Four-quadrant spatial phase-shifting Fourier transform digital holography for recording of cosine transform coefficients

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A new one-step four-quadrant spatial phase-shifting Fourier transform digital holography is presented for recording of cosine transform coefficients, because cosine transform is a real-even symmetric Fourier transform. This approach implements four quadrant spatial phase shifting at a time using a special phase mask, which is located in the reference arm, and the phase distributions of its four-quadrants are 0, $\pi/2$, π, and $3\pi/2$ respectively. The theoretical analysis and computer simulation results show that cosine transform coefficients of real-valued image can be calculated by capturing single four-quadrant spatial phase-shifting Fourier transform digital hologram.

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Optical cosine transform of image was proposed owing to its highly parallel operation[1]. However, since the cosine transform coefficients is real number of negative or positive, only capturing the intensity of cosine transform coefficients is not sufficient to reconstruct the image. Using holography to record the coherent field is an efficient method to solve this problem.

Phase-shifting digital holography is one of the modern holographies, in which in-line arrangement with phase-shifting technique is adopted to decrease the maximum spatial frequency of the interference pattern of charge coupled device (CCD) plane and to eliminate the direct current (DC) component and conjugate images[2−4], and it has been widely applied in microscopy, encryption, pattern recognition[5], watermarking of three-dimensional (3D) objects[6], and surface contouring[7]. In four-step phase-shifting digital holography, wave-front amplitude and phase on CCD plane can be derived from four holograms sequentially, which are recorded using reference wave with different phase shifts of 0, $\pi/2$, π, and $3\pi/2$ respectively. The four-step phase-shifting digital holography cannot be used for instantaneous measurement of moving objects because a relatively long time is required to record the holograms by using temporal phase shifts. For instantaneous measurement of moving objects, a one-step parallel quasi-phase-shifting digital holography was proposed[8], in which the technique implements four kinds of phase shifting at the same time using a phase shifting array of 2×2 phase retarder.

In this letter, we propose the four-quadrant spatial phase-shifting Fourier transform digital holography for recording of cosine transform coefficients of real-valued image, because cosine transform is the real-even symmetric Fourier transform. This approach implements one-step four-quadrant spatial phase shifting using a special phase mask, and phase distributions of its four quadrants are 0, $\pi/2$, π, and $3\pi/2$ respectively, which is located in the reference arm.

Figure 1 shows the optical implement of two-dimensional (2D) Fourier transform based on a lens and a spatial light modulator (SLM). A real-even symmetric image (shown in Fig. 2) can be created from the original image with computer by horizontal and vertical reflection, and can be transmitted to the SLM which is placed at the front focal plane of the lens. Since cosine transform is real-even symmetric Fourier transform, the cosine transform coefficients of original image can be obtained from Fourier transform coefficients of real-even symmetric image. The complex amplitude transmittance of SLM is given by

$$g(x,y)=\begin{cases} f(x,y) & \text{if} \quad x\geq 0, y\geq 0 \\ f(-x,y) & \text{if} \quad x\leq 0, y\geq 0 \\ f(x,-y) & \text{if} \quad x\geq 0, y\leq 0 \\ f(-x,-y) & \text{if} \quad x\leq 0, y\leq 0 \end{cases}, \tag{1}$$

where $f(x,y)$ and $g(x,y)$ are original image and real-even symmetric image. The Fourier transform of real-even symmetric image is denoted as $G(u,v)$, which can be obtained at the back focal plane when the SLM is

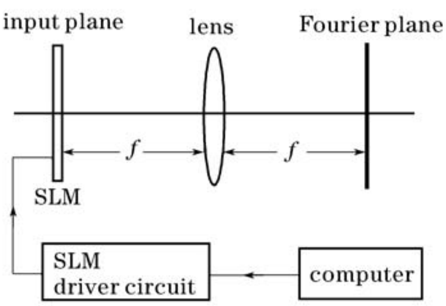

Fig. 1. Optical realization of Fourier transform.

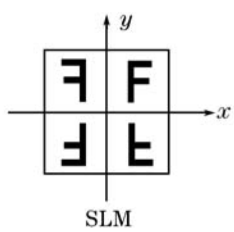

Fig. 2. Real-even symmetric image from original image.

illuminated by collimated planar laser. Because Fourier transform of a real-even function is still a real-even function, $G(u,v)$ is a real-even function, which can be represented as

$$G(u,v) = G(-u,v) = G(u,-v) = G(-u,-v). \qquad (2)$$

Cosine transform coefficients of original image can be obtained by extraction part of $G(u,v)$, when $u \geq 0$ and $v \geq 0$.

