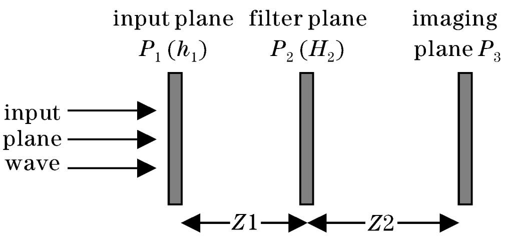

Fig. 1. Optical setup of the cascaded Fresnel hologram system.

Using Fresnel diffraction theory, we obtain the electric field arriving at the filter plane P_2 located at a distance z_1 from the input plane P_1

$$G_1(x_2, y_2) = \iint A_1(x_1, y_1) h_1(x_1, y_1)$$

$$\times \frac{j\pi}{\lambda z_1}[(x_2 - x_1)^2 + (y_2 - y_1)^2]\mathrm{d}x\mathrm{d}y, \quad (1)$$

where λ represents the illumination wavelength, $g_1(x_1, y_1) = A_1(x_1, y_1)h_1(x_1, y_1)$. For simplicity, we rewrite Eq. (1) as

$$G_1(x_2, y_2) = \mathrm{FrT}\{g_1(x_1, y_1); z_1\}, \quad (2)$$

where FrT represents the Fresnel transform, z_1 denotes the distance of the Fresnel diffraction. Then $G_1(x_2, y_2)$ is multiplied by $H_2 = \exp[i\gamma(x_2, y_2)]$ and totally Fresnel transformed to the imaging plane, the electric field in the plane P_3 can be written as

$$c(x_3, y_3) = \mathrm{FrT}\{\mathrm{FrT}\{g_1(x_1, y_1); z_1\}H_2(x_2, y_2); z_2\}. \quad (3)$$

While now the image in the imaging plane P_3 is not the expected image to be hidden, so the projections-onto-constraint-sets (POCS) algorithm[5] is employed to adjust the phase function $h_1(x_1, y_1)$ of $g_1(x_1, y_1)$ in the Fresnel diffraction procedure just as in Refs. [3—5], in which the Fourier transforms are replaced by the Fresnel transforms. The POCS algorithm has been employed in several areas of signal processing (with many other designations), which is similar to the iterative Fourier transform algorithm (IFTA). Now the system's output expected is

$$c(x_3, y_3) = I_3(x_3, y_3)\exp[j\psi(x_3, y_3)], \quad (4)$$

where $I_3(x_3, y_3)$ denotes the amplitude of the output image and $\psi(x_3, y_3)$ the phase of $c(x_3, y_3)$. From Eq. (3) the input function $g_1(x_1, y_1)$ is given by

$$g_1(x_1, y_1) = \mathrm{IFrT}\left\{\frac{\mathrm{IFrT}\{c(x_3, y_3); z_1\}}{H_2(x_2, y_2)}; z_2\right\}, \quad (5)$$

where IFrT is the inverse Fresnel operator. The POCS algorithm based on Fresnel transformation shown in Fig. 2, starts with the complex function $g_1(x_1, y_1)$ which is the result of the host image multiplied by a random-phase function $h_1(x_1, y_1) = \exp[j\phi(x_1, y_1)]$. The algorithm consists of the following four steps: 1) The complex function $g_1(x_1, y_1)$ is transformed by the cascaded Fresnel diffraction, defined in Eq. (3), into the domain (x_3, y_3). 2) The obtained function $c(x_3, y_3)$ is projected onto the constraint set, which means the amplitude of the function $c(x_3, y_3)$ is replaced with the predefined image, here is the expected image to be hidden. 3) And then back through the inverse cascaded Fresnel diffraction defined by Eq. (5) into the domain (x_1, y_1). 4) The function obtained $g_1'(x_1, y_1)$ is projected onto the constraint sets in the domain (x_1, y_1), which means the modulus of $g_1'(x_1, y_1)$ is replaced with the host image and the significant area is limited in the initial area of the function $g_1(x_1, y_1)$, out of the significant area is padded with zeros. Then a new input function $g_1(x_1, y_1)$ is formed. The algorithm continues to circulate between two domains until the error between the actual and the desired output functions is no longer meaningfully reduced.

