Manipulation role of the relative phase and incoherent pumping on a light pulse propagation

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In an open Λ type system with the spontaneously generated coherence (SGC), when the probe and driving fields have different frequencies, the switching of the group velocity of the probe pulse from subluminal to superluminal is realized not only by adjusting values of the relative phase between the probe and driving fields but also by varying values of the incoherent pumping rate. For the subluminal propagation, the system always exhibits the probe absorption, however, the superluminal propagation is always companied with gain of the probe field.

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In a medium, when the peak of the light pulse travels at the group velocity, the group velocity could be slower (subluminal light) or faster (superluminal light) than the velocity of light in a vacuum and even negative. In recent years, there have been many studies on both subluminal and superluminal light propagation[1−13]. For superluminal light, the real information is not carried at the group velocity and causality is preserved[1]. For subluminal light, there are some interesting possibilities for practical applications such as optical delay lines and optical memories. In the viewpoint of potential applications of light propagation, a question of interest is whether one can have a controlling parameter in a single experiment for switching from subluminal to superluminal propagation. Agarwal *et al.*[7] have presented that the variation of a coupling field connecting the two lower metastable states in Λ system can lead to a weak pulse propagation changing from subluminal to superluminall. Bortman-Arbiv *et al.*[8] have shown that the probe pulse propagation switching from subluminal to superluminal can be realized by changing the relative phase between the probe and driving fields. Sahrai *et al.*[13] have demonstrated that the tunable control of the group velocity of a weak probe pulse from subluminal to superluminal can be obtained by phase variation of one of the control fields in an extended Λ type system with two extra control fields and an extra energy level. However, these studies all are for the closed system. In this paper, we investigate the effects of the relative phase between the probe driving fields and the incoherent pumping on the group velocity of the probe field (GVPF) in an open Λ type three-level system with the spontaneously generated coherence (SGC), when the weak probe field and the strong driving field have different frequencies.

The open Λ-type atomic system is illustrated in Fig. 1. The spontaneous emission rates from upper level $|1\rangle$ to the two closely spaced lower levels $|2\rangle$ and $|3\rangle$ are $2\gamma_2$ and $2\gamma_1$, respectively. The atomic injection rates for levels $|2\rangle$ and $|3\rangle$ are J_2 and J_3, respectively. The atomic exit rate from the cavity is γ_0. The weak probe fields with Rabi frequency G_p and the center frequency ω_p, and an incoherent pumping field with a rate $2R$ are applied between states $|1\rangle$ and $|3\rangle$; the strong driving fields with Rabi frequency G_c and the center frequency ω_c are coupled to the states $|1\rangle$ and $|2\rangle$. In the rotating wave frame the density matrix equations of motion for the system can be written as

$$\dot{\rho}_{11} = -2(\gamma_1+\gamma_2)\rho_{11} + 2R\rho_{33} - \gamma_0\rho_{11} + iG_\mathrm{c}\rho_{21} - iG_\mathrm{c}^*\rho_{12} + iG_\mathrm{p}\rho_{31} - iG_\mathrm{c}^*\rho_{13}, \tag{1}$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{11} - \gamma_0\rho_{22} + iG_\mathrm{c}^*\rho_{12} - iG_\mathrm{c}\rho_{21} + J_2, \tag{2}$$

$$\dot{\rho}_{33} = 2\gamma_1\rho_{11} - (2R+\gamma_0)\rho_{33} + iG_\mathrm{p}^*\rho_{13} - iG_\mathrm{p}\rho_{31} + J_3, \tag{3}$$

$$\dot{\rho}_{23} = -[R + i(\Delta_\mathrm{p}-\Delta_\mathrm{c})]\rho_{23} + 2p\sqrt{\gamma_1\gamma_2}\rho_{11} + iG_\mathrm{c}^*\rho_{13} - iG_\mathrm{p}\rho_{21}, \tag{4}$$

$$\dot{\rho}_{12} = -(\gamma_1+\gamma_2+i\Delta_\mathrm{c})\rho_{12} + iG_\mathrm{p}\rho_{32} - iG_\mathrm{c}(\rho_{11}-\rho_{22}), \tag{5}$$

$$\dot{\rho}_{13} = -(\gamma_1+\gamma_2+R+i\Delta_\mathrm{p})\rho_{13} + iG_\mathrm{c}\rho_{23} - iG_\mathrm{p}(\rho_{11}-\rho_{33}), \tag{6}$$