Optical setup of four-quadrant spatial phase-shifting Fourier transform in-line digital holography for recording of cosine transform coefficients is shown in Fig. 3. It is based on a Mach-Zehnder interferometer architecture. In Fig. 3, M_1, M_2 are reflection mirrors, BS1 and BS2 are beam splitters, f is focal length of imaging lens. The phase distribution of phase mask is depicted in Fig. 4, phase distributions of four quadrants of phase mask are 0, $\pi/2$, π, and $3\pi/2$ respectively. An expanded and collimated linear polar laser beam is divided into the reference beam and object beam. The object beam illuminates the SLM placed at the front focal plane of a Fourier lens. The reference beam illuminates the phase mask, whose phase distributions of four quadrants are 0, $\pi/2$, π, and $3\pi/2$ respectively. The phase mask is placed at a distance of $2f$ in front of the imaging lens, and the imaging lens is placed at a distance of $2f$ in front of CCD sensors. The phase mask is imaged onto the CCD sensor by an imaging lens so that the phase distributions of reference wave are 0, $\pi/2$, π, and $3\pi/2$ respectively on the four quadrants of CCD sensor. The object beam and reference beam interfere with each other on the CCD plane placed at the back focal plane of the Fourier lens to form an in-line Fourier

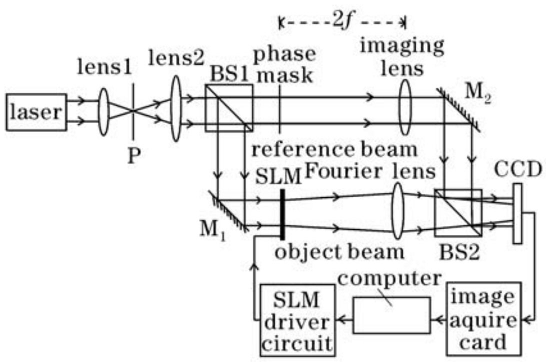

Fig. 3. Optical setup of four-quadrant spatial phase-shifting Fourier transform in-line digital holography for recording of cosine transform coefficients.

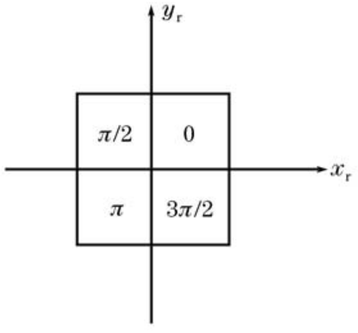

Fig. 4. Phase distribution of phase mask.

transform digital hologram which can be recorded by the CCD camera and transmitted to computer.

CCD plane is described as u-v plane. We denote the complex reference wave at the CCD plane as

$$R(u,v) = A_{\mathrm{r}} \exp\{i[\phi_{\mathrm{r}}(u,v) + \theta]\}, \qquad (3)$$

where A_{r} and θ are constant amplitude and initial phase of reference wave and can be substituted by 1 and 0 respectively. $\phi_{\mathrm{r}}(u,v)$ is the phase delay function of phase mask, which can be represented as

$$\phi_{\mathrm{r}}(u,v) = \begin{cases} 0 & \text{if} \quad u \geq 0, v \geq 0 \\ -\pi/2 & \text{if} \quad u < 0, v \geq 0 \\ -\pi & \text{if} \quad u < 0, v < 0 \\ -3\pi/2 & \text{if} \quad u \geq 0, v < 0 \end{cases}. \qquad (4)$$

Fourier transform of the real-even symmetric image is given by

$$G(u,v) = A_{\mathrm{o}}(u,v) \exp[i\phi_{\mathrm{o}}(u,v)], \qquad (5)$$

where $A_{\mathrm{o}}(u,v)$ and $\phi_{\mathrm{o}}(u,v)$ are amplitude and phase of the object wave respectively. The hologram intensity at the CCD plane is given by the coherent superposition of object wave and reference wave.

$$I(u,v) = |G(u,v) + R(u,v)|^2$$
$$= |G(u,v)|^2 + A_{\mathrm{r}}^2 + 2A_{\mathrm{r}}A_{\mathrm{o}}(u,v)\cos[\phi_{\mathrm{o}}(u,v) - \phi_{\mathrm{r}}]. \qquad (6)$$

By capturing a single hologram, the amplitude and the phase of the object wave at the CCD plane can be calculated by

$$A_{\mathrm{r}}A_{\mathrm{o}}(u,v) = \frac{1}{4}\left\{[I(u,v) - I(-u,-v)]^2 + [I(u,-v) - I(-u,v)]^2\right\}^{\frac{1}{2}}, \qquad (7)$$

$$\phi_{\mathrm{o}}(u,v) = \arctan\left[\frac{I(u,-v) - I(-u,v)}{I(u,v) - I(-u,-v)}\right], \qquad (8)$$

where A_{r} is constant amplitude of reference wave and can be substituted by 1, and the cosine transform coefficients can be obtained by

$$C(u,v) = \begin{cases} G(u,v) & \text{if } u \geq 0 \text{ and } v \geq 0 \\ 0 & \text{otherwise} \end{cases}. \qquad (9)$$

