Effects of PMD on dispersion managed soliton links with coarse-step approach

Ming Xu (徐 铭) and Jianhua Ji (吉建华)

Advanced Technology Research Center of Information Engineer College, Shenzhen University, Shenzhen 518060

Received June 30, 2005

Statistical distributing and magnitude of the first and second-order polarization mode dispersion (PMD) vectors are evaluated and the timing displacements of its effects on dispersion-managed soliton (DMS) are also given with help of coarse-step approach, such as Jones matrix (JME) and coupled nonlinear Schrödinger equations (CNLSE). The results showed that the coarse-step approach can not only simulate the statistical characteristics of PMD, but also compute the nonlinear pulse characteristic of timing and energy jitters evolution affected by PMD. The presented results are very useful to simplify the measurement of second-order PMD and instructively reveal the degree of PMD effects in DMS systems.

OCIS codes: 060.5530, 060.2330, 060.2420, 320.7110.

Recently, the dispersion-managed solitons (DMSs) are of great interest in soliton communication systems, because they provide some prior advantages compared with conventional solitons, such as higher pulse energy and signal noise ratio, lower averaged dispersion line and timing jitter. However, in the dispersion managed systems, lower averaged dispersion line can lead to polarization mode dispersion (PMD) becoming even more obvious at higher bit rates and in longer transmission distance, which may induce to lots of dispersion waves and the differential group-delay (DGD) accumulating.

The performance degradation due to PMD effects in DMS systems has been quantified with experiment or simulated, but its statistical characteristic has not been analyzed[1−3]. At the same time the statistical properties of first- and second-order PMDs have already been derived, measured, and simulated[4,5], but it is in linear systems or at very low bit rates. Here, we will simplify the relationships between the first-order and second-order PMDs and simulate its effects on DMS transmission systems with coarse-step approach, which is very suited to dispersion managed systems.

We characterize PMD by the first- and second-order PMD vectors $\mathbf{\Omega}(\omega)$ and $\mathbf{\Omega}_\omega(\omega)$ in Stokes space

$$\mathbf{\Omega}(\omega) = \Delta\tau(\omega)\mathbf{q}(\omega), \tag{1}$$

$$\mathbf{\Omega}_\omega(\omega) = \Delta\tau_\omega(\omega)\mathbf{q}(\omega) + \Delta\tau(\omega)\mathbf{q}_\omega(\omega), \tag{2}$$

here $\mathbf{q}(\omega)$ aligned with the principal state of polarization (PSP) and $\mathbf{q}_\omega(\omega)$ is unit Stokes vector and differential with respect to the optical angular frequency ω, $\Delta\tau(\omega)$ is the differential group delay (DGD). Whereas $\mathbf{\Omega}_{\omega/\!/}$ = PCD = $\Delta\tau_\omega(\omega)\mathbf{q}(\omega)$ is parallel to $\mathbf{\Omega}(\omega)$ and leads to polarization-dependent chromatic dispersion (PCD), and $\mathbf{\Omega}_{\omega\perp} = \Delta\tau(\omega)\mathbf{q}_\omega(\omega)$ is perpendicular to $\mathbf{\Omega}(\omega)$ and the cause of PMD depolarization. In a first-order approximation both $\Delta\tau$ and $\mathbf{q}$ are frequency independent, which may induce the pulse widen or two polarized modes time displacement. second-order effects are represented by the linear DGD frequency dependence $\Delta\tau_\omega(\omega)$ and by a linear PSP rotation with frequency dependence $\mathbf{q}_\omega(\omega)$. $\Delta\tau_\omega(\omega)$ may induce the pulse evolved into dispersive waves or time displacement of two polarized modes. $\mathbf{q}_\omega(\omega)$ may induce the two polarized modes coupling randomly and lead to pulse power overshoot or pulse energy receded.

Now we use the Jones matrix (JME) method to evaluate these PMD components. The fiber is represented by lots of 2 × 2 complex transfer matrix of $\mathbf{T}(\omega) = \mathrm{e}^{-(\alpha+j\bar{\beta}(\omega))z}\mathbf{M}(\omega)$, where α and $\bar{\beta}$ are the fiber attenuation and the mean propagation constant, respectively. Here we use the conventional model of PMD in fibers, which is modeled as a cascade of many small segments N with constant birefringence. Assuming all the segments have the identical length $h_{\rm i}$, which is the mode-coupling length. The orientation θ of the birefringence and DGD varies randomly without correlation between adjacent segments, which satisfied with uniformity and Gaussian distribution, respectively. Then the sum of JME can be gotten as[5]

