Diffractive-optical processing of temporal signals, part I: basic principles

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The topic of this paper is the utilization of time for optical information processing. As clock rates in computing and communication systems increase and reach the THz border, optical techniques for signal filtering, shaping and clock distribution become attractive. We discuss the use of optics in temporal processing and consider in particular diffractive solutions. In part one of this paper, we discuss the basic concepts of temporal optics.

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Most optical and spectroscopic instruments are described in terms of spatial coordinates only. The temporal aspects are often neglected. Those instruments are operating in a time-stationary fashion. Some exceptions do exist but they are often considered not to be fundamental. However, that attitude has changed during recent decades for two reasons: laser light, with its narrow temporal frequency spectrum is available now. And the optical communications technology requires and provides high-speed optical devices and processors.

Time does play an important role in optics. A few scientists paid attention to the temporal aspects of light already long before it became a central issue around 1970 in the context of optical communications. In his paper, "Die Freiheitsgrade von Strahlenbündeln" (The degrees of freedom of bundles of light rays) published in 1914, Laue[1] introduced implicitly what would now be called the "time-bandwidth product" of an optical signal. Even earlier, Talbot described a diffraction experiment in white light, and discovered the so-called "Talbot bands" (not to be confused with Talbot's self-imaging)[2]. This experiment has been largely ignored by most text book authors. It is worth reviewing, however, for historical reasons and because it provides a simple example of the role of the group velocity in interferometry[3,4].

In order to categorize the different aspects of "time in optics", we look at the scheme shown in Table 1 utilizing the analogy to spatial optics. From this matrix, we can deduce four categories for the role of time in optics: first, topics that can directly be described in the t-domain, second, topics for which a frequency description in ν_t is appropriate, third, the space-time isomorphism with analogous phenomena in both domains, and fourth, the coupling of the spatial and temporal domain.

Table 1. Categorization of Spatial and Temporal Optical Phenomena. The Mathematical Description Can Be Done by Using the Spatial and Temporal Coordinate, Respectively, or by Going to the Fourier Domain

	Time	Space
Direct	$s(t)$	$u(x)$
Fourier-Inverse	$\tilde{s}(\nu_t)$	$\tilde{u}(\nu_x)$

Before briefly discussing the four categories, we will start by comparing light propagation in the spatial and temporal dimension. Propagation in space is described by Fresnel diffraction (Fig. 1). Diffraction represents spatial dispersion and is described mathematically by a quadratic phase term in the exponent of the diffraction integral

$$u_0(x,y) = \iint \tilde{u}_0(\nu_x,\nu_y) \exp\left[2\pi i\left(\nu_x x + \nu_y y\right)\right] \mathrm{d}\nu_x \mathrm{d}\nu_y, \quad (1)$$

$$u(x,y,z) = \iint \tilde{u}_0(\nu_x,\nu_y) \times \exp\left\{2\pi i\left[\nu_x x + \nu_y y - \left(\frac{\lambda z}{2}\right)\left(\nu_x^2+\nu_y^2\right)\right]\right\} \mathrm{d}\nu_x \mathrm{d}\nu_y, \quad (2)$$

here, the term including $\nu_x^2+\nu_y^2$ describes diffraction or, as we might say, "spatial dispersion". Propagation in the temporal domain (for example, of a light wave in an optical fiber, see Fig. 2) is described by

$$s_0(t) = \iint \tilde{s}_0(\nu_t) \exp\left(-2\pi i \nu_t t\right) \mathrm{d}\nu_t, \quad (3)$$

$$s(t,L) = \iint \tilde{s}_0(\nu_t) \exp\left\{-2\pi i\left[\nu_t\left(t+L/c\right) - D\nu_t^2\right]\right\} \mathrm{d}\nu_t. \quad (4)$$

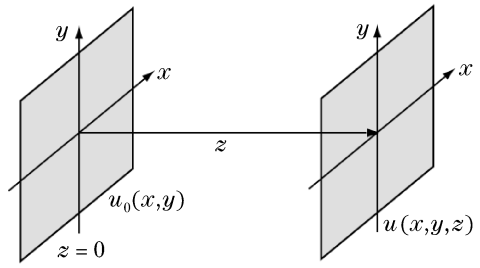

Fig. 1. Fresnel propagation of an optical wavefield from plane $z=0$ to $z>0$. The complex amplitude at $z=0$ is $u_0(x,y)$, at $z>0$ it is denoted by $u(x,y,z)$.