To demonstrate the method described above, we simulate its experiment process with computer. The experimental setup of our simulation is shown in Fig. 3. The CCD array is placed at the back focal plane of a Fourier lens. The CCD has an array of 1024×1024 pixels, and the size of each pixel is 10×10 (μm). The light source is a He-Ne laser with wavelength of 632.8 μm. The original image is a pure amplitude (real-valued) object with a size of 512×512 pixels as shown in Fig. 5(a). Figure 5(b) is the real-even symmetric image on the SLM. Figure 5(c) is the four-quadrant spatial phase-shifting Fourier transform digital hologram recorded by the CCD camera. Figure 5(d) is the cosine transform coefficients obtained by four-quadrant spatial phase-shifting Fourier transform

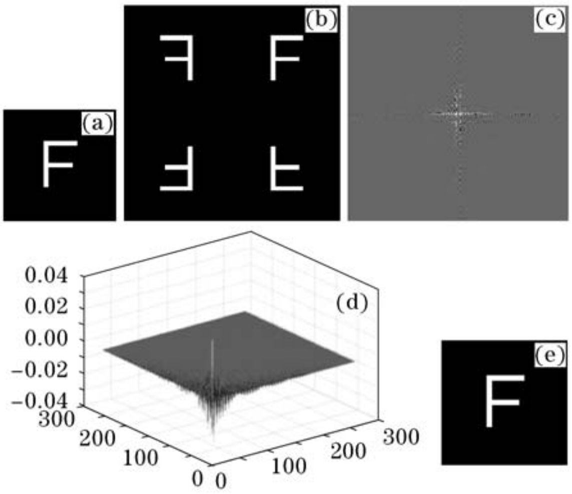

Fig. 5. Computer simulation results. (a) Original image; (b) real-even symmetric image extended from original image; (c) four-quadrant spatial phase-shifting Fourier transform digital hologram at CCD plane; (d) cosine transform coefficients calculated from four-quadrant spatial phase-shifting Fourier transform digital hologram; (e) reconstruction image.

digital holography. Figure 5(d) shows that the most energy of cosine transform coefficients tends to be concentrated in a few low frequency coefficients with positive and negative values, and high frequency coefficients are very close to zero. Figure 5(e) shows the reconstruction image by using inversion cosine transform.

The method proposed here requires that the optical axis center of the real-even symmetric Fourier transformation should be aligned quite well with those of four-quadrant spatial phase mask. However, it is difficult to satisfy this criterion in actual; there are some errors generally. In order to find the influence of these errors on the results of the cosine transformation, numerical simulations are performed and discussed as follows.

Figure 6 shows the reconstruction images using our method when the alignment errors between optical axis center of Fourier transformation and those of four-quadrant spatial phase mask along x and y axes are 100, 200, 300, and 400 μm. It can be found that with the increase of the error, the low-frequency information is lost more seriously, leading to worse quality of the reconstruction image. When the error is 400 μm, only the boundary of the image is left. When the error is larger than 800 μm, it is hard to recognize the original image from the rebuilding results.

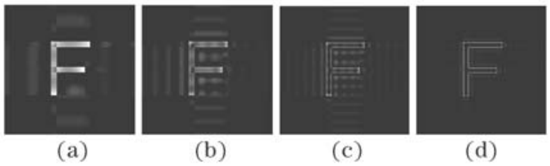

Fig. 6. Reconstruction images under different alignment errors (100 (a), 200 (b), 300 (c), 400 μm (d)) between optical axis center of Fourier transformation and those of four-quadrant spatial phase mask along x and y axes.

We can use CCD to evaluate this error in actual; then it provides an alignment method. The reference wave path is blocked at first; the optical axis center of the Fourier transformation can be aligned according to the real-even symmetry of the cosine transformation. Then open the reference path and record the coherent field. The coefficients of the cosine transformation can be obtained according to the method presented above. With the increase of the alignment error, the energy of cosine transform coefficients is lost more seriously; so the center alignment can be adjusted by monitoring the energy of cosine transform coefficients obtained by CCD.

In conclusion, we have proposed the four-quadrant spatial phase-shifting Fourier transform digital holography to record the cosine transform coefficients in this letter. The theoretical analysis and computer simulation have shown that the cosine transform coefficients of real-valued image can be calculated by capturing single spatial phase-shifting Fourier transform digital hologram.

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