$$\mathbf{M}(\omega) = \prod_{i=1}^{N}\mathbf{M}_i(\omega) = \prod_{i}^{N}\mathbf{R}_i^{-1}(\omega)\mathbf{D}_i(\omega)\mathbf{R}_i(\omega)$$
$$= \prod_{i=1}^{N}\begin{pmatrix}\cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i\end{pmatrix}^{-1}\begin{pmatrix}\mathrm{e}^{k} & 0 \\ 0 & \mathrm{e}^{-k}\end{pmatrix}$$
$$\times\begin{pmatrix}\cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i\end{pmatrix}, \tag{3}$$

where $\mathbf{R}(\omega)$ is the rotating matrix of PSP and $\mathbf{D}(\omega)$ takes into account the different propagation speeds on the two PSPs. $k = j(\omega\Delta\tau/2 + \varphi_i)$, φ_i is perturbation of temperature which is uniformly distributing. For one segment DGD $\Delta\tau = \Delta\beta' h_{\rm i}$, $\Delta\beta'$ is inverse group velocity difference between the polarization components. After a number of randomly oriented segments, we can obtain the statistical average of the squared DGD as $\langle\Delta\tau^2\rangle = \Delta\beta'^2 h_{\rm i} z$. Since the DGD is a stochastic variable with a Maxwellian distribution function, the relation between its average and square average is $\langle\Delta\tau^2\rangle = \langle\Delta\tau\rangle^2 3\pi/8$. Usually, this is incorporated in the PMD coefficient of the fiber which defined as the average DGD per square root transmission distance: $D_{\rm PMD}^{\rm 1st} = \langle \Delta\tau \rangle / \sqrt{z} = \Delta\beta' \sqrt{8h_{\rm i}/3\pi}$. So $k = j(\sqrt{3\pi/8} D_{\rm PMD}^{\rm 1st} \omega \sqrt{h_{\rm i}}/2 + \varphi_i)$.

From the JME method, we can analyze the statistical characteristics of PMD components. But when we consider the PMD effects on the pulse of linear or nonlinear, the coupled nonlinear Schrödinger equations (CNLSE) must be also applied. The propagations of orthogonal polarized optical pulse components U and V in a dispersion-managed line are described in constantly birefringence fiber by[1]

$$i\left(\frac{\partial U}{\partial Z} + \delta_{\rm g}' \frac{\partial U}{\partial T}\right) + \frac{d(Z)}{2}\frac{\partial^2 U}{\partial T^2} + Q(|U|^2 + m|V|^2)U + \frac{1}{3}U^* V^2 \exp(-4i\Delta\beta Z) = 0, \quad (4a)$$

$$i\left(\frac{\partial V}{\partial Z} - \delta_{\rm g}' \frac{\partial V}{\partial T}\right) + \frac{d(Z)}{2}\frac{\partial^2 V}{\partial T^2} + Q(|V|^2 + m|U|^2)V + \frac{1}{3}V^* U^2 \exp(-4i\Delta\beta Z) = 0, \quad (4b)$$

where U and V are the x and y components of normalized electrical field, $\delta_{\rm g}' = \Delta\beta' L_{\rm D}/2T_0$. We have used the common soliton normalizations $Z = \frac{z}{L_{\rm D}}$, $T = \frac{t}{T_0}$, $Q(Z) = \gamma \exp(2G_0 Z)$, where $L_{\rm D} = \frac{T_0^2}{\beta_2}$ is dispersion length, β_2 and γ are group velocity dispersion (GVD) and nonlinear coefficient respectively, $T_0 = t_0/1.665$ for Gaussian pulse, t_0 is full pulse width at half of maximum power, G_0 is gain of amplifier. $d(Z) = -\beta_2(z)L_{\rm D}/T_0^2$ is the normalized varying group-velocity dispersion parameter in dispersion managed links, m represents coupling parameters between U and V, which usually equals to 3/2 for linear polarization fiber. The terms of $\exp(-4i\Delta\beta Z)$ can be ignored under the condition of pulse width $t_0 > 1$ ps. From the Eq. (4), the $\delta_{\rm g}'$ dependent or independent ($\delta_{\rm g}' = 0$) on frequency may induce the pulse components time delay and let time displacement of pulse two polarization modes, which can be seemed as effects of second-order or first-order PMD. Term included m factor may induce the two polarized modes coupling such as second-order of PMD effects. But owing to a nonlinear binding force of XPM that keeps their polarization components together, the soliton can withstand this splitting with moderate PMD. For DMS having the varying dispersion $d(Z)$, the novel mechanism of DMS is that they can trap some part of the radiation[6].