As we have mentioned above, the projection $P_1[\cdot]$ on the constraint set in the imaging plane is

$$P_1[c_n(x_3, y_3)] = A_3(x_3, y_3)\exp[j\psi(x_3, y_3)], \quad (6)$$

where $A_3(x_3, y_3)$ is a real positive function representing the output image or the hidden image. In the input plane

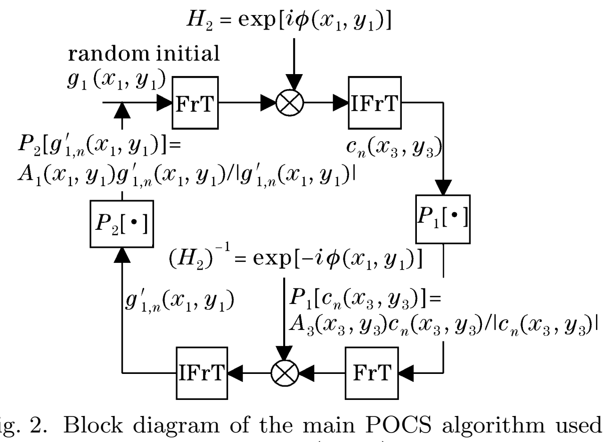

Fig. 2. Block diagram of the main POCS algorithm used to compute the phase function $h_1(x_1, y_1)$.

we recall that $h_1(x_1, y_1)$ should be the phase function of the input function $g_1(x_1, y_1)$, and therefore the projection $P_2[\cdot]$ on the constraint set is

$$P_2[g_1'(x_1, y_1)] = \begin{cases} A_1(x_1, y_1)\exp[j\phi(x_1, y_1)], & \text{if } (x_1, y_1) \in W \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

where $\phi(x_1, y_1)$ denotes the phase distribution of $h_1(x_1, y_1)$, that is, $\exp[j\phi(x_1, y_1)] = g_1'(x_1, y_1)/|g_1'(x_1, y_1)|$, and W is a window function. Note that $H_2(x_2, y_2)$ is chosen only once before the beginning of the iterations. After $H_2(x_2, y_2)$ defined, it becomes part of the Fresnel hologram kernel function and never to be changed during the circulating process.

The average mean-square error e_n between the intensity of the output function before and after the projection, used to evaluate the convergence of the algorithm to the desired image in the nth iteration, is defined as

$$e_n = \frac{1}{M}\iint \left| |P_1[c_n(x_3, y_3)]|^2 - |c_n(x_3, y_3)|^2 \right|^2 \mathrm{d}x\mathrm{d}y, \quad (8)$$

where M is the entire area of the imaging plane.

Computer simulations are performed to verify the validity of the technique proposed. As shown in Fig. 3(a), the letter "R" in the central square is the image desired, which comprised of 24×24 pixels in the imaging plane. The input, filter and imaging planes are 128×128 pixels. The function $g_1(x_1, y_1)$ in the input plane is made to cover only the central area of 60×60 pixels, assumed actual size is 4×4 (mm) and designated as the window W. Since the architecture is lensless, the beam propagating through the system may be somewhat divergent, and the field sizes of the significant planes must be mismatched[6]. Hence z_1 and z_2 should be chosen carefully to reduce the size mismatching according to the propagation distance. Here we choose both z_1 and z_2 equal to 60 mm, satisfied with Nyquist sampling condition.

The Fresnel diffraction calculation is based on the fractional Fourier transform algorithm[7]. Wavelength λ of the input plane wave is 600 nm. The phase function $h_1(x_1, y_1)$ of $g_1(x_1, y_1)$ obtained from the POCS-based iterative algorithm is shown in Fig. 3(b). The host image, the amplitude of the function $g_1(x_1, y_1)$ is shown in Fig. 3(c). Only when the host image multiplied by the right phase function $h_1(x_1, y_1)$, which is shown in Fig. 3(d), is placed in the input plane and $H_2(x_2, y_2)$ in the filter plane shown in Fig. 3(e), we can obtain the reconstructed image, the image of the letter "R", which is shown in Fig. 3(f). If the phase function $h_1(x_1, y_1)$ is wrong, the image in the imaging plane is white noise not the expected image, which is shown in Fig. 3(g). Figure 4 shows the convergence of the mean square error with the number of iterations to the minimum.

