Invariance of spectrum and polarization of electromagnetic Gaussian Schell-model beams propagating in free space

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Received November 7, 2005

The propagation of polychromatic electromagnetic Gaussian Schell-model (EGSM) beams in free space is investigated. It is shown that the spectral degree of polarization, spectral degree of coherence, and normalized spectrum change generally on propagation. The conditions of keeping the spectral invariance and keeping polarization invariance for the polychromatic EGSM beams are derived respectively. The results indicate that the constraints on the parameters of EGSM source to keep polarization invariance on propagation are more rigorous than those to keep invariance of the normalized spectrum.

OCIS codes: 030.1640, 260.5430, 300.6170.

In recent years there have been growing interests in the changes of the polarization, the coherence and the spectrum of a random electromagnetic beam propagating in free space or in a turbulent atmosphere[1−8]. It has been found that the polarization degree of a random electromagnetic beam may change on propagation, even if in free space[1,3]. A unified theory has been developed, which makes it possible to study the changes not only in the degree of polarization, but also in the coherence and the spectrum[5]. More recently a class of electromagnetic Gaussian Schell-model (EGSM) beams has been studied, because these beams are the simplest known model of random electromagnetic beams[4]. Moreover an approach for generating such beams was proposed[6]. In this letter the propagation of polychromatic EGSM beams in free space, and the changes either in the polarization or in the normalized spectrum on propagation are investigated.

Let us choose that a planar, secondary, electromagnetic Gaussian Schell-model (EGSM) source at $z = 0$ plane can be expressed by 2×2 (electric) cross-spectral density matrix [5]

$$\underline{\underline{W}}^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega) = W_{ij}^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega) = \langle E_i^*(\mathbf{r}_1',\omega)E_j(\mathbf{r}_2',\omega)\rangle, \quad (i,j=x,y) \qquad (1)$$

here

$$W_{ii}^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega) = \sqrt{S_i^{(0)}(\mathbf{r}_1',\omega)}\sqrt{S_i^{(0)}(\mathbf{r}_2',\omega)}\exp\left[-\frac{(\mathbf{r}_1'-\mathbf{r}_2')^2}{2\delta_{ii}^2(\omega)}\right] = S_i^{(0)}(\omega)\exp\left[-\frac{\mathbf{r}_1'^2+\mathbf{r}_2'^2}{4\sigma_i^2(\omega)}\right]\exp\left[-\frac{(\mathbf{r}_1'-\mathbf{r}_2')^2}{2\delta_{ii}^2(\omega)}\right], \quad (i=x,y) \qquad (2)$$

and

$$W_{xy}(\mathbf{r}_1',\mathbf{r}_2',\omega) = W_{yx}^*(\mathbf{r}_1',\mathbf{r}_2',\omega) = |B_{xy}|\sqrt{S_x^{(0)}(\mathbf{r}_1',\omega)}\sqrt{S_y^{(0)}(\mathbf{r}_2',\omega)}\cdot\exp\left[-\frac{(\mathbf{r}_1'-\mathbf{r}_2')^2}{2\delta_{xy}^2(\omega)}\right]$$
$$= |B_{xy}|\sqrt{S_x^{(0)}(\omega)}\sqrt{S_y^{(0)}(\omega)}\cdot\exp\left[-\left(\frac{\mathbf{r}_1'^2}{4\sigma_x^2(\omega)}+\frac{\mathbf{r}_2'^2}{4\sigma_y^2(\omega)}\right)\right]\exp\left[-\frac{(\mathbf{r}_1'-\mathbf{r}_2')^2}{2\delta_{xy}^2(\omega)}\right], \qquad (3)$$

where $\sigma_x(\omega)$, $\sigma_y(\omega)$, $\delta_{xx}(\omega)$, $\delta_{yy}(\omega)$, and $\delta_{xy}(\omega)$ are positive constants, and are generally dependent on frequency ω. $1 \geq |B_{xy}| \geq 0$. $S_x^{(0)}(\omega)$ and $S_y^{(0)}(\omega)$ are also positive constants. $\mathbf{r}_1'$ and $\mathbf{r}_2'$ are two position vectors at the source plane. The asterisk in Eq. (1) stands for the complex conjugate. Under paraxial approximation, EGSM source can radiate a EGSM beam, whose cross-spectral density of the beam at the z plane ($z > 0$) can be given by[3]