where $\rho_{ij}\,(i\neq j)$ is the atomic polarization between states

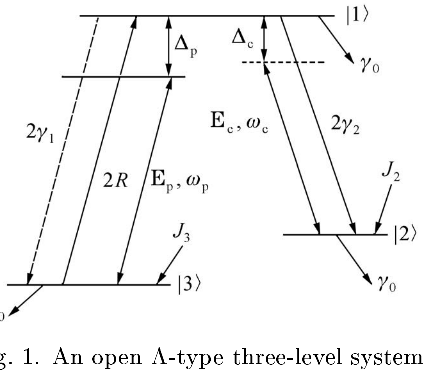

Fig. 1. An open Λ-type three-level system.

$|i\rangle$ and $|j\rangle$, $\rho_{ii}(i = 1-3)$ is the population of the state $|i\rangle$. $\Delta_{\rm p}(=\omega_{\rm p}-\omega_{13})$ and $\Delta_{\rm c}(=\omega_{\rm c}-\omega_{12})$ are the detunings of the probe and driving fields from their relevant atomic transitions, respectively. $p\sqrt{\gamma_1\gamma_2}$ represents SGC effect resulting from the cross coupling between the spontaneous emission $|1\rangle \rightarrow |3\rangle$ and $|1\rangle \rightarrow |2\rangle$. $p \equiv \overrightarrow{\mu}_{12}\cdot\overrightarrow{\mu}_{13}/|\overrightarrow{\mu}_{12}||\overrightarrow{\mu}_{13}| = \cos\theta$, θ represents the angle between the two dipole moments $\overrightarrow{\mu}_{12}$ and $\overrightarrow{\mu}_{13}$. When the dipoles $\overrightarrow{\mu}_{12}$ and $\overrightarrow{\mu}_{13}$ are parallel, $p = 1$, and SGC effect is maximized, whereas when the diploes $\overrightarrow{\mu}_{12}$ and $\overrightarrow{\mu}_{13}$ are orthogonal, then $p = 0$ and there is no SGC effect. The SGC effects have been widely discussed[14−20]. Because of the existence of SGC, the properties of system can be changed by varying the relative phase of the applied fields. If defining $\phi_{\rm p}$ and $\phi_{\rm c}$ as the phase of the probe and driving fields, respectively, then $G_{\rm c} = g_{\rm c}\exp(-i\phi_{\rm c})$, $G_{\rm p} = g_{\rm p}\exp(-i\phi_{\rm p})$ ($g_{\rm c}$ and $g_{\rm p}$ are real parameters). Redefining atomic variables in Eq. (1) as $\tilde{\rho}_{ii} = \rho_{ii}$, $\tilde{\rho}_{12} = \rho_{12}\exp(i\phi_{\rm c})$, $\tilde{\rho}_{13} = \rho_{13}\exp(i\phi_{\rm p})$, and $\tilde{\rho}_{23} = \rho_{23}\exp(i\Phi)$, where $\Phi = \phi_{\rm p}-\phi_{\rm c}$, we obtain equations for the redefined density-matrix elements which are identical to Eqs. (1)—(6) except that $2p\sqrt{\gamma_1\gamma_2}$, $G_{\rm c}$ and $G_{\rm p}$ are replaced by $p\sqrt{\gamma_1\gamma_2}\exp(i\Phi)$, $g_{\rm c}$ and $g_{\rm p}$, respectively. Under the steady state condition, in the limit of a weak probe, we obtain the linear analytical expression of $\tilde{\rho}_{31}$ as

$$\begin{aligned}\tilde{\rho}_{31} = \{&ig_{\rm p}[(R-i\Delta_{\rm p}+i\Delta_{\rm c})n_{13} - ig_{\rm c}\tilde{\rho}_{12}^{(0)}] \\ &-i2p\sqrt{\gamma_1\gamma_2}{\rm e}^{-i\Phi}g_{\rm c}\tilde{\rho}_{11}\}/ \\ &[(\gamma_1+\gamma_2+R-i\Delta_{\rm p})(R-i\Delta_{\rm p}+i\Delta_{\rm c})+g_{\rm c}^2],\end{aligned} \quad (7)$$