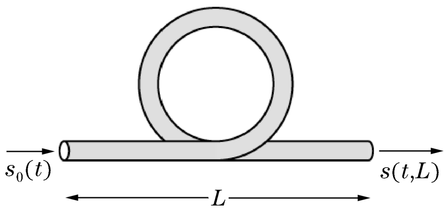

Fig. 2. Propagation of an optical wave over a distance L (for example, in an optical fiber) results in a corresponding time delay. In addition, one observes dispersion.

The expression L/c is just a delay, the second term in the exponent, $D\nu_t^2$, is the temporal dispersion. It is the analog to the quadratic expression of Eq. (2). Note: we consider only quadratic and no higher phase terms for the sake of simplicity.

The first category relates to the fundamental properties of light. An optical wave is a function of three spatial and the temporal coordinates and here we explicitly consider its properties with respect to t. One may consider one of the fundamental theorems of optics, Fermat's principle, as an example of the temporal properties of light: it states that the optical path of a light ray is a minimum[5]

$$\frac{1}{c}\int_{P_1}^{P_2} n\mathrm{d}s = \int_{t(P_1)}^{t(P_2)} \mathrm{d}t = \Delta t, \qquad (5)$$

here, the integral on the left side represents the optical path, the integral on the right side the duration for the light propagation. The time coordinate may come into play in an implicit manner (as in the case of Fermat) by the propagation delay of a light signal. Or it may play an explicit role as in the case of a modulated light signal. Von Laue, already mentioned above, went on to discuss the temporal modulation of light. He distinguished between "true" and "pseudo"-modulation of light depending on the ratio of the modulation frequency and the optical (carrier) frequency of the light signal[6]. The first "true" modulation of light was achieved by Connes *et al.* in 1962[7].

Connes' work may also be viewed as belonging to the second category about the spectral aspects of light signals. The whole complex of optical spectroscopy belongs here with its many different aspects and long history. Spectroscopic techniques have been developed for centuries because the power spectrum $|\tilde{u}(\nu_t)|^2$ of a signal $u(t)$ is much easier to observe than the signal itself. However, as we will discuss later, many interferometric devices known from spectroscopy can be useful also for direct processing of optical signals in the time domain. The most widely known spectrometer is probably the grating diffraction interferometer. It is also considered in the experiment about the aforementioned "Talbot bands". Talbot noticed that one of the two first diffraction orders is modulated by a fringe pattern if a glass plate is put halfway into the illuminating beam before the grating (Fig. 3). The other first order band remains unchanged. An explanation describing the experiment as a "curious dispersion" effect is found in Ref. [3]. Of course, we should mention the most prominent and timely case of time optics experiment, the "frequency comb", which has

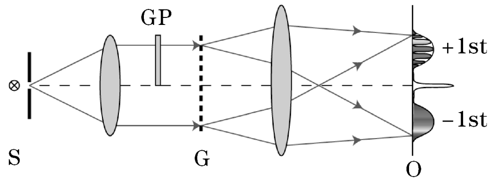

Fig. 3. Experimental setup for observing Talbot bands: a plane wave illuminated a grating spectrometer. By inserting a glass plate (GP) halfway into the aperture, the spectrum in the +1st order is modulated by interference fringes while the −1st order remains (visibly) unchanged.

recently been honored by the 2005 Nobel prize awarded to Hänsch[8]. Frequency comb techniques will allow one, for example, to control the amplitude of ultrafast laser pulses.

A light-wave is a spatio-temporal phenomenon and there exists an isomorphism between the spatial and the temporal effects. The essence of this analogy[9] can be stated as follows: the temporal frequency corresponds to the spatial frequency in the lateral direction, the time t corresponds to the z-coordinate indicating the optical axis of light propagation. The space-time duality can be used, for example, to explain pulse compression schemes[10−13], temporal imaging[14], and filtering[15].