A key problem in computer holography is the realization of a transparency that controls both the amplitude and a phase of transmitted wave at each point in accord with a computed complex function. Here we propose the method of kinoform to implement for its excellent performance. A kinoform is a fully transparent plate which

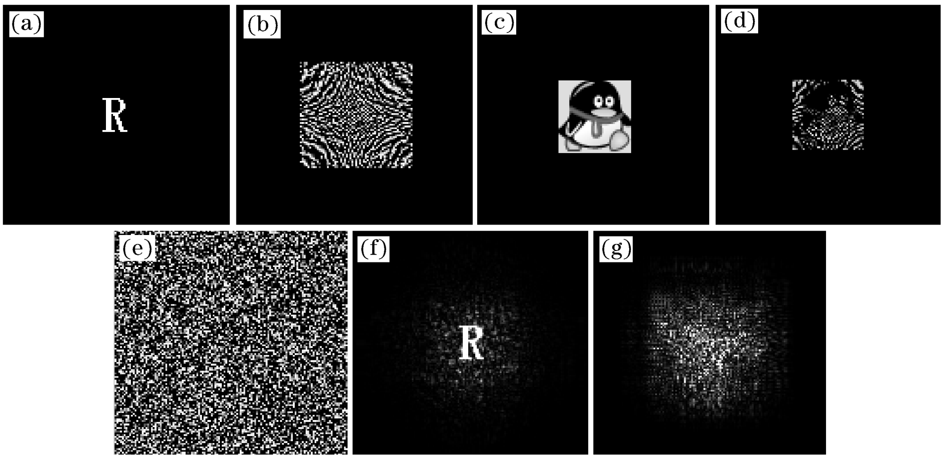

Fig. 3. Result of the computer simulation experiment. (a) The image to be hidden; (b) the phase function $h_1(x_1, y_1)$ of $g_1(x_1, y_1)$; (c) the host image; (d) the host image multiplied by the phase function $h_1(x_1, y_1)$; (e) the phase-only function $H_2(x_2, y_2)$ generated by a mini POCS algorithm; (f) the reconstructed image in the imaging plane; (g) the reconstructed image in the imaging plane with the wrong phase function $h_1(x_1, y_1)$ of the input function $g_1(x_1, y_1)$.

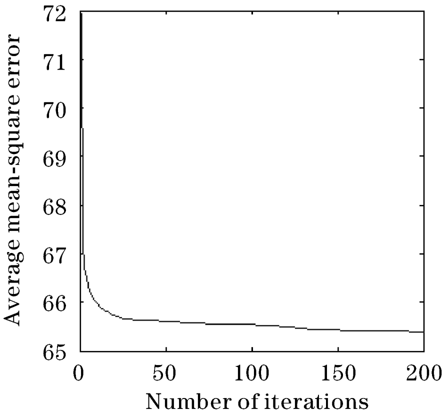

Fig. 4. Average mean-square error versus the number of iterations.

does not degrade incident light intensity. In the ideal case all the incident light illuminating the kinofrom can be used to reconstruct a single image with the efficiency of diffraction 100%[8].

When holograms are generated by computer the phase is many times quantized, which means that the phase at each point in the hologram cannot assume any one of a continuous range of values $(-\pi, \pi)$, but only those from a discrete set, say N equally spaced values. Here we derive the effects of phase quantization in CFDHs according to the Goodmn and Silvestri theory[9,10] of phase quantization and present simulated results of phase quantization in CFDHs.

After phase quantization of $g_1(x_1, y_1)$ and $H_2(x_2, y_2)$, we can obtain the reconstruction in the plane P_3

$$\begin{aligned}\hat{c}(x_3, y_3) &= \mathrm{FrT}\{\hat{G}_1(x_2, y_2)\hat{H}_2(x_2, y_2); z_2\}\\ &= \mathrm{sinc}^2(1/N) I_3(x_3, y_3)\exp[j\psi(x_3, y_3)]\\ &\quad +\mathrm{sinc}^2(1/N)\mathrm{FrT}\{\sum_{m,m\neq 0}[(-1)^m/(mN+1)]G_{1m}(x_2, y_2)H_2(x_2, y_2); z_2\}\\ &\quad +\mathrm{sinc}^2(1/N)\mathrm{FrT}\{\sum_{r,r\neq 0}[(-1)^r/(rN+1)]G_1(x_2, y_2)H_2(x_2, y_2); z_2\}\\ &\quad +\mathrm{sinc}^2(1/N)\mathrm{FrT}\{\sum_{m,m\neq 0}[(-1)^m/(mN+1)]G_{1m}(x_2, y_2)\sum_{r,r\neq 0}[(-1)^r/(rN+1)]H_{2r}(x_2, y_2); z_2\},\end{aligned} \tag{9}$$