Now we consider that the pulse transmits one segment of constant birefringence fiber to others, the Jones matrix must be included. The approach of combining Eqs. (3) and (4) is named as so-called coarse-step approach[1]. This approach presupposes that polarization effects like repolarization are dominated but nonlinearity and dispersion play no significant role in these effects, which is reasonable in modern-day dispersion-managed systems because the local dispersion is large and the average dispersion is kept small. The new orthogonal polarized optical pulse components can be expressed as

$$\begin{bmatrix} U' \\ V' \end{bmatrix} = \mathbf{M}_i(\omega) \begin{bmatrix} U \\ V \end{bmatrix}. \quad (5)$$

We consider a dispersion-managed link made of an arrangement of $l_1 = 35$ km anomalous dispersion fiber $D_2 = 17$ ps/(nm·km) and $l_2 = 5$ km of normal dispersion fiber $D_2 = -118.2$ ps/(nm·km), yielding an averaged dispersion of $\bar{D} = 0.1$ ps/(nm·km), and assuming they have same loss and Kerr coefficients, $\alpha = 0.2$ dB/km, $n_2 = 3.2 \times 10^{-20}$ m^2/w, $A_{\rm eff} = 50$ μm^2. When we simulate the statistic of PMD, let $l = 160$ km, $N = 1000$, and the wavelength range from 1500 to 1580 nm. For simplify considering, we simulate the characteristic of statistical PMD, the $z_{\rm h} = 0.01$ km and $h_{\rm i} = l/N = 160/1000 = 0.16$ km. In long haul distance, we adopt the step $h_{\rm i} = z_{\rm h} =$ 0.1 km. To obtain adequate resolution in simulating $\Delta\tau_\omega$ and $\mathbf{q}_\omega$, we use the interleave steps of wavelength of 0.002 nm. $D_{\rm PMD}^{\rm 1st}$ is valued with 3, 1, 0.5, 0.3, and 0.1 ps/km$^{1/2}$. For $\langle \Delta\tau \rangle \Delta\lambda \le 1.0$ ps·nm, let JME step size be 0.008 nm. We now investigate the propagation of Gaussian pulse of $t_0 = 5$ ps, bit duration at 40 Gb/s in the system mentioned above with coarse step approach and assuming that the effects of dispersion slope are neglected.

Figure 1 shows the statistical distribution of DGD. It can be seen that the DGD has a Maxwellian distribution, and the mean DGD is 42.477 ps. The DGD value is so big because the dispersion management link is a upgrading system on the large PMD value common fiber. Figure 2 is the DGD and PCD varying with wavelength in 1500—1580 nm, which is the first and second-order PMD characteristic values. It can be seen that the PCD has larger than 420 ps/nm at some wavelengths, about 10 times DGD which is the same as the Refs. [4] and [5]. It must considere not only the first-order PMD but also the second-order PMD compensated in so larger PCD value system, particularly in 40 Gb/s system. Figures 3(a), (b), (c), and (d) are the statistical distributing of PCD, $|\mathbf{q}_\omega|$, $\Delta\tau_\omega$ and $\Omega_{\omega\perp}$, the root mean square (RMS) values are 161.8 ps/nm, 29.8276 ps, 407.15 ps/nm, and 1186.3 ps^2, respectively, but $\Delta\tau_\omega$ has a zero mean value. $\langle \Delta\tau^2 \rangle^2 / \langle (\Delta\tau_\omega)^2 \rangle$ is about 27.0804 ± 0.2895 consisted with the theory results[9] by other approach, which may prove our right simulating. From those figure, we can get

$$\sqrt{\langle \Omega_{\omega\perp}^2 \rangle} / \langle \Delta\tau \rangle^2 = 0.6423 \pm 0.0053 \sqrt{\left\langle \Omega_{\omega /\!/}^2 \right\rangle} / \langle \Delta\tau \rangle^2 = 0.2242 \pm 0.0041,$$

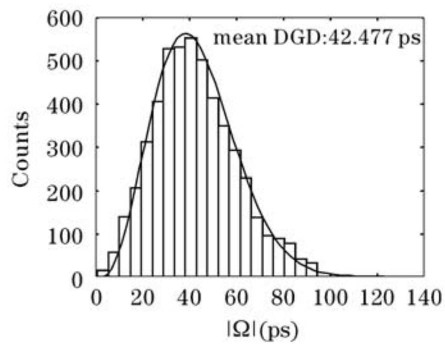

Fig. 1. Statistical distribution of DGD. Solid curve: Maxwellian PDF.

