Feature-oriented fuzzy bidirectional flow for image enhancement

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A fuzzy bidirectional flow is presented, which performs a fuzzy backward (inverse) diffusion along the gradient direction to the isophote lines (edges), while does a certain forward diffusion along the tangent direction on the contrary. Controlled by the image gradient magnitude, the fuzzy membership function guarantees image textures with a natural transition between two different areas. To preserve image features, the nonlinear diffusion coefficients are locally adjusted according to the directional derivatives of the image.

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Image enhancement is an important operation in the general fields of image processing and computer vision. Traditional image sharpening methods mainly increase the gray level difference across edges, while their width remains unchanged[1]. This is effective for enhancing edges that are narrow and of low contrast. However, for wide and blurry edges, increasing their contrast brings only very limited effect.

Over the past years the use of partial differential equations (PDEs) has grown significantly in these fields[2−6]. Initially proposed by Perona and Malik[4], the nonlinear anisotropic diffusion (AD) filters have been widely used in image denoising, enhancement, and sharpening. The gray levels of an image $u(x,y,t):\Omega\times[0,+\infty)\to R$, are diffused according to

$$\frac{\partial u(x,y,t)}{\partial t}=\mathrm{div}(g(|\nabla u(x,y,t)|)\nabla u(x,y,t)),\qquad(1)$$

where the scalar diffusivity $g(|\nabla u|)$, chosen as a non-increasing function, governs the behaviour of the diffusion process. By formally developing the divergence term, Eq. (1) can be put in the terms of second derivatives taken in the directions of the gradient vectors ($\overrightarrow{n}$) and in the orthogonal tangent ones ($\overrightarrow{t}$) (see Fig. 1)

$$\frac{\partial u}{\partial t}=(g'(|\nabla u|)\,|\nabla u|+g(|\nabla u|))u_{nn}+g(|\nabla u|)u_{tt},\qquad(2)$$

where u_{nn} and u_{tt} are the second normal and tangent derivatives of the image, respectively.

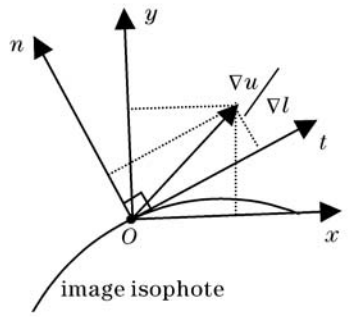

Fig. 1. Decomposition of the directional derivative.

Another PDE-based enhancement process was proposed by Alvarez and Mazorra[5], which couples an anisotropic diffusion with the shock filter (we call it ADSF)[6], yielding

$$\frac{\partial u}{\partial t}=-\mathrm{sign}(G_\sigma\times u_{nn})\,|\nabla u|+cu_{tt},\qquad(3)$$

where c is a positive constant, G_σ is a Gaussian kernel of standard deviation σ. The first term on the right side creates solutions approaching piecewise constant regions separated by shocks at the zero-crossings of the smoothed second derivative of the image along $\overrightarrow{n}$. The second term is an anisotropic diffusion along the level-set lines $\overrightarrow{t}$ (see Fig. 2).

Noticing the expression

$$\mathrm{sign}(s)=s/\,|s|\,,\quad s\neq 0,\qquad(4)$$

we put forward a unified bidirectional flow (BDF) equation covering Eqs. (1) and (3),

$$\frac{\partial u}{\partial t}=\alpha(-c_n(u_n,u_{nn},u_{tt})u_{nn})+\beta(c_t(u_n,u_{nn},u_{tt})u_{tt}),\qquad(5)$$

where α and β are the backward and forward flow control coefficients, $c_n(s)$ and $c_t(s)$ are the diffusion coefficients of their arguments, which should be properly designed to preserve features of the image such as edges, corners and

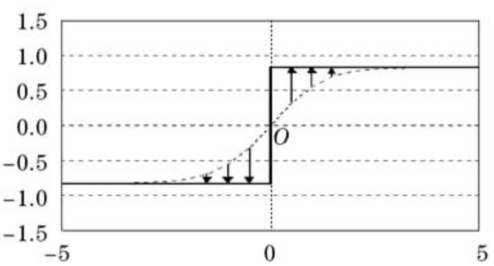

Fig. 2. Edge sharpening process (the solid line) compared with the original edge (the broken line).

fine part. We will discuss this issue below. In a word, we need two opposing forces of diffusion, acting simultaneously on the image: one is a backward force, to sharpen edges along $\overrightarrow{n}$, and the other is a forward one, used for suppressing noise and jaggies to smooth contours along $\overrightarrow{t}$.

