Modulation instability of broad optical beams in nonlinear media with general nonlinearity

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The modulation instability of quasi-plane-wave optical beams is investigated in the frame of generalized Schrödinger equation with the nonlinear term of a general form. General expressions are derived for the dispersion relation, the critical transverse spatial frequency, as well as the instability growth rate. The analysis generalizes the known results reported previously. A detailed discussion on the modulation instability in biased centrosymmetric photorefractive media is also given.

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During the past decade or so, optical spatial solitons have been the focus of considerable attention[1−3]. Much progress has been made in the recent research, including the discoveries of new nonlinear materials that can support spatial solitons[4−11]. When the nonlinear refractive index deviates from Kerr nonlinearity, it seems very interesting to employ a more general form of the intensity-dependent nonlinearity and thus the propagation of the solitary beam should obey the generalized nonlinear Shrödinger equation (GNLSE), which has been considered in some papers for analyzing the properties of spatial bright, dark, and grey solitons[12,13]. On the other hand, soliton occurrence is always in connection with the modulation instability (MI), which is a universal process that is inherent to most nonlinear wave systems in nature[14−18]. Owing to MI, small amplitude and phase perturbations in space tend to grow exponentially and a broad optical beam disintegrates to filament. It usually occurs in the same parameter region where soliton occurrence is observed, and thus is considered as a precursor of soliton formation. To date, theoretical and experimental researches on MI have been presented for Kerr media[14] and biased photorefractive crystals[15−18]. However, MI in biased centrosymmetric photorefractive media[19] still remains an open problem. In this letter, a linear stability analysis is presented to investigate the MI for the GNLSE. General formulas for both the dispersive relation and the instability gain are derived. Finally, we use the general results to discuss the propagation of a broad beam in a biased centrosymmetric photorefractive medium under the steady-state condition and give a detailed analysis of MI in this kind of media.

As is well known, the propagation of (1+1)-dimensional quasi-plane-wave optical beams in a nonlinear medium can be described by the GNLSE under the condition of slowly varing envelope approximation[12,13], namely

$$i\frac{\partial u(x,z)}{\partial z}+\frac{1}{2k}\frac{\partial^2 u(x,z)}{\partial x^2}+f\left(|u(x,z)|^2\right)u(x,z)=0, \quad (1)$$

where $u(x,z)$ is the slowly varying complex electric field amplitude, $k=k_0n_\mathrm{b}$, k_0 is the wave number in the vacuum, n_b is the linear refractive index of the medium, the real function $f(|u|^2)=k_0\Delta n(|u|^2)$, and Δn is the photo-induced index perturbation in a general form.

For a broad plane beam, the envelope u is expected to remain a constant over a large range of x. Therefore, Eq. (1) has an exact solution of the form

$$u=r^{1/2}\exp(if(r)z), \quad (2)$$

where the positive quantity r represents the dimensionless optical beam intensity. To further investigate the stability of this solution, we look for a solution of Eq. (1) in the form of small-amplitude perturbation of the background, e.g.,

$$u(x,z)=(r^{1/2}+\sigma(x,z))\exp(if(r)z), \quad (3)$$

where $\sigma(x,z)$ represents the weak complex perturbation and $|\sigma(x,z)|^2\ll r$. Substituting Eq. (3) into Eq. (1) and linearizing in σ, we obtain

$$i\frac{\partial\sigma}{\partial z}+\frac{1}{2k}\frac{\partial^2\sigma}{\partial x^2}+rf'(r)\left(\sigma+\sigma^*\right)=0, \quad (4)$$

where $f'(r)\equiv \mathrm{d}f/\mathrm{d}x|_{x=r}$. Usually, Eq. (4) has the following perturbation solution[20]

$$\sigma(x,z)=c\cos(\Gamma z-\Omega x)+id\sin(\Gamma z-\Omega x), \quad (5)$$

where Γ and Ω are the wave number and spatial frequency of the perturbation wave, respectively. Inserting Eq. (5) into Eq. (4), and then we obtain the real and imaginary part equations as

$$\Gamma c+\frac{1}{2k}\Omega^2 d=0, \quad (6a)$$

$$\left(2rf'(r)-\frac{\Omega^2}{2k}\right)c-\Gamma d=0. \quad (6b)$$

The nontrivial solutions of Eq. (6) exist only when the dispersion relation is

$$\Gamma^2=-\Omega^2\left(\frac{rf'(r)}{k}-\frac{\Omega^2}{4k^2}\right). \quad (7)$$

