Adaptively wavelet-based image denoising algorithm with edge preserving

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A new wavelet-based image denoising algorithm, which exploits the edge information hidden in the corrupted image, is presented. Firstly, a canny-like edge detector identifies the edges in each subband. Secondly, multiplying the wavelet coefficients in neighboring scales is implemented to suppress the noise while magnifying the edge information, and the result is utilized to exclude the fake edges. The isolated edge pixel is also identified as noise. Unlike the thresholding method, after that we use local window filter in the wavelet domain to remove noise in which the variance estimation is elaborated to utilize the edge information. This method is adaptive to local image details, and can achieve better performance than the methods of state of the art.

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Removing noise from image is a basic task in many applications such as image restoration, video surveillance, and object recognition etc.. It is very difficult to find a suitable compromise between noise reduction and significant image detail preservation. To achieve good performance in the sense, a denoising algorithm should be adapted to local image details. Many efforts have been done to reduce noise while overcoming the drawback of blurring edges. The wavelet transform compresses the image information into relatively few, large coefficients that represent the main image details. Initially Donoho[1] proposed a wavelet shrinkage method to process each coefficient separately where the coefficient is set to zero or preserved decided by a threshold.

With the motivation of making algorithm preservation image details without over-smoothing image, many researchers developed adaptively denoising algorithms based on wavelet. Chang *et al.*[2] proposed an adaptive wavelet threshold method, in which the wavelet coefficients in each subband are modeled as generalized Gaussian distribution, whose parameters are estimated locally. To reduce complexity, Mihcak *et al.*[3] modeled wavelet image coefficients as zero-mean Gaussian random variables with high local correlation that is a simple spatially adaptive statistical model. In contrast to these methods, Cai *et al.*[4] identified the edge coefficients from the noisy ones specifically to improve the local adaptive property. But it is still imprecise because they classify the coefficients only with local variance.

Implicitly utilizing the edge information, Scharcanski *et al.*[5] proposed a new adaptive edge preservation image denoising method which models a coefficient as Gaussian random variable to be edge or noise. Unfortunately, the existence of noise prevents the precise discrimination of edges so that the noise information would be retained in image. Observed the fact that wavelet coefficients related to noise are less correlated across scales than coefficients related to edges, Xu *et al.*[6] multiplied the adjacent scales of wavelet coefficients to magnify the edges and suppress noise. Their method is a spatially selected filtering technique, where the product result is used to identify the edge characteristics by iterative selection. Zhang *et al.*[7] improved it by estimating the threshold more accurately. However, their binary decision of being edge or not by a threshold from the correlation coefficients may be too coarse to satisfy the edge preservation.

In this paper, we combine the two ideas, one of which is to estimate the variance in a local window and the other is to exploit the edge map identified from each subband, into the new denoising method. The significant points of the new method come from two aspects. The first one is the edge identification that is operated by a canny-like edge detector in each subband and the refinement in the multiplication domain of adjacent scale of coefficients. Another one is the local variance estimation in a rectangle window, which embeds the edge information in the estimation of a noise coefficient.

In all image denoising methods based on wavelet, whatever deterministic or probability method, the basic task is to discriminate the signal coefficients from noise ones. Edge coefficients compact most energy of each subband, which motivates the researchers to develop denoising technique by looking edges as signal and the others as noise[5−7]. With the existence of noise, the accurate extraction of edges is not an easy job.

To overcome the shortcoming of the method presented by Xu *et al.*[6], we propose a new edge detection technique by combining the edge implication from the canny-like detector and the correlation image of the subbands of adjacent scale.

The most edge detection methods identify the local maximum of first derivative as edge. And the edge detection methods have to compromise between the dislocation and the sensitivity to noise. To solve this problem, Canny[8] presented three principles of good edge detector. In this work, we put emphasis on good detection and good localization rather than low spurious response. Since we need identify the edge coefficients from the noisy ones for the denoising occasion, only one edge map for an image cannot satisfy the requirement. Like the spatial domain, we detect the edge with the first derivative in each wavelet subband to form edge map.

We presume that image pixels are corrupted by additive white Guassian noise (AWGN) with known variance σ_n^2. Note that after the orthogonal wavelet transform the noise can still be supposed as Gaussian white in each subband. Let $X_s^d(i,j)$ represent the wavelet coefficients of the ideal image, and the wavelet coefficients of the noisy image given by $Y_s^d(i,j) = X_s^d(i,j) + n_s^d(i,j)$, where d is the direction and s is the scale.