To describe the fuzziness of information received from nature by human being, Zadeh[7] put forward the fuzzy sets theory, denoting the fuzzy set S in the region R as

$$S = \int \frac{\mu_S(x)}{x}, \quad x \in R, \tag{6}$$

where $\mu_S(x) \in [0,1]$ is called the membership function of S on R. Chen *et al.*[8] extended the above further to the generalized fuzzy sets, where they denoted the generalized membership function $\mu_S(x) \in [-1,1]$ to substitute $\mu_S(x) \in [0,1]$.

In Eq. (3), however, indicating edges by the zero-crossing using the symbol function $\mathrm{sign}(x)$ is a binary decision process, by which, unfortunately, the obtained result is a false piecewise constant image in some areas. Therefore, we substitute $\mathrm{sign}(x)$ in these areas by a hyperbolic tangent membership function $\mathrm{th}(x)$, which tends to 1 when a pixel tends to the upper edge, while tends to -1 when the pixel tends to the lower edge. At the same time, we use Gaussian smoothing to the second normal derivatives of the image to decide the zero-crossing, which can allow a robust sharpening process against noise. Combining fuzzy control of the changes of gray levels of the image beside the edge center and the Gaussian smoothing, we propose a fuzzy bidirectional flow (FBDF) equation

$$\begin{cases} v = G_\sigma * u, \ l{=}k\,|v_n| \\ \frac{\partial u}{\partial t} = \alpha(-c_n \mathrm{th}(l v_{nn})) + \beta(c_t u_{tt}) \end{cases} \tag{7}$$

with Neumann boundary condition, where l is a parameter to adapt the gradient of the membership function to different image areas controlled by the image gradient magnitude and the constant k, and then we adopt the following diffusion coefficients to suppress overshoots effectively and excess smoothness to fine part of an image

$$c_n = |u_n|/(1 + l_1 u_{nn}^2), \quad c_t = 1/(1 + l_2 u_{tt}^2), \tag{8}$$

where l_1 and l_2 are constants.

We use the flux limitation technique[9] in computing fluid dynamic to implement our scheme (Eqs. (7) and (8)). A number of images have been used to test it. Examples shown in Figs. 3—5 are the peppers and boats images. Here for the two images, we adopt the following

Fig. 3. Test images: left, peppers; right, boats.

Fig. 4. Enhancement of the peppers image by different methods: from top-left to bottom-right, noisy blurred image, images enhanced by AD, ADSF and FBDF, respectively.

Fig. 5. Enhancement of part of the boats image by different methods: from top-left to bottom-right, noisy blurred image, images enhanced by AD, ADSF and FBDF, respectively.

parameters: $[k, l_1, l_2] = [400, 9 \times 10^{-4}, 4 \times 10^{-4}]$, $[\alpha, \beta] = [3, 2]$. Generally the parameters need to be selected adaptively according to image characteristics and its degradation level. In Figs. 4 and 5, the noisy blurred images are enhanced by AD, ADSF and FBDF respectively. It can be seen that, although it denoises the image well specially in the smoother segments, AD produces a blurry image with relatively unsharp edges, showing that its ability to sharpen edges is not very great because of its somewhat unsuitable diffusion coefficients. As for ADSF, a binary decision process results in a false piecewise constant image which looks unnatural with a discontinuous transition between two different areas, though it sharpens edges very well. The best visual quality is obtained by enhancing images using FBDF, which preserves most features of the image with a natural transition between two different areas, and produces pleasing sharp edges and smooth contours while denoising the images effectively.

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