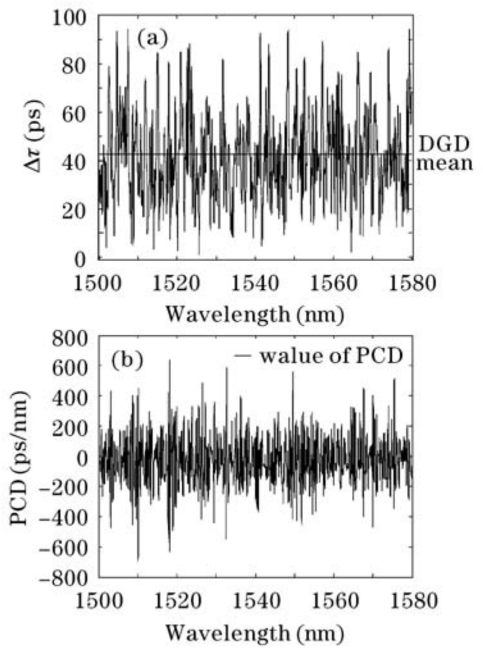

Fig. 2. DGD and PCD varying with the wavelength.

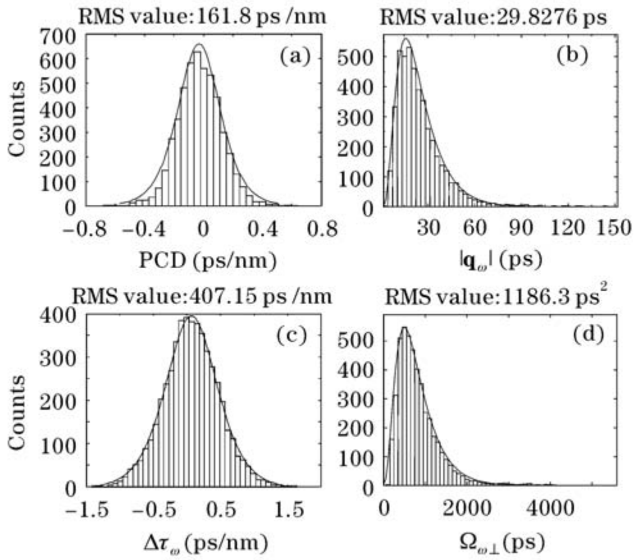

Fig. 3. Statistical distributions of PCD (a), $|\mathbf{q}_\omega|$ (b), $\Delta\tau_\omega$ (c), and $\Omega_{\omega\perp}$ (d). Solid curves of (a) and (c) are sech^2 probability density functions (PDFs).

$$\sqrt{\langle \mathrm{PCD}^2\rangle}/\langle\Delta\tau\rangle^2 = 0.0891 \pm 0.0022\sqrt{\langle|\mathbf{q}_\omega|^2\rangle}/\langle\Delta\tau\rangle$$

$$= 0.7102 \pm 0.0089.$$

The statistical distributing of both PCD and $\Delta\tau_\omega$ is the curve of sech^2, which is similar to the energy density of soliton. When we know the DGD, all of the vectors of second-order of PMD can be computed from above relations. The first-order and second-order PMD which may induce the pulse widen, energy recededs and time displacements of two polarized components are described in Figs. 4 and 5. Figure 4 is evolution of the normalized pulse energy and width of DMS along the normalized distance for different PMDs. Figure 5 is evolution of mean square of time displacement of DMS two polarization modes along the propagation line for different PMDs. It can be seen that PMD affects the energy and width of DMS distinctly. For example, when $D_{\mathrm{PMD}}^{\mathrm{1st}} = 1$ ps/km$^{1/2}$ and the transmission distance $24 \times 195 = 4680$ km, the pulse energy is 0.2 time of initial value, and pulse width is 5 times of initial value. Though these results have favorable performance than the linearly systems or common soliton systems[1] but may be very serious in high-speed transmission system, in which the bit slot is commonly 5 time to width of soliton. But in $D_{\mathrm{PMD}}^{\mathrm{1st}} =$ 0.1 ps / km$^{1/2}$, the PMD cannot hardly affect on DMS, which is very like with conventional soliton[8]. This situation is the same as evolution of mean square of time displacement of DMS two polarization modes. So, we must consider some control instrument in this high speed and long distance transmission system at $D_{\mathrm{PMD}}^{\mathrm{1st}} > 0.3$ ps/km$^{1/2}$. Figure 6 shows that the DMS can be more effects restraining the PMD inducing time displacement than linear pulse. The scheme of DMS will be more important in high speed and long distance communications.