We use the Sobel mask to implement the derivative operator, which has less sensitivity to the noise than the others. To obtain a better separation between edge and noise, the gradient image can be thersholded. In order to detect the weak edges we use two thresholds. Specifically, like the Canny's method[8], the pixel with the derivative value larger than the high threshold T_h is stated as edge point and as the seed point of edge, while all the pixels in the neighborhood of a seed with a derivative value larger than the low threshold T_l are also marked as edge points. The two thresholds are calculated like the Canny operator[8]. The expression of edge indicator can defined as

$$\mathrm{edge}_s^d(i,j) = \begin{cases} 1, & |\nabla Y_s^d(i,j)| > T_\mathrm{h}(s,d) \\ & \quad \text{or } B_s^d(i,j) \text{ is true} \\ 0, & \text{other} \end{cases}, \tag{1}$$

where

$$B_s^d(i,j) = \begin{cases} \text{true}, & \text{if } K_s^d(i,j) \text{ is true} \\ & \quad \text{and } |\nabla Y_s^d(i,j)| > T_\mathrm{l}(s,d) \\ \text{false}, & \text{else} \end{cases}, \tag{2}$$

and $K_s^d(i,j)$ is true means that here exists $|\nabla Y_s^d(m+i, n+j)| > T_\mathrm{h}(s,d)$ in the 8-neighbourhood of (i,j).

The edge identification method in the previous subsection would produce many false edge pixels. To remove those fake pixels, the properties of wavelet transform can be exploited. Firstly, the edges are generally consecutive which connect each other to form a line in each subband. According to the constraint, the isolated edge pixels detected by the canny-like detector can be converted as noise ones. Secondly, the edge will produce large coefficients across scales while then noise will decrease rapidly, so multiplying the subbands of adjacent scales may enhance the edges and suppress noise. Many experiments suggest that it is sufficient to achieve the effect by employing two adjacent scales.

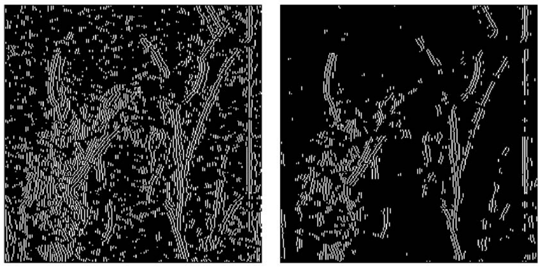
The point-wise products of the adjacent scales are given by

$$P_s^d(i,j) = Y_s^d(i,j) \cdot Y_{s+1}^d(i,j), \tag{3}$$

exploiting edge point will be highlighted, we can refine the edge detection result by

$$\mathrm{edge}_s^d(i,j) = \begin{cases} 1, & \text{if } |P_s^d(i,j)| > T_\mathrm{p}(s,d) \\ & \quad \text{and } \mathrm{edge}_s^d(i,j) = 1 \\ 0, & \text{else} \end{cases}, \tag{4}$$

where $T_\mathrm{p}(s,d)$ is a threshold calculated by the specific case of the magnification of each subband.



(a) (b)

Fig. 1. Edge pixels in one subband of Lena edge map by both thresholding method (a) and edge refinement method (b).

The practical suband example of Lena by the edge detection and refinement described in the above sections is shown in Fig. 1. The refinement excludes many false edges from the coarse edge detection result.

Whatever edge detection method is taken, it is unavoidable to make noise coefficients preserved as edges, especially in the case of low signal-to-noise ratio (SNR). In the other hand, the weak edge may be identified as noise. So, many denoising techniques that take the hard threshold method that keep edge coefficients unchanged and evaluate the noise related coefficients zero will not remove noise and preserve edge information effectively.

To improve the denoising performance, we introduce the local domain denoising method which utilizes the local statistical property of wavelet coefficients. The basic idea is that the estimation of variance of noise coefficient takes the respective class of coefficients rather than all the coefficients in the local window[3], in the mean while, an edge related coefficient is retained without any change.

In the related work[2−4], the wavelet coefficient is shrunk by a ratio of the ideal signal variance to the noisy signal variance according to the least minimum mean square error (LMMSE) rule. The shrinkage formula is

$$\hat{X}_s^d(i,j) = \frac{\left[\sigma_s^d(i,j)\right]^2}{\left[\sigma_s^d(i,j)\right]^2 + \sigma_n^2} Y_s^d(i,j), \tag{5}$$

where $\left[\sigma_s^d(i,j)\right]^2$ is the variance of ideal coefficient, and σ_n^2 is the variance of noise. Following the method of Ref. [9], σ_n^2 is estimated from the absolute median M_x of the highest frequency subband coefficients

$$\sigma_n = \frac{M_x}{0.6745}. \tag{6}$$

Because the variance of ideal coefficient is unknown, we can only estimate it by the noisy coefficients. The previous research estimates the $\sigma_s^d(i,j)$ based on the local neighborhood $N_s^d(i,j)$, that is

$$\left[\tilde{\sigma}_s^d(i,j)\right]^2 = \max\left(0, \frac{1}{(2M+1)^2} \sum_{k=i-M}^{k=i+M} \sum_{l=j-M}^{l=j+M} [Y_s^d(k,l)]^2 - \sigma_n^2\right), \tag{7}$$

where $2M+1$ is the width of the local square window.