$$\underline{\underline{W}}(\mathbf{r}_1,\mathbf{r}_2,z,\omega) = W_{ij}(\mathbf{r}_1,\mathbf{r}_2,z,\omega) = \iint \mathrm{d}\mathbf{r}_1' \iint \mathrm{d}\mathbf{r}_2{}' \cdot W_{ij}(\mathbf{r}_1',\mathbf{r}_2',\omega)\cdot K^*(\mathbf{r}_1,\mathbf{r}_1',z,\omega)\cdot K(\mathbf{r}_2,\mathbf{r}_2',z,\omega)$$
$$= (\frac{k}{2\pi z})^2 \iint \mathrm{d}\mathbf{r}_1' \iint \mathrm{d}\mathbf{r}_2{}' W_{ij}(\mathbf{r}_1',\mathbf{r}_2',\omega)\cdot\exp\left[-ik\frac{(\mathbf{r}_1-\mathbf{r}_1')^2-(\mathbf{r}_2-\mathbf{r}_2')^2}{2z}\right], \qquad (4)$$

here $\mathbf{r}_1$ and $\mathbf{r}_2$ are two position vectors in the z plane.

Substituting Eqs. (2) and (3) into Eq. (4), one can obtain 2×2 (electric) cross-spectral density matrix at the z plane ($z > 0$), so that the spectrum, the spectral degree of polarization and the spectral degree of coherence of the beam at the z plane are given respectively by[7]

$$S(\mathbf{r},\omega) = Tr\underline{\underline{W}}(\mathbf{r},\mathbf{r},\omega), \qquad (5)$$

and

$$P(\mathbf{r},\omega) = \sqrt{1-\frac{4\mathrm{Det}\underline{\underline{W}}(\mathbf{r},\mathbf{r},\omega)}{\left[\mathrm{Tr}\underline{\underline{W}}(\mathbf{r},\mathbf{r},\omega)\right]^2}}, \qquad (6)$$

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\mathrm{Tr}\underline{\underline{W}}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{\mathrm{Tr}\underline{\underline{W}}(\mathbf{r}_1,\mathbf{r}_1,\omega)}\sqrt{\mathrm{Tr}\underline{\underline{W}}(\mathbf{r}_2,\mathbf{r}_2,\omega)}}, \quad (7)$$

where $\mathbf{r}$ is position vector in z plane, Tr denotes the trace, and Det denotes the determinant of the matrix. Equation (7) expresses the coherence of a pair of field points $\mathbf{r}_1$ and $\mathbf{r}_2$.

The spectrum, polarization, and coherence of the source are, in general, dependent on the frequency ω, and the spectrum of the source generally depends on the source point. Now we give some examples to illustrate the changes in polarization, spatial coherence and spectrum of the beams on propagation. For brevity, we assume that

$$\sigma_x(\omega)=\sigma_y(\omega)=\sigma_0, \quad (8)$$

and

$$S_x^{(0)}(\omega)=S_y^{(0)}(\omega)=S^{(0)}(\omega). \quad (9)$$

Equation (8) indicates that the beam spots of x- and y-polarization of EGSM source are the same and are independent of frequency ω. Equation (9) denotes that the spectra of x- and y-polarization of the source are also the same. Under these assumptions, the degree of polarization of across the source plane is uniform, which is given by

$$P^{(0)}(\mathbf{r}')=|B_{xy}|\,. \quad (10)$$

The normalized spectrum across the source is also assumed to be uniform, which is of Gaussian type of entered frequency ω_0 and spectral width Γ

$$S^{(0)}(\omega)=S_0\exp\left\{-\frac{(\omega-\omega_0)^2}{2\Gamma^2}\right\}, \quad (11)$$

here S_0 is a constant.