where

$$\tilde{\rho}_{11} = [2\Gamma_{12}(R_2Q_2+2RQ_3)g_{\rm c}^2 + 2R\gamma_0Q_3D_1]/D,$$

$$\tilde{\rho}_{12}^{(0)} = [g_{\rm c}\Delta_{\rm c} + ig_{\rm c}\Gamma_{12}(R_{12}J_3+R_3J_2)]/D,$$

$$n_{13} = \tilde{\rho}_{11}^{(0)} - \rho_{33}^{(0)}, \quad \tilde{\rho}_{33}^{(0)} = [2\gamma_1\tilde{\rho}_{11}^{(0)} + J_3]/R_2,$$

$$\tilde{\rho}_{11}^{(0)} = [2\Gamma_{12}(R_2J_2+2RJ_3)g_{\rm c}^2 + 2R\gamma_0J_3D_1]/D,$$

$$\Gamma_{12} = \gamma_1+\gamma_2, \quad \Gamma_{13} = \gamma_1+\gamma_2+R,$$

$$R_{12} = 2R(2\gamma_2-\gamma_0), \quad R_1 = 2R+\gamma_1+\gamma_0,$$

$$R_2 = 2R+\gamma_0, \quad R_3 = (2R+\gamma_0)(2\gamma_2+\gamma_0)+2\gamma_0\gamma_1,$$

$$D_1 = (\gamma_1+\gamma_2)^2+\Delta_{\rm c}^2, \quad D = (4\Gamma_{12}R_1g_{\rm c}^2+R_3D_1)\gamma_0,$$

$$Q_2 = J_2+A, \quad Q_3 = J_3-A-\frac{\gamma_0}{2R}(A+B),$$

$$B = ig_{\rm p}(\tilde{\rho}_{13}^{(0)}-\tilde{\rho}_{31}^{(0)}), \quad \tilde{\rho}_{13}^{(0)} = \frac{i2p\sqrt{\gamma_1\gamma_2}{\rm e}^{-i\Phi}g_{\rm c}\tilde{\rho}_{11}^{(0)}}{\Gamma_{12}(R-i\Delta_{\rm c})+g_{\rm c}^2},$$

$$A = ig_{\rm p}g_{\rm c}\frac{i\Gamma_{12}(\tilde{\rho}_{32}^{(0)}+\tilde{\rho}_{23}^{(0)})+\Delta_{\rm c}(\tilde{\rho}_{32}^{(0)}-\tilde{\rho}_{23}^{(0)})}{D_1},$$

$$\tilde{\rho}_{23}^{(0)} = \frac{2p\sqrt{\gamma_1\gamma_2}{\rm e}^{-i\Phi}\Gamma_{13}\tilde{\rho}_{11}^{(0)}}{\Gamma_{13}(R-i\Delta_{\rm c})+g_{\rm c}^2}.$$

In our notation, if ${\rm Im}\tilde{\rho}_{31} > 0$, the system exhibits gain for the probe field; if ${\rm Im}\tilde{\rho}_{31} < 0$, the probe laser field is attenuated. The dispersion of the probe laser field is proportional to ${\rm Re}\tilde{\rho}_{31}$. The expression of GVPF can be written as[8]

$$\frac{c}{v_{\rm g}} - 1 = 2\pi\frac{N\mu_{13}^2}{\gamma_1\hbar} \times \left[{\rm Re}\frac{\tilde{\rho}_{31}\gamma_1}{g_{\rm p}} + \frac{\omega_{\rm p}}{\gamma_1}{\rm Re}\frac{\partial(\tilde{\rho}_{31}\gamma_1/g_{\rm p})}{\partial(\omega\gamma_1^{-1})}\bigg|_{\omega=\omega_{\rm p}}\right], \quad (8)$$

where N is the atomic density. If $(c/v_{\rm g})-1 > 0$, GVPF is smaller than c and the propagation is subluminal. If $(c/v_{\rm g})-1 < 0$, GVPF is larger than c (even negative) and the propagation is superluminal.

In the following we analyze the effects of the relative phase and incoherent pumping rate on GVPF by using the numerical results from Eqs. (7) and (8) when the weak probe field and the strong driving field have different frequencies. The unit of $(c/v_{\rm g})-1$ is $2\pi N\mu_{13}^2/\hbar$, and other parameters are measured in γ.