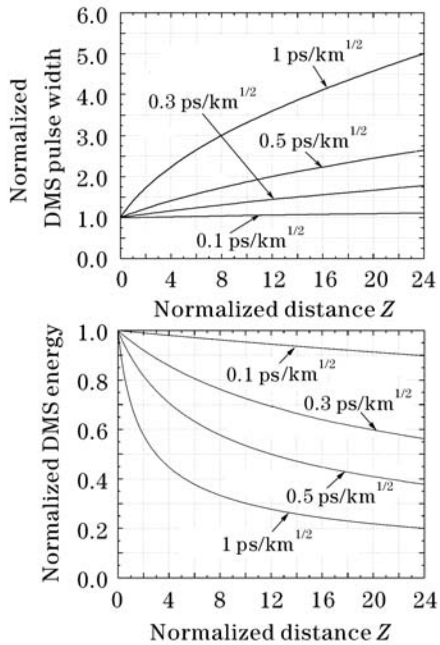

Fig. 4. Evolution of the normalized pulse width and energy of DMS along the normalized distance for different $D_{\mathrm{PMD}}^{\mathrm{1st}}$.

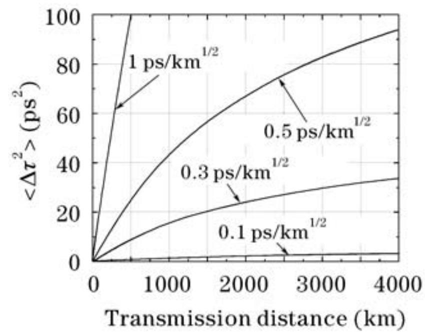

Fig. 5. Evolution of mean square of time displacement of DMS two polarization modes along the propagation lines for different $D_{\mathrm{PMD}}^{\mathrm{1st}}$.

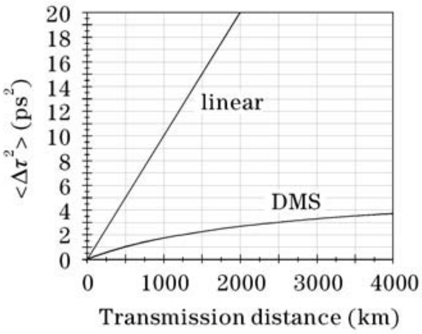

Fig. 6. Evolution of mean square of time displacement of two polarization modes (DMS and linear pulse) along the propagation line for PMD value $D_{\rm PMD}^{\rm 1st} = 0.1$ ps/km$^{1/2}$.

In the all, the results had declared that the statistical characteristics of PMD in DMS transmission systems are similar in linear and common soliton systems, but the DMS can restrain the PMD effects more effectively than common systems. We must consider some control instrument at $D_{\rm PMD}^{\rm 1st} > 0.3$ ps/km$^{1/2}$ in high speed such as 40 Gb/s and several thousand kilometers systems. The results of coarse-step approach are very consistent with other simulating method and theory results[1,6]. This means that coarse-step approach can not only simulate the statistical characteristics of PMD, but also can compute the nonlinear pulse characteristic of transmission evolution affected by PMD, such as timing and energy jitters. The presented results are very useful to simplify the measurement of second-order PMD and instructively revealed degree of PMD effects in DMS systems.

This work was supported by the National Natural Science Foundation of China (No. 60132040), and the Guangdong Natural Science Foundation Project (No. 04300855). M. Xu's e-mail address is xum@szu.edu.cn.

References

1. C. Xie, M. Karlsson, P. A. Andrekson, H. Sunnerud, and J. Li, IEEE J. Sel. Top. Quantum Electron. **8,** 575 (2002).
2. H. Sunnerud, M. Karlsson, C. Xie, and P. A. Andrekson, J. Lightwave Technol. **20,** 2204 (2002).
3. C. Xie, A. R. Grant, L. F. Mollenauer, and X. Liu, IEEE Photon. Technol. Lett. **16,** 111 (2004).
4. P. Ciprut, B. Gisin, R. Dassy, J. P. Von der Weid, F. Prieto, and W. Zimmer, J. Lightwave Technol. **16,** 757 (1998).
5. G. J. Foschini and C. D. Poole, J. Lightwave Technol. **9,** 1439 (1991).
6. T. I. Lakoba, Opt. Lett. **25,** 1789 (2000).
7. D. Wang and C. R. Menyuk, J. Lightwave Technol. **17,** 2520 (1999).
8. L. F. Mollenauer, K. Smith, and J. P. Gordon, Opt. Lett. **14,** 1219 (1989).
9. G. J. Foschini, L. E. Nelson, R. M. Jopson, and H. Kogelnik, J. Lightwave Technol. **19,** 1882 (2001).