The disadvantage of the estimation method is obvious that the variance may be large enough for a noise coefficient to be preserved as signal when it is around the edge. We always hope that the noise can be removed completely and the weak edge wrongly discriminated as noise can be reserved. The result is much affected by the estimation precision of variance. So, incorporated with edge map identified in the previous section, the variance estimation of a noise coefficient just utilizes all the noise related coefficients in its neighboring domain

$$\sigma_{s,d}^2(i,j) = \frac{1}{(2M+1)^2}\sum_{k=i-M}^{k=i+M}\sum_{l=j-M}^{l=j+M}\{[1-\mathrm{edge}_s^d(k,l)]\cdot Y_s^d(k,l)\}^2. \quad (8)$$

Thus the variance of ideal coefficient is given by

$$\left[\tilde{\sigma}_s^d(i,j)\right]^2 = \max\left(0, \sigma_{s,d}^2(i,j) - \sigma_n^2\right). \quad (9)$$

Replace $\sigma_s^d(i,j)$ with estimation $\tilde{\sigma}_s^d(i,j)$, the restored value of noisy coefficient can be obtained.

The adaptive image denoising algorithm is summarized as follows: Step I: decompose the image by undecimated wavelet transform. Step II: detect the edge coefficients in each subband by the approach described in section II, and form the edge map for each subband. Step III: for every coefficient in each subband except for the lowest subband: If it is an edge coefficient, keep its value unchanged; Else, estimate the ideal variance and restore the ideal coefficient with Eq. (5). Step IV: the restored coefficients are operated by inverse wavelet transform, and the denoising image is obtained.

We applied our method to two 512×512 images with artificial noise: the Lena and Barbara. The Daubecies-8 filter bank is used as wavelet base, and image is decomposed with three levels.

For objective evaluation, we choose the peak-signal-to-noise-ratio PSNR = $10\lg(255^2/\text{mean squared error})$ as the performance measure. Quantitative results for the two test images are summarized in Table 1.

The first method is the Matlab's image denoising technique wiener2. The second method is the hard threshold for overcomplete wavelet decomposition. The third method chooses the locally adaptive window-based denoising using MAP from Ref. [3], and the window is 7×7.

Table 1. PSNR [dB] Results for Several Methods

	Noise Standard Deviation σ_n			
	10	15	20	25
	Lena			
Wiener2	33.58	31.14	28.96	27.18
Hard Threshold	33.90	32.09	30.79	29.68
LAWMAP[3]	34.24	32.27	30.92	29.90
This Paper	34.89	33.16	31.95	30.89
	Barbara			
Wiener2	29.76	28.12	26.74	25.42
Hard Threshold	31.10	28.78	27.00	25.77
LAWMAP[3]	32.50	30.13	28.57	27.40
This Paper	33.44	31.15	29.60	28.30

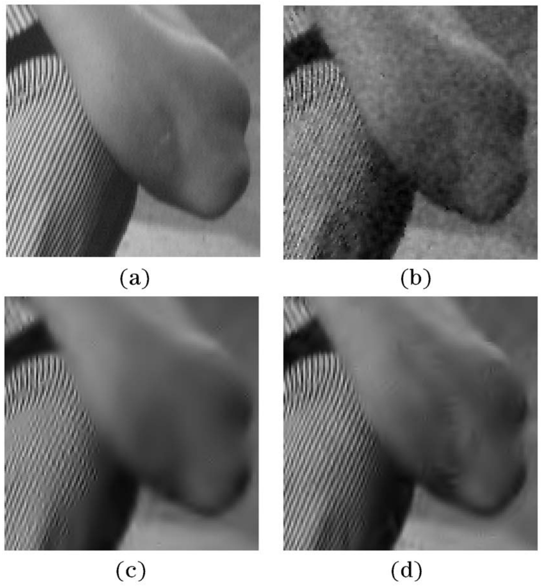


Fig. 2. Comparison of denoising results on Barbara (local 128×128, $\sigma = 20$). (a) Original image; (b) wiener2; (c) hard threshold; (d) our method.

The PSNR results of different methods are presented in Table 1. For each noise level, our method outperforms the other filtering methods of the state of arts. Compared with the best result of LAWMAP, the PSNR values are improved about 1 dB. With the adaptability, our method is much superior to the hard threshold method for Barbara which contains much edge information. Figure 2 provides a visual comparison of Barbara denoised using the hard threshold method and our method, and the presented result is the local view of the image. Our method preserves the features of the original image better than the hard threshold method, and the resulted image of the later method is over-smoothed than ours. In the other hand, our method produces ring effect that is mainly resulted from the noise pixels retained as the edge pixels. So we need make the edge detection method more accurate.

A new adaptive denoising algorithm based on wavelet is described in this paper. Our denosing procedure consists of three steps. Firstly, a canny-like edge detector identifies all the candidate edge pixels. Next, the refinement of edge information is operated in the constraint of the edge property in wavelet domain. Finally, the edge information is incorporated to local window filter to remove the noise better.

The experimental results show that the method achieves much better performance than some other denoising techniques, both quantitatively and qualitatively, and it is adaptive to the local image details.

Since many noise pixels are still wrongly labeled as edges, the edge detection method still needs to be improved. Although the denoising technique avoids the blurring of image, it produces some ring effects that result in many fake edges. Avoiding this effect is the next effort to be conducted.

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