The spectral degree of polarization at the field points can be obtained as

$$P(\mathbf{r},z,\omega)=\sqrt{1-\frac{4\frac{\Delta_{xx}^2(z,\omega)}{\Delta_{yy}^2(z,\omega)}\exp\left[-\frac{r^2}{2\sigma_0^2}\left(\frac{1}{\Delta_{xx}^2}+\frac{1}{\Delta_{yy}^2}\right)\right]-4\left|B_{xy}\right|^2\frac{\Delta_{xx}^4(z,\omega)}{\Delta_{yy}^4(z,\omega)}\exp\left[-\frac{r^2}{\sigma_0^2\Delta_{xy}^2(z,\omega)}\right]}{\exp\left[-\frac{r^2}{\sigma_0^2\Delta_{xx}^2(z,\omega)}\right]+\frac{\Delta_{xx}^4(z,\omega)}{\Delta_{yy}^4(z,\omega)}\exp\left[-\frac{r^2}{\sigma_0^2\Delta_{yy}^2(z,\omega)}\right]+2\frac{\Delta_{xx}^2(z,\omega)}{\Delta_{yy}^2(z,\omega)}\exp\left[-\frac{r^2}{2\sigma_0^2}\left(\frac{1}{\Delta_{xx}^2}+\frac{1}{\Delta_{yy}^2}\right)\right]}}. \quad (12)$$

As shown from Eq. (12), the polarization will generally change on propagation, and the polarization in the field point is generally dependent on frequency ω. A grey-scale plot for the degree of polarization of the light beam propagating in free space is given in Fig. 1. The parameters for calculation are chosen as $\sigma_0=0.1$ mm, $\delta_{xx}=0.1$ mm, $\delta_{yy}=0.1$ mm, $\delta_{xy}=0.17$ mm, and $\omega=3\times10^{15}$ s^{-1}. We find that the polarization of the beam changes on propagation in free space. It is shown in Fig. 1 that the on-axis polarization increases with the increase of z, and tends to a maximum degree of polarization as the propagation distance is larger than 800 mm (in Fig. 1, the maximum degree of polarization is 0.6). Moreover for a fixed z plane, the maximum degree of polarization occurs at on-axis point, and the degree of polarization decreases with the increase of the transverse distance from the axis r.

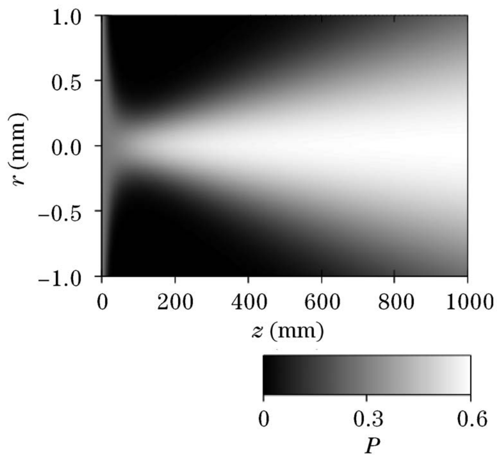

Fig. 1. Grey-scale plot for the distribution of the degree of polarization of the EGSM beam propagating in free space. The parameters for calculation are chosen as $\sigma_0=0.1$ mm, $\delta_{xx}=0.1$ mm, $\delta_{yy}=0.1$ mm, $\delta_{xy}=0.17$ mm, $\omega=3\times10^{15}$ s^{-1}.

Now we turn to the spectrum of the EGSM beams on propagation. We can obtain the spectrum in the field as given by

$$S(r,z,\omega)=S^{(0)}(\omega)M(r,z,\omega), \quad (13)$$

where the spectral modifier

$$\begin{aligned}M(r,z,\omega)=&\frac{1}{1+\frac{z^2c^2}{(\omega\sigma)^2}\left(\frac{1}{4\sigma^2}+\frac{1}{\delta_{xx}(\omega)}\right)}\\&\times\exp\left\{-\frac{r^2}{2\sigma^2\left[1+\frac{z^2c^2}{(\omega\sigma)^2}\left(\frac{1}{4\sigma^2}+\frac{1}{\delta_{xx}(\omega)}\right)\right]}\right\}\\&+\frac{1}{1+\frac{z^2c^2}{(\omega\sigma)^2}\left(\frac{1}{4\sigma^2}+\frac{1}{\delta_{yy}(\omega)}\right)}\\&\times\exp\left\{-\frac{r^2}{2\sigma^2\left[1+\frac{z^2c^2}{(\omega\sigma)^2}\left(\frac{1}{4\sigma^2}+\frac{1}{\delta_{yy}(\omega)}\right)\right]}\right\}, \quad (14)\end{aligned}$$

which is generally dependent on both the field points (r and z) and the parameters (σ_0, δ_{xx}, δ_{yy}) of the source. We can find from Eqs. (13) and (14) that the normalized spectrum of the field points is generally different from that of the source, and the spectrum changes with different field points. This phenomenon is called the Wolf effect, due to the source violating the scaling law[8]. If the parameters of the light beams along x- and y-polarization are the same, Eq. (13) is simplified to the scalar case, which has been discussed in Ref. [9]. The changes in spectrum of light on propagation within scalar approximation have been extensively studied[10,11].