The coupling of the temporal and spatial coordinates occurs naturally due to the propagation delay. It has been exploited in several ways. The first was probably Denisyuk *et al.*[16] who started efforts on the recording of the propagation of an optical wave as later made popular by Abramson[17] and Bartelt *et al.*[18]. The frequency resolved optical gating (FROG) method for measuring ultrafast optical pulses is another example[19]. The space-time-coupling is also the reason for time limitations in high-speed optical parallel processors[20,21]. As the basis of their considerations in Ref. [21], Lohmann *et al.* discussed imaging as a degeneracy of Fermat's principle. Degeneracy means, that by using a lens, not only one ray but many rays travel from a particular object point P_1 to the image point P_2 with identical delays. This concept of "Fermat degeneracy" can be extended further from single lens to telecentric imaging systems to serve as a model for the analysis of temporal and spatial optical systems. As we will discuss later in part II, space-time-coupling can also be utilized in order to implement temporal interconnection such as the perfect shuffle[22,23] and filtering operations.

The handling of information has three aspects: transmission, processing, and storage. (We omit the aspect of display here.) The variables may be the spatial coordinates or the time coordinate. Furthermore, the information can be represented in an analog (continuous or discrete) or in a digital format. All these possibilities open up a large combinatoric space. Much literature about the various subjects can be found[24−26].

Here, we are concerned with the processing of temporal optical signals. Our interest is to provide solutions for the frequency range beyond the THz border where electronics just runs out of bandwidth. The fastest electronic transistors can be operated at about 600 GHz. On the other hand, optical communications systems based on fiber-optics can go much farther. So, in order to move beyond the electronics-based regime of optical fiber communications, optical techniques for switching, processing, and storage have to be extended to the THz domain[27,28]. The processing tasks may involve issues such as pulse shaping, filtering, clocking, encryption, etc..

Short optical pulses with durations in the ps/fs regime are used for many purposes in optical information technology. The temporal processing may serve different purposes as mentioned above: pulse compression, pulse shaping, filtering, ultra-precise time measuring by means of the "frequency comb", etc.. The processing of an optical signal requires suitable devices. Their realization depends on the pulse width. For fs-pulses optical interferometers are used, some of them were already cited above, like the Treacy interferometer. The Treacy interferometer is an example of an double grating setup used in configuration where the +1st order is combined with the −1st order of the second grating. One can observe that quite often, optical filtering is achieved by double grating devices. However, other possibilities exist as well: resonator-type interferometers like the Fabry-Perot interferometer or integrated waveguide-optical ring resonators[29,30].

The different filter devices can be categorized in terms of linear systems theory: we can distinguish between finite impulse response (FIR) and infinite impulse response (IIR) filters. Fabry-Perot[7] and ring resonator structures are of IIR-type, Mach-Zehnder and grating interferometers are examples for FIR-filters. The structure of a FIR-filter is shown in Fig. 4. The input signal, $s_{\rm in}(t)$, is split up into n branches. The nth branch is delayed by $n\tau$ and weighted with a coefficient a_n. The output signal $s_{\rm out}(t)$ is hence given as the convolution ($*$) of the input with the temporal impulse response $h(t)$

$$s_{\rm out}(t) = \sum_{n=0}^{N} s_{\rm in}(t - n\tau) = s_{\rm in}(t) * h(t). \quad (6)$$

Instead of using the temporal impulse response, a linear filter can also be equivalently described by its transfer function

$$\tilde{H}(\nu) = \int_{-\infty}^{+\infty} h(t)\exp(2\pi i\nu t)\mathrm{d}t. \quad (7)$$

A simple example for an optical FIR-filter is a diffraction grating. In the direction of the 1st order, for example, the time delay $\tau = \lambda/c$.

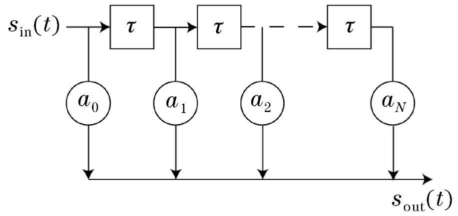

Fig. 4. Schematic representation of a tapped-delay line (or finite impulse response) filter. The incoming signal $s_{\rm in}(t)$, is divided into N branches, each branch gets delayed and weighted differently. Finally all branches are recombined to form the output signal, $s_{\rm out}(t)$.

In the first part of this paper we have provided an overview of the different aspects of time and space in information optics. In particular, we have categorized the field into four areas which explain in a systematic way the variety of phenomena. The most prominent and timely case of time optics experiment, the "frequency comb", has recently been honored by the 2005 Nobel prize awarded to Th. Hänsch[8].

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