If Γ has an imaginary part, the weak perturbation σ grows exponentially during the propagation of the broad beam. Therefore, according to Eq. (7), an instability region only appears at $f'(r) > 0$ (i.e., this type of MI should occur in the self-focusing medium) and the critical transverse spatial frequency of the perturbation Ω_{crit} is $\Omega_{\mathrm{crit}} = [4rkf'(r)]^{1/2}$. For $\Omega < \Omega_{\mathrm{crit}}$, the initial small perturbation amplitude $|\sigma(x,z)|$ grows exponentially and the MI takes place. Contrarily, when the medium is of self-defocusing type, i.e., $f'(r) < 0$, the broad beam is modulationally stable. Therefore, spatially localized solutions with vanishing boundary conditions are possible only for the case when the plane wave solution is modulationally unstable, i.e., only for the focusing nonlinearity, while the dark- or grey-soliton solution with nonvanishing background can exist only for the case of the defocusing nonlinearity. When the MI develops, the instability growth rate is given by

$$g(\Omega) = \mathrm{Im}(\Gamma) = |\Omega| \left(\frac{rf'(r)}{k} - \frac{\Omega^2}{4k^2} \right)^{1/2}. \quad (8)$$

Obviously, the MI growth rate $g(\Omega)$ reaches the maximum when $\Omega = [2rkf'(r)]^{1/2}$ and it is given by

$$g_{\max} = rf'(r). \quad (9)$$

The exponential growth of small perturbation always leads to periodical filaments. This should be expected in self-focusing medium. It can be physically explained in this way: when a broad light beam carrying small noise propagates in a self-focusing medium, those regions with slightly higher intensity have slightly higher refractive indices. During the beam propagation, the higher index regions attract more light energy nearby and yield even higher indices that attract more light energy farther. If this self-focusing effect is stronger than the diffraction effect, the light begins to localize. This more localized light then causes the diffraction effect to grow. When the diffraction effect finally balances the localization effect, the MI patterns are formed.

The result given by Eqs. (7)—(9) generalizes the known results[15,17] for MI in biased photorefractive crystals with and without photovoltaic effects. In the following, we will use the general results, namely Eqs. (7)—(9), to give a detailed discussion on the MI in a biased centrosymmetric photorefractive medium. In this case, we have[6,19]

$$f\left(|u^2|\right) = k_0 \Delta n_0 \frac{(1+\rho)^2}{\left(1+|u|^2\right)^2}, \quad (10)$$

where $\Delta n_0 = n_{\mathrm{b}}^3 g_{\mathrm{eff}} \varepsilon_0^2 (\varepsilon_{\mathrm{r}} - 1)^2 E_{\mathrm{ext}}^2/2$ represents the change in the refractive index driven by the dc Kerr effect in the absence of light at a uniform external field E_{ext}, ρ is the dimensionless ratio of optical beam intensity to dark irradiance far from the center, g_{eff} is the effective quadratic electro-optic coefficient, ε_0 and ε_{r} are the vacuum dielectric coefficient and relative dielectric coefficient, respectively. When the quasi-plane-wave optical beam has the form of Eq. (2), we have $\rho = r$. Substituting Eq. (10) into Eq. (7), the dispersion relation of MI in this centrosymmetric photorefractive medium is obtained as

$$\Gamma^2 = -\Omega^2 \left(-\frac{2r\Delta n_0}{(1+r)\,n_{\mathrm{b}}} - \frac{\Omega^2}{4k^2} \right), \quad (11)$$

and the MI growth rate is

$$g(\Omega) = |\Omega| \sqrt{-\frac{2r\Delta n_0}{(1+r)\,n_{\mathrm{b}}} - \frac{\Omega^2}{4k^2}}. \quad (12)$$