where $G_{1m}(x_2, y_2) = \mathrm{FrT}\{|g_1(x_1, y_1)|\exp[i(Nm+1)\phi(x_1, y_1)]; z_1\}$, $G_1(x_2, y_2) = \mathrm{FrT}\{g_1(x_1, y_1); z_1\}$, m is 0, ± 1, ± 2, $\cdots$, $\pm\infty$, N is the number of phase quantization levels, the sign ˆ means the phase quantization of complex function. So the reconstructed function $\hat{c}(x_3, y_3)$ of CFDHs after phase quantization consists of a summation of several different contributions, similar to that of Ref. [10], the first term in the equation above, in the case of $m = 0$, is the primary image, the other three terms are "false images". The concept of "false image" follows that in Ref. [10], which is usually noise here. As the number N of quantization levels increases, the strength of the primary image increases, approaching that of the expected function $I_3(x_3, y_3)$, while the strength of the "false images" decreases. Next we will demonstrate the effects of the phase quantization in CFDHs with previous simulated procedure. The phases of two sub-holograms are quantized into 3, 5 and 15 levels, the simulated results are shown in Fig. 5. We can see that the reconstructed image is degraded with the number N of phase quantization decreasing. When $N > 5$, the reconstructed image can be told. According to the nowadays technics level,

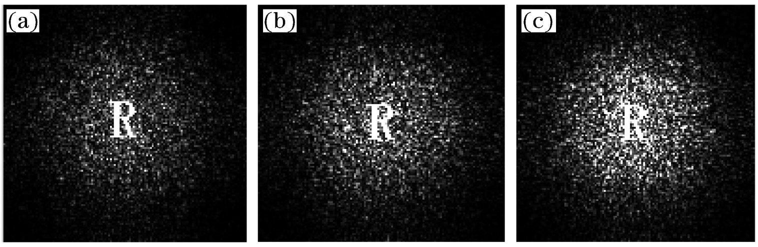

Fig. 5. Reconstructed image after phase quantization with different numbers N of quantization. (a) $N = 15$; (b) $N = 5$; (c) $N = 3$.

the CFDH is practical.

To summarize, we have introduced a CFDH technique, which means the reconstructed image is produced by two sub-holograms located at different positions along the illuminating beam. The two sub-holograms are two phase-random functions statistically independent on each other, generated by a computer using the POCS algorithm based on Fresnel transformation. The theory analysis of the Fresnel digital hologram is briefly given, together with the numerical simulation experiment of its application to hiding an image or information, which shows the method is practical. The kinoform is proposed to implement CFDH, its analysis of phase quantization is briefly given, which shows when the number $N > 5$ of phase quantization level the reconstructed image can be told. The method proposed can be used for security optical verification, watermarks, product authenticity verification as well as hiding a halftone picture.

S. Deng's e-mail address is dengshaogeng@hotmail.com.

References

1. P. Refregier and B. Javidi, Opt. Lett. **20,** 767 (1995).
2. H. Kim, D.-H. Kim, and Y. H. Lee, Opt. Express **12,** 4812 (2004).
3. D. Abookasis, O. Montal, O. Abramson, and J. Roson, Appl. Opt. **44,** 3019 (2005).
4. Y. Li, K. Kreske, and J. Rosen, Appl. Opt. **39,** 5295 (2000).
5. H. Stark, *Image Recovery Theory and Application* (1st edn.) (Academic, New York, 1987) p.29, p.277.
6. G. Situ and J. Zhang, Opt. Lett. **29,** 1584 (2004).
7. D. Mas, J. Garcia, C. Ferreira, L. M. Bernardo, and F. Marinho, Opt. Commun. **164,** 233 (1999).
8. Z. Yu and G. Jin, *Computer-Generated Holograms* (1st edn.) (in Chinese) (Tsinghua University Press, Beijing, 1984).
9. J. W. Goodman and A. M. Silvestri, IBM J. Res. Develop. **14,** 478 (1970).
10. W. J. Dallas, Appl. Opt. **10,** 673 (1971).