Figure 2 illustrates ${\rm Re}\rho_{31}$, ${\rm Im}\rho_{31}$ and $(c/v_{\rm g})-1$ as functions of the relative phase Φ for different values of the incoherent pumping rate R. From Eq. (8) we know that $(c/v_{\rm g})-1$ is only relative to ${\rm Re}\rho_{31}$ and not to ${\rm Im}\rho_{31}$. But in order to reflect the property of the atomic medium corresponding subluminal or superluminal propagation of the light pulse, so in Fig. 2 we plot the curves ${\rm Im}\rho_{31}$ corresponding to $(c/v_g)-1$. From Fig. 2 we can see that: 1) ${\rm Re}\rho_{31}$, ${\rm Im}\rho_{31}$, and $(c/v_{\rm g})-1$ all are periodical functions of the relative phase Φ with the periodicity 2π. GVPF is continuous tunable over a value range even from subluminal to superluminal (Fig. 2(b)) and this can be realized just by changing values of the relative phase Φ while

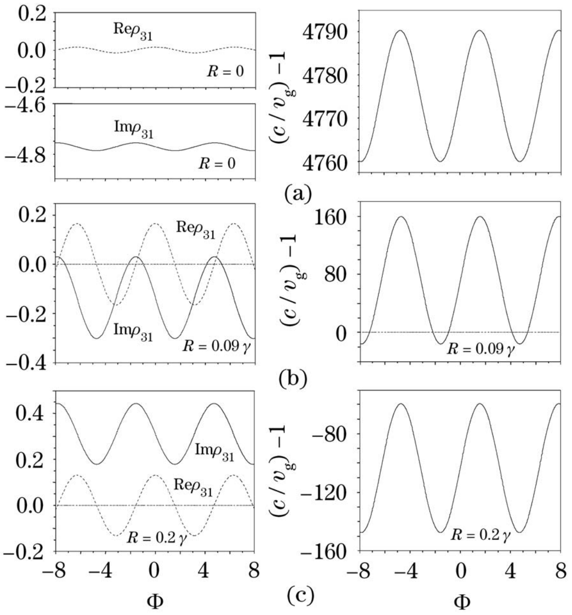

Fig. 2. ${\rm Im}\rho_{31}$, ${\rm Re}\rho_{31}$, and $(c/v_{\rm g})-1$ as functions of the relative phase Φ between the probe and driving fields for different values of the incoherent pumping rate R with $\gamma_1 = \gamma_2 = 0.1\gamma$, $\omega_{13} = 0.05\gamma$, $\Delta_{\rm c} = 0$, $g_{\rm p} = 1\gamma$, $g_{\rm c} = 100\gamma$, $\gamma_0 = 0.01\gamma$, $S = 2$, and $\Delta_{\rm p} = 100\gamma$.

other parameters are kept unvarying. 2) When $R = 0$ (Fig. 2(a)), i.e., the incoherent pumping does not exist, we can only obtain the subluminal propagation. With R increasing, the value range of $\mathrm{Re}\rho_{31}$ and the maximal value of $(c/v_{\mathrm{g}}) - 1$ decrease. When the incoherent pumping rate reaches to some value, for example, $R = 0.09\gamma$ (Fig. 2(b)), both the subluminal and superluminal propagations are possible by varying the value of relative phase Φ. When the incoherent pumping rate continues increasing and reaches to some value, for example, $R = 0.2\gamma$ (Fig. 2(c)), only the superluminal propagation occurs, the subluminal propagation disappears. 3) For subluminal propagation the system always exhibits the probe absorption. However, the superluminal propagation is always companied with gain of the probe field.

In summary, we have studied the GVPF in the open Λ type system with SGC when the weak probe field and the strong driving field have different frequencies. By using the numerical results from the steady analytical solutions of the density matrix equations of motion of the system, we analyzed the effects of the incoherent pumping rate and relative phase between the probe and driving fields on the probe gain (or absorption) and GVPF. We find that: 1) the switching of GVPF from subluminal to superluminal can be realized by varying the values of the incoherent pumping rate or the relative phase between the probe and driving fields. 2) The gain (absorption), dispersion, and group velocity of the probe field all are periodical functions of the relative phase Φ with the periodicity 2π. 3) The subluminal propagation is always companied with the absorption while the superluminal propagation with gain. And this is much different from the result that in a closed V type system with the SGC the superluminal propagation is always accompanied with absorption of the probe field[8].

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