The above equation clearly shows that the MI gain is possible when $\Delta n_0 < 0$ (i.e., $g_{\mathrm{eff}} < 0$). As an example, we take the centrosymmetric photorefractive medium to be potassium lithium tantalate niobate (KLTN). The typical parameters[6] are $\lambda = 500$ nm, $n_0 = 2.2$, $\varepsilon_{\mathrm{r}} = 4000$, and $\Delta n_0 = -5 \times 10^{-4}$ when $E_{\mathrm{ext}} = 2$ kV/cm. Thus, we can plot the MI gain $g(\Omega)$ as a function of Ω/k for several values of r, as shown in Fig. 1. Obviously, the critical transverse spatial frequency $\Omega_{\mathrm{crit}} = k\left(-8r\Delta n_0/[n_{\mathrm{b}}(1+r)]\right)^{1/2}$ increases with the intensity r monotonously. When $\Omega < \Omega_{\mathrm{crit}}$, MI will happen. The maximum MI gain $g_{\max} = -2rk\Delta n_0/(1+r)$ as well as the corresponding spatial-frequency also increases with the light intensity r.

From Eq. (12), the MI growth rate also depends on Δn_0 and thus on the external applied electric field E_{ext}. Figure 2 shows the MI gain $g(\Omega)$ as a function of Ω/k for four different values of E_{ext} when $r = 1$ and for the same other system parameters given above. Obviously, the maximum MI gain $g_{\max}$ increases linearly with E_{ext}^2 and its

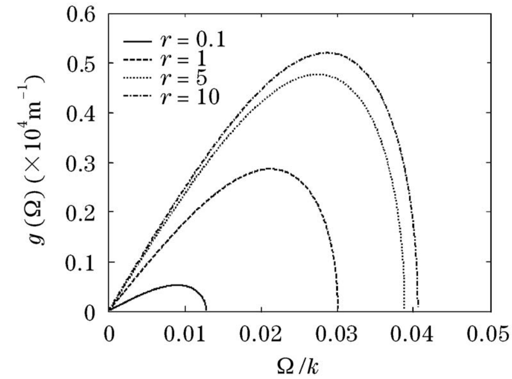

Fig. 1. MI gain as a function of Ω/k for $r = 0.1$ (solid line), 1 (dashed line), 5 (dotted line), and 10 (dashed-dotted line) in a biased KLTN at $E_{\mathrm{ext}} = 2$ kV/cm.

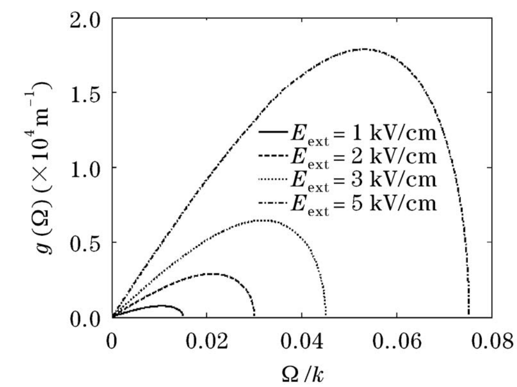

Fig. 2. MI gain as a function of Ω/k for $E_{\mathrm{ext}} = 1$ kV/cm (solid line), 2 kV/cm (dashed line), 3 kV/cm (dotted line), and 5 kV/cm (dashed-dotted line) in a biased KLTN at $r = 1$.

associated spatial frequency increases linearly with E_{ext}. This dependence allows one to externally control the spatial period of the generated filaments.

In conclusion, we have shown the modulation instability of broad optical beams within the frame of the nonlinear Schrödinger equation with a nonlinear refractive index change of a general type. General expressions are derived for the dispersion relation, the critical transverse spatial frequency as well as the MI growth rate. It is found that MI region appears only in the self-focusing medium. When the nonlinear medium is of self-defocusing type, the modulation is stable. No MI is observed and thus dark- or gray-soliton solution with nonvanishing background is possible to form. We make a detailed discussion on the MI in a biased centrosymmetric photorefractive medium. In this type of media, the MI growth rate increases monotonously with the external bias field and the normalized optical beam intensity. It is worthy of mention that MI is not restricted to nonlinear optics and therefore the results obtained in this letter are also very useful in other nonlinear physics such as fluid dynamics, plasma physics and protein chemistry, etc..

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