It may be more interesting to find the conditions for ensuring the invariance of polarization and normalized spectrum on propagation. We first of all study the conditions for the invariance of normalized spectrum of the EGSM beams. It is seen from Eqs. (13) and (14) that, to ensure invariance of spectrum, the spectral modifier $M(r,z,\omega)$ should be independent of frequency ω. From Eq. (14), we readily obtain the conditions for ensuring the spectral invariance on propagation, as follows

$$\delta_{xx}(\omega)=\frac{2\sigma_0}{\sqrt{4\alpha_x^2\omega^2\sigma_0^2-1}}, \quad (15)$$

$$\delta_{yy}(\omega)=\frac{2\sigma_0}{\sqrt{4\alpha_y^2\omega^2\sigma_0^2-1}}, \quad (16)$$

where α_x and α_y are two constants. Equations (15) and (16) are the scaling law for EGSM source. In deriving Eqs. (15) and (16), the assumptions (Eqs. (8) and (9)) have been employed. Equations (15) and (16) indicate that only both x- and y-polarization of the EGSM source satisfy the scaling law, the normalized spectrum of the EGSM beams will keep the same throughout the space, from the near zone to the far field. When the field can be described by scalar field, the scaling law for the EGSM beam is changed into that for scalar GSM beam[9].

In order to keep invariance invariance for the polarization of EGSM beams, the degree of polarization (Eq. (12)) must be independent of frequency ω and the field points. Moreover the degree of polarization in the field points must be equal to that of the source, i.e.,

$$P(r,z,\omega)=|B_{xy}|. \quad (17)$$

Substituting Eq. (17) into Eq. (12), one obtains that the parameters of the EGSM source must satisfy

$$\sigma_x(\omega)=\sigma_y(\omega)=\sigma_0, \quad (18)$$

$$\delta_{xx}(\omega)=\delta_{yy}(\omega)=\delta_{xy}(\omega)=\frac{2\sigma_0}{\sqrt{4\alpha_0^2\omega^2\sigma_0^2-1}}, \quad (19)$$

where α_0 is a constant.

Contrasting Eqs. (18) and (19) to Eqs. (15) and (16), we readily find that the constraints on the invariance of the polarization are more rigorous than those for the invariance of the normalized spectrum. It is seen from Eqs. (15) and (16) along with assumption Eq. (8) that to keep spectral invariance on propagation, the dependences of the $\delta_{xx}(\omega)$ and $\delta_{yy}(\omega)$ on the frequency ω are given by Eqs. (15) and (16). However in order that the polarization is unchanged on propagation, three parameters $\delta_{xx}(\omega)$, $\delta_{yy}(\omega)$, and $\delta_{xy}(\omega)$ must be the same and their dependence on the frequency ω must be given by last formulas of Eqs. (18) and (19). This indicates that even when Eqs. (15) and (16) are satisfied, i.e., the normalized spectrum keeps invariance on propagation, the polarization may change if Eqs. (18) and (19) are not satisfied. In order to keep the invariance for the polarization, more rigorous conditions on the parameters of the source (i.e., Eqs. (18) and (19)) must be satisfied.

In conclusion, the propagation of polychromatic EGSM beams in free space is investigated. The results show that the spectral degree of polarization, spectral degree of coherence, and spectrum generally change on propagation. The conditions of keeping the spectral invariance and polarization invariance for the polychromatic EGSM beams, respectively, are derived. The results indicate that the constraints on the invariance of polarization are more rigorous than those for invariance of spectrum.

I am indebted to Prof. Emil Wolf at the University of Rochester (USA) for his helpful discussions and encouragement. This work was supported by the National Natural Science Foundation of China under Grant No. 60477041. J. Pu's e-mail address is jixiong@hqu.